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(please email me if you would like a link to the resources in this presentation)

Increasing AP Exam Free Response Scores!

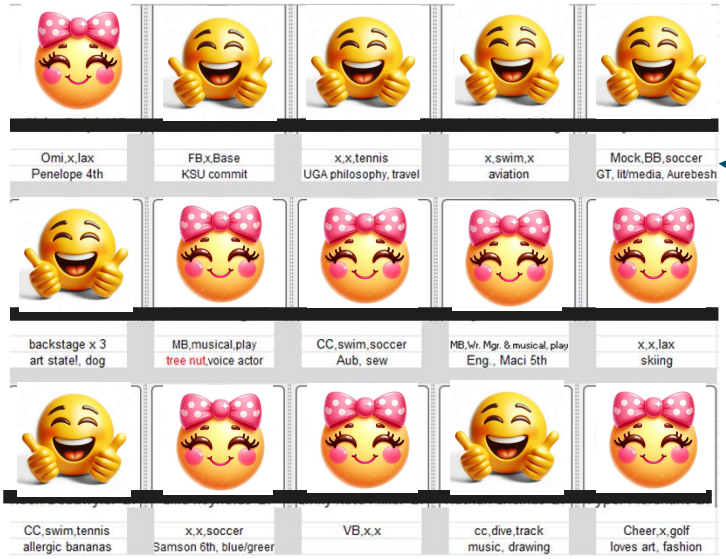
My students perform very well on the free response questions on the AP exam! I will share some factors as to why…maybe this will help your students.

	FR-my students	FR-global	% above
2025	**40.2**	**23.8**	69
2024	**37.2**	**23.1**	61
2023	**32.3**	**20.4**	58
2022	**33.3**	**19.6**	70
***2021**	**20.5**	**13.9**	47
2019	**26.1**	**17.1**	53



1. Build a strong rapport early and often.
   a) Establish high expectations, a high level of support, a nonjudgemental atmosphere.
   b) Demonstrate genuine interest in who they are beyond the classroom.
2. Know the historical categories of the AP Exam Free Response questions.
3. Understand the grading standards of the AP Exam Free Response questions.
4. Consistently teach and model a “sufficient but efficient” approach to questions.
5. Teach free response standards as content is covered.
6. Incorporate perpetual reinforcement.
   a) Cumulative tests with free response style partial questions.
   b) Cumulative out of class graded work (quizzes or problem sets).
7. Purpose to finish the course material four weeks before the exam to review.
8. Have specific days to review and practice each free response question type.
9. Give a comprehensive free response test.
   a) 1-2 weeks before the exam over two class periods
   b) Covers the foundational concepts of the free response questions
   c) Counts as two test scores
   d) Motivation for students to finish well

1. ***Build a strong rapport early and often***
   a) ***Establish high expectations, a high level of support, a nonjudgemental atmosphere***
   b) ***Demonstrate genuine interest in who they are beyond the classroom***



Student headshot

Student names

Student activities/info

Communicate to them how hard you are working to make them successful!

2 & 3. Know the historical categories of the AP Exam Free Response questions as well as the most common sub-questions that are asked. Understand the grading standards of the AP Exam Free Response questions.

1. Data Table
2. Graph Analysis
3. Differential Equations
4. Implicit Equations
5. Particle Motion
6. Area/Volume
7. Rate In – Rate Out
8. Der. & Int. basics



Resources I use and will share with you if you email me!

THE DATA TABLE
Calculator or NO calculator

Given a table of data (usually with time and either a rate or an amount function), on a closed interval with an initial amount given (typically at t=0), be able to:

a) Estimate the derivative of the given function (by calculating slope using the closest two points and the slope formula, (change in function) divided by (change in time). If you are asked to explain the meaning of your answer: "At t=___ minutes, the (rewrite what the function stands for from the given information) is (increasing or decreasing…depending on whether your estimate is positive or negative) at a rate of (your estimate) and add the correct units.
b) Use estimation technique (left, right, midpoint, trapezoid) to estimate the total amount. Remember from a data table you need to calculate change in x for each interval. If you are using midpoint method, you will use every other x-value for the bases and the y-value of the point that you skip will be the height of the rectangles.
c) Know how to tell if each technique is an over or underestimate (left and right by increasing/decreasing; trapezoid and midpoint by concavity.)
d) Explain the meaning of the integral of the function in the context of the problem. (Change in amount, final amount, or average value.)
e) Given an equation of another function that models the function given in the table or a different function (working against the function in the table), find an amount or a rate or an average value.
f) Use the Intermediate Value Theorem to prove that the given function takes on a certain y-value. Ensure that you state that the function is continuous (typically because differentiable) and why. Find the y-values of the endpoints. Show that the certain y-value is between the y-values of the endpoints.
g) Use the Mean Value Theorem to prove that the derivative of the function takes on a certain value. Ensure that you state that the function is differentiable and therefore continuous. Find the average rate of change between the endpoints of the interval (using slope formula).

26 #1, 25 #3, 24 #1, 23 #1, 22 #4, 21 #1, 19 #2, 18 #4, 17 #1, 16 #1, 15 #3, 14 #4, 13 #3, 12 #1

2026 #1 Calculator

a.

$M'(7.5) \approx \frac{M(10)-M(5)}{10-5} = \frac{16-7}{10-5} = \frac{9}{5}$ birds per day per day — Answer with setup — **Point 1 (P1)**; Units — **Point 2 (P2)**

b.

i. $\int_0^{30} M(t)\,dt \approx M(5)\cdot(10-0) + M(15)\cdot(20-10) + M(25)\cdot(30-20)$ — Form of midpoint Riemann sum — **Point 3 (P3)**

$= 7\cdot 10 + 6\cdot 10 + 2\cdot 10 = 150$ — Approximation with supporting work — **Point 4 (P4)**

ii. $\int_0^{30} M(t)\,dt$ represents the number of male birds of this species that arrive at the nesting area from $t = 0$ days to $t = 30$ days. — Interpretation — **Point 5 (P5)**

c.

$\int_{15}^{45} F(t)\,dt$ — Integral — **Point 6 (P6)**

$= 641.859164$ — Answer — **Point 7 (P7)**

4. Consistently teach and model a "sufficient but efficient" approach to questions

Everything you write on your boards as you teach should be modeling to the students your expectations. This should also be a sufficient but efficient approach.

Emphasize:

- Correct mathematical notation and communication
- Conceptual understanding versus memorization
- At some point allow for non-simplification of computation



5. Teach free response standard as content is covered.

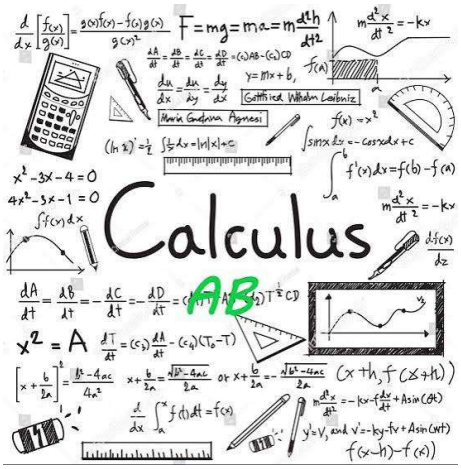
Every time a free response concept is taught in the flow of the curriculum:

- Teach the concept to the scoring standard
- Incorporate partial free response questions to the lesson and in all following assessments
- Ensure the concept is taught with and without a calculator

This can start on the first day with 1.1 and data tables!

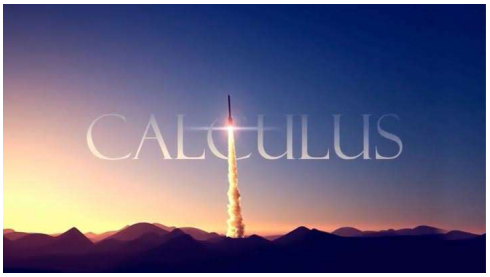
Units 1-5 opportunities to incorporate free response concepts:

- 1.1 Estimating & Interpreting Instantaneous Rate of Change – Data Table
- 1.16 IVT – Data Table
- 3.1 Derivative rules from generic functions
- 3.2 Implicit Differentiation
- 4.2 Particle Motion
- 4.3 Rate in – rate out
- 4.4 Related rates from old FR questions
- 4.6 Tangent line approximation - Implicit
- 4.7 L'Hospital's Rule
- 5.1 MVT – Data Table
- 5.4 1st Derivative Test – Graph Analysis
- 5.5 Candidate's Test
- 5.6 Concavity – Graph Analysis
- 5.7 2nd Derivative Test – Graph Analysis and Implicit
- 5.9 Connecting 1st and 2nd Derivatives – Graph Analysis
- 5.12 Implicit



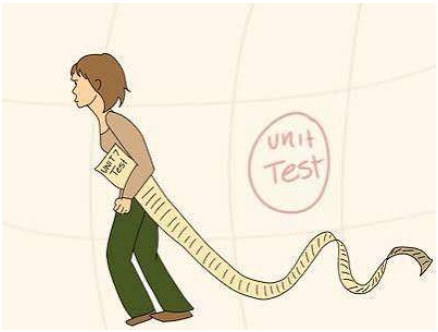
Units 6-8 opportunities to incorporate free response concepts:

- 6.2 Riemann Sums – Data Tables
- 6.7 FTC – Graph Analysis
- 7.1 Differential Equations
- 7.4 Reasoning Using Slope Fields – Differential Equations
- 7.7 Finding Particular Solutions – Differential Equations
- 8.1 Average Value – Particle Motions and Data Table and Rate In/Out
- 8.2 Integration - Particle Motion
- 8.3 Integration in Applied Contexts – Rate In/Out
- 8.4-5 Area between curves
- 8.7-8.8 Volume by Cross-Section
- 8.9-8.12 Volume by Revolution



6. Incorporate perpetual reinforcement

a) Cumulative tests
b) Cumulative out of class graded work



Most all my unit tests are *cumulative*!

How I do quizzes:

- take-home (generally a week outside of class)
- cumulative in content
- open-note
- with a partner
- great training ground for AP exam practice

Consider “problem sets” in lieu of quizzes

7. Purpose to finish the course material to leave four weeks before the exam to review
8. Have specific days to review and practice each free response question type

April 1 – May 10

2027 AP Calculus Review

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
28	29 **No School (Easter Break)**	30 11 Review (6)	31 Unit 11 test (5)	1 Unit 11 test (6)	2 Free Response #1 (5-6)	3
4	5 Free Response #2a (5-6)	6	7 Free Response #2b (5-6)	8 Free Response #3 (5-6)	9 Free Response #4 (5)	10
11	12 Free Response #4 (6)	13 Free Response #5 (5-6)	14 Free Response #6 (5-6) MC Q#1 due (3-5)	15	16 Multiple Choice practice (5-6)	17
18	19 No School (b/c academics are such a small part of school)	20 Multiple Choice practice (5-6)	21 Review Test Day 1 (5)	22 Review Test Day 1 (6)	23 Review Test Day 2 (5-6)	24
25	26 Discuss FR and MC practice (5-6)	27	28 In class FR group practice	29 Sr. Honors Day MC Q#2 due (3-5) MC Practice	30 Sr. Chapel	1
2	3 Practice Exam or Review 8:00-3:00	4 Practice Exam or Review 8:00-3:00	5 Practice Exam or Review 9:00-3:00	6 Practice Exam or Review 8:00-3:00	7 Practice Exam or Review 8:00-3:00	8 Optional Practice Exam or review 8:30-12:00

Review Test counts as TWO test scores.
Practice Exam is optional but will replace lowest test score but not the Review Test.
****AP Calculus Exam is on Monday, May 10th, in the morning.***

9. *Give a comprehensive free response test*

a) 1-2 weeks before the exam
b) Covers the foundational concepts of the free response questions
c) Takes two class periods
d) Counts as two test scores
e) Helps students finish well



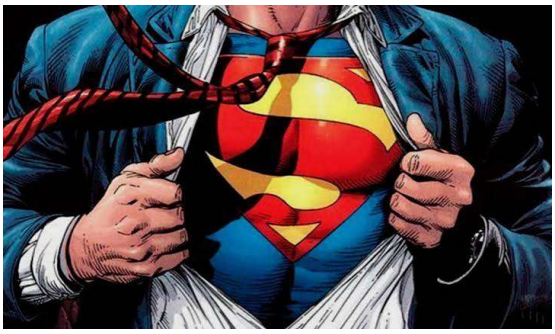
My end of course free response review test is in the resource folder.

t (minutes)	4	8	11	15	20	26
R(t) (gal/min)	11	12	15	20	30	45

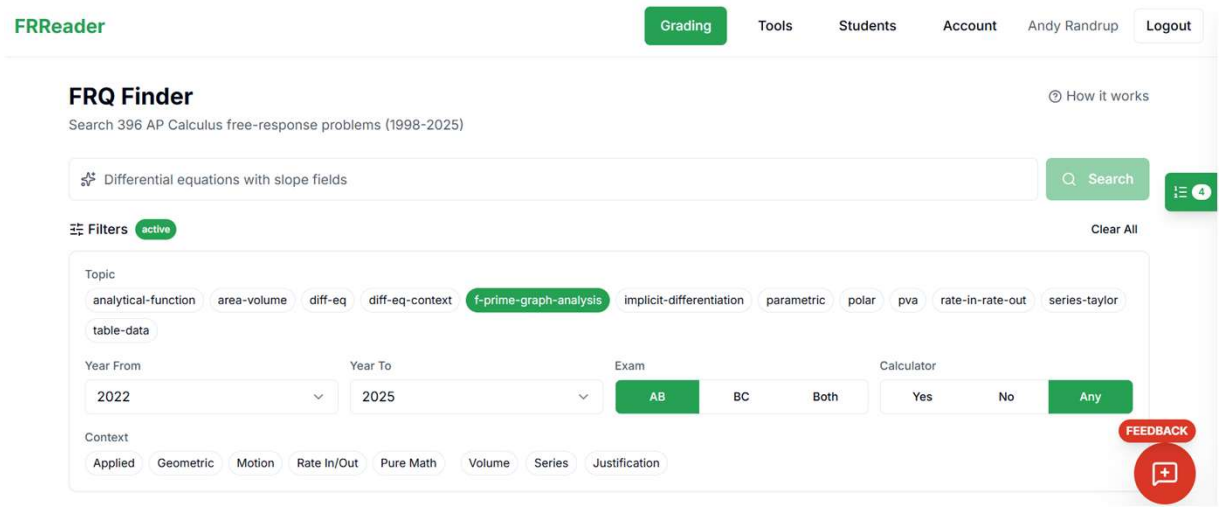
Water is being released out of a lake through a dam, in gallons per minute, according to the increasing, concave up, differentiable function *R(t)* for $t \geq 0$. At $t = 4\ minutes$, there are 45,000 gallons of water in the lake.

a) Estimate $R'(12)$. Using correct units, interpret the meaning of your answer in the context of the problem.
b) Estimate $R'(15)$. Using correct units, interpret the meaning of your answer in the context of the problem.
c) Must there be a time $4 \leq t \leq 26$ such that $R(t) = 26$? Give a reason for your answer.
d) Must there be a time $4 \leq t \leq 26$ such that $R'(t) = \frac{17}{11}$? Give a reason for your answer.
e) If ***A(t)*** represents the number of gallons of water in the lake, write an equation of a tangent line to ***A(t)*** at $t = 4\ minutes$.
f) If ***A(t)*** represents the number of gallons of water in the lake, evaluate $\lim_{t \to 4} \frac{0.2A(t) - 9000}{16 - t^2}$.
g) Use a Right Reimann sum with five sub-intervals to estimate $\int_4^{26} R(t)dt$.
    1. Using correct units, interpret the meaning of your answer in the context of the problem.
    2. Is your estimate greater or less than the actual value of the integral? Give a reason for your answer.
h) Use four Trapezoids to estimate $45{,}000 - \int_4^{20} R(t)dt$.
    1. Using correct units, interpret the meaning of your answer in the context of the problem.
    2. Is your estimate for $\int_4^{20} R(t)dt$ an over or under estimate? Give a reason for your answer.
i) Use a Left Reimann sum with three sub-intervals to estimate $\frac{1}{15}\int_{11}^{26} R(t)dt$.
    1. Using correct units, interpret the meaning of your answer in the context of the problem.
    2. Is your answer an over or under estimate? Give a reason for your answer.
j) If a rainstorm is adding water to the lake at a rate modeled by the function $S(t) = 5e^{0.11t}$ gallons per minute, using the value found in part ***g***, estimate the total number of gallons of water in the lake at $t = 26\ minutes$ to the nearest gallon.
k) If the actual rate in gallons per minute the water is being released from the lake through the dam is $D(t) = .065t^2 - .37t + 11$, how many gallons of water are in the lake at $t = 26\ minutes$ (with no rainstorm)?
l) Using the ***D(t)*** and ***S(t)*** functions above, how many gallons of water are in the lake at

Super FRQs!



Also in the resource folder.



https://frreader.com/frq-finder