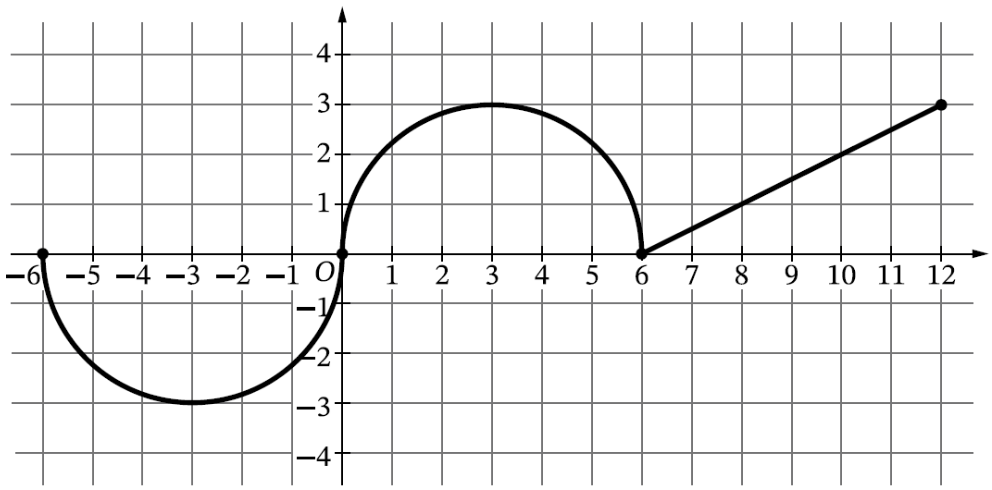
Problem Overview:

The function f is continuous on the closed interval $-6 \le x \le 12$. The function g is given by $g(x) = \int_6^x f(t)\,dt$.

A graph of f, consisting of two semi-circles and a line segment is shown below.



Graph of f

Part a:

Students were asked to find $g'(8)$ and give a reason for the answer.

Part b:

Students had to find all values of x in the open interval $-6 < x < 12$ at which the graph of g has a point of inflection. Once again, a reason had to be provided.

Part c:

Students had to find $g(12)$ and $g(0)$.

Part d:

Students had to find the value of x on the closed interval $-6 \le x \le 12$ at which g attains an absolute minimum and justify the answer.

Comments on student responses and scoring guidelines:

Part a worth 2 points

P1 was earned for considering $g'(x) = f(x)$, which could be expressed in ways such as $g' = f$ or $g'(8) = f(8)$. **P2** was earned for the answer. Some students used a version of an area argument. Note that the area of the triangle bounded above by the graph of f, below by the *x*-axis, and on the right by $x = 8$ is 1, which is the answer. If it was clear that this approach was being used, **P2** was not earned.

In what may have come from an integral of some type, readers saw $f(8) - f(6) = 1 - 0$. This only works because $f(6) = 0$. The mere response $f(8) = 1$ was also seen and was considered an implied connection to g'. Responses such as these, arriving at the correct answer, earned **P2** but not **P1**. A more interesting approach was occasionally shown by replacing $f(t)$ in $g(x) = \int_6^x f(t)\,dt$ with an expression for the line segment for $x \geq 6$. Such an approach was usually successful. Some responses interpreted the given graph as the graph of g and incorrectly gave the slope of the line segment as the value requested.

Part b worth 2 points

P3 was earned only for the correct answer: $x = -3$, $x = 3$, and $x = 6$. Consideration of the endpoints of the interval did not impact scoring. To earn **P4** a reason tied to the graph of f had to be provided. Thus "$g'' = f'$ changes sign there" did not earn **P4** since this does not refer to the graph of f. "The slope of f changes sign there" earned **P4**; however "the slope of f changes from negative to positive there" did not earn the point since this is not what happens at $x = 3$.

A number of students presented just two of the correct values of x (and no others) and remained eligible for **P4**, earning the point if the reasoning for those two values was correct.

Part c worth 2 points

P5 was earned for $g(12) = 9$ and **P6** was earned for $g(0) = -\frac{9\pi}{2}$, with or without supporting work. Intervening work was treated as scratch work. Unlabeled answers did not earn either of these two points.

Part d: worth 3 points

P7 was earned for consideration of $g' = 0$ and could be shown in several ways such as:

- $g' = 0$
- $f = 0$
- Using the phrase "the critical points of $g(x)$ …."
- Considering or mentioning a sign change of either g' or f

P8 was earned for justifying the answer, which amounted to comparing the values of $g(-6)$, $g(0)$, $g(6)$ and $g(12)$. A response comparing to any other values of g did not earn **P8**. If $g(6)$ is not considered, it must be explained why that value could not be a minimum, as in "g' does not change sign at $x=6$" or "g' does not change from negative to positive at $x=6$." Many students did not earn **P8** despite earning one or more of the other two points in this part of the question. Attempts to discuss the change in sign of g' at $x=0$ very rarely earned this point. Most such arguments did not completely discuss the intervals over which g' assumed positive and negative values.

P9 was earned for the answer, $x=0$. A number of students did not read the question carefully and gave the answer as $g(0)$ rather than the value of x at which this minimum was attained. Also, this answer had to be in the presence of some supporting work, whether or not totally correct for **P8**. "$x=0$" alone did not earn **P9**.

Observations and recommendations for teachers:

(1) Defining a function as an integral of another is a common setup on the AP exam. The first thing to do is write the derivative of the defined function using the Fundamental Theorem. This provides valuable information for a variety of possible questions. In the case of part a, this tells us that the given graph shows values of g'.

(2) When the graph of $f(x)$, the derivative of a function $g(x)$, is presented, students often have difficulty determining locations of points of inflection on the (unknown) graph of $g(x)$ itself. Points of inflection occur where concavity changes, yes. But students are expected to use the given graph in explaining why concavity is changing. For example, "$g''=f'$ changes sign" is not referring to the graph of $f(x)$ and would not earn **P4**. Reference had to be made to the given graph of $f(x)$ (not merely "the graph"). And so, locations of points of inflection need to be justified by using wording to describe g', with language mentioning the given graph of f. Such phrasings that appropriately describe g' in terms of f are "the slopes of f change sign" or "f changes from increasing to decreasing or vice versa."

(3) In order to find the values of $g(12)$ and $g(0)$, the basic skill needed was the evaluation of $\int_6^x f(t)\,dt$ for appropriate values of x. In the context of this question, students needed to know how this integral relates to areas of regions between the graph of the curve and the *x*-axis. Most responses showed appropriate work with areas, although work did not have to be shown. With two answers in this part of the question, it was very important to label the answers. No points were earned for unlabeled answers. Students had more difficulty finding $g(0)=\int_6^0 f(t)\,dt$, mostly because of failure to negate the values of any areas found. If the lower limit in the definite integral is less than the upper limit, the integral can be rewritten since $\int_6^0 f(t)\,dt=-\int_0^6 f(t)\,dt$.

(4) Part d asks for the location of the absolute minimum of the function $g(x)$ on the closed interval $-6 \le x \le 12$. The Extreme Value Theorem guarantees both an absolute max and min somewhere on a closed interval, because $g(x)$ is continuous. First, the values at the endpoints should be determined. Any other *possible* absolute min (since $g(x)$ is differentiable) must occur where $g'(x) = 0$. Showing an attempt at finding such values of x is important in a complete approach to answering this part of question. Finally, comparing values of $g(x)$ at these values of x will reveal the location(s) of the absolute minimum of $g(x)$.

(5) Searching for absolute extrema of a function on a closed interval is straightforward (especially if the function is differentiable). The possible locations are endpoints and where the derivative is 0. Comparing the values of the function at these points is sometimes referred to as the "candidates' test." Trying to make local arguments like the first or second derivative tests into global arguments is very difficult, if possible, and should be avoided in favor of the candidates' test.