

2 **Problem Overview**

3 Students were told that the function $C(t) = 7.6 \arctan(0.2t)$ models the amount of land (in acres) that is
4 invaded by a species of plant. Students were given that the derivative of C is $C'(t) = 38/(25 + t^2)$.

5 **Part a**

6 Students were asked to find the average number of acres from $t = 0$ to $t = 4$ and to show the setup for
7 their calculations.

8 **Part b**

9 Students were asked to find the time at which the instantaneous rate of change of C is the same as the
10 average rate of change of C , and to show the setup for their calculations.

11 **Part c**

12 Students were asked to write and evaluate an expression that describes the end behavior of the rate of
13 change of the number of acres invaded by the plant.

14 **Part d**

15 Students were told that after time $t = 4$, the function

16
$$A(t) = C(t) - \int_4^t 0.1 \ln(x) dx$$

17 describes the number of acres invaded by the plant. Students were asked to find the maximum value of A
18 on the interval $4 \leq t \leq 36$ and to justify their answer.

19 **Comments on Student Responses and Scoring Guidelines**20 **Overall Comments**

21 If the student “misspelled” C as f , C' as f' , or t as x , Readers assumed the correct spelling for parts
22 (a), (b), and (c). However, in part (d), there were two functions and two derivatives, so any misspelling of
23 function names resulted in points not being earned.

24 The problem gave the student a note at the beginning reminding the student that the calculator should be
25 in radians. Therefore, answers in degrees did not earn any points.

26 **Part a**

27 The first part could earn the student P1 and P2. The average number of acres is the average value of $C(t)$,
28 which is

29
$$\frac{1}{4} \int_0^4 C(t) dt = 2.778.$$

30 P1 was earned for presenting a definite integral of $C(t)$ along with evidence of division by 4. The division
31 by 4 need not be attached to the integral. P2 was earned for the correct answer. Thus, if the student
32 computed $\int_0^4 C(t) dt = 11.112896$ correctly and never divided by 4, neither P1 nor P2 were earned.
33 Likewise, if the student used $C'(t)$ instead of $C(t)$, neither P1 nor P2 were earned.

34 **Part b**

35 This part of the question could earn the student points P3 and P4. The minimal solution earning both points
36 is

37
$$\frac{C(4) - C(0)}{4 - 0} = C'(t) \Rightarrow t = 2.154.$$

38 P3 was earned for presenting an appropriate average rate of change. This could be presented in many ways
39 since $C(0) = 0$, such as

40
$$\frac{C(4)}{4}, \quad \frac{\int_0^4 C'(t) dt}{4}, \quad \frac{5.128 - 0}{4 - 0}, \quad \text{or} \quad \frac{5.128}{4}.$$

41 Simply presenting the value of the average rate of change, 1.282, without it being attached to a quotient
42 did not earn P3. To earn P4, the student must present an appropriate equation from which the solution is
43 $t = 2.154$.

44 **Part c**

45 Both P5 and P6 were available in this part. The minimal solution earning both points is

46
$$\lim_{t \rightarrow \infty} C'(t) = 0.$$

47 P5 was earned for writing the limit expression, and P6 was earned for the value 0. Some allowance was
48 made for misinterpreting the question as asking for the number of acres instead of the rate of change of the
49 number of acres: if the response presented $C(t)$ in the argument of the limit instead of $C'(t)$, P5 was still
50 earned, but the response was not eligible for P6. Any work between the limit expression and the value was
51 not read, including work for a mis-application of L'Hôpital's rule.

52 **Part d**

53 In the last part, points P7, P8, and P9 were available. The minimal response which earned all three points
54 is the following.

t	$A(t)$
4	5.128
11.441	7.316, maximum
36	1.743

$$A'(t) = 0$$

$$t = 11.4417$$

Students could earn P7 by writing any equation equivalent to $A'(t) = 0$, such as $C'(t) - 0.1 \ln(t) = 0$ or $C'(t) = 0.1 \ln(t)$. P7 could also be earned by writing that the sign of $A'(t)$ changes or using the phrase “critical points of $A(t)$.” A response of “ $A(t)$ changes from increasing to decreasing” did not earn P7 as this was considered a *precalculus* reason.

To earn P8, the student must make a global argument for the absolute maximum, ideally by using a candidate’s test. Only the first place after the decimal point was read for these function values in the candidate’s test. The table of function values for each candidate was allowed to have one incorrect value. If more than one value was incorrect, the response did not earn P8. If a local argument was used (first or second derivative tests), P8 was not earned, but was eligible for P9. If a local argument was used, the student could recover P8 if the response went on to say that this is the only critical on the interval, or if the endpoints were used in describing the intervals where $A'(t)$ is positive and negative. Sign charts were not read.

Only an answer of $t = 11.441$ or $t = 11.442$ with supporting work earned P9.

An example of a resolved local argument using the first derivative test that earned all three points is

Since $A' > 0$ for $4 < t < 11.442$ and $A' < 0$ for $11.442 < t < 36$,
and $t = 11.442$ is the only critical point, A has a maximum at $t = 11.442$.

An example of a resolved local argument using the second derivative test that earned all three points is

$A'(t) = 0$ for $t = 11.442$, $A''(11.442) < 0$, and $t = 11.442$
is the only critical point on $4 < t < 36$, A has a maximum at $t = 11.442$.

The response did not need to show an expression for $A''(t)$.

Observations and Recommendations for Teachers

(1) The vast majority of students did not earn P1 nor P2 because they either presented an integral with $C'(t)$ or they computed an average rate of change of C (or C'). Teachers should break the “key word” habits of students: students see the word “average” and do not consider what comes after that word. Indeed, some students found values of $C(1)$, $C(2)$, $C(3)$, and $C(4)$, added them, and divided by 4, clearly interpreting “average” as “mean.”

(2) In many responses to part (a), the student solved the equation $C(t) = C'(t)$, but this did not earn either point. Strangely, some students who computed an average rate of change of C in part (a) computed the average value of C in part (b). Again, students should know the meaning of “average value” and “average rate of change.”

(3) One exception was made to the three-place rounding rule. In part (b), many students correctly found the average rate of change of C , but wrote it as 1.28, rather than 1.282. The equation $C'(t) = 1.28$ then has solution $t = 2.16$. As only the final answer must be rounded to three places, Readers were allowed to read $t = 2.16$ as earning P4 if the intermediate value of 1.28 was presented and used in the equation.

(4) A great many responses in part (b) solved the equation $C'(t) = 1.282$ by hand. On a calculator active question, this is simply a waste of time. Students should be taught what work needs to be shown on the calculator active problems as well as the calculator prohibited problems.

(5) Notational errors in part (c) were the main culprit in students not earning points P5 and P6. Notational travesties such as

$$\lim_{\infty \rightarrow 0} C'(t), \quad \frac{C(t)}{t \rightarrow \infty}, \quad \lim_{t \rightarrow \infty} = C'(t), \quad \text{and} \quad \lim_{C \rightarrow \infty} t$$

did not earn P5. Some students – BC students apparently – set up and evaluated an improper integral. However, if they correctly evaluated an improper integral of $C'(t)$, which is equivalent to writing $C(t)$ as the argument of the limit, the student earned P5.

(6) When it comes to P6, this Reader saw many students evaluate the limit of $C(t)$ as $t \rightarrow \infty$. This limit is $7.6 \cdot \pi / 2 = 11.938$. While this does not earn P6 in any way, it was disheartening to see so many responses round this to 12.

(7) Many responses for part (d) amounted to “ $t = 11.442$ is the maximum because that’s the highest point on the graph.” Again, even on a calculator active problem, justifications must be based on calculus, not calculators.

(8) In part (d), without specific reference to $A'(t)$ P7 was not earned. That is, “the derivative is zero at $t = 11.442$ ” does not earn P7 since we do not know which derivative is being referred to: A' or C' ? Some responses read like recipes and did not earn P7: “The derivative is zero or undefined at a maximum. There is a maximum at $t = 11.442$.” Students should remember to answer the question being asked, and to communicate effectively by using function names from the problem and appropriate calculus vocabulary.

(9) P8 was the most difficult point for students to earn because so few made a global argument for the maximum, and some of these included wildly incorrect function values in their candidate’s table. Such free-response questions asking the students to find a global extrema on an interval are quite common on the AP Exam; teachers should ensure that students be exposed to these kinds of questions and have opportunities to practice them.