2025 AB-6 Chuck Garner, Rockdale Magnet School

Problem Overview

Students were given the equation $y^3 - y^2 - y + \frac{1}{4}x^2 = 0$ and told that it defines a curve G.

Part a

Students were asked to show that $\frac{dy}{dx} = \frac{-x}{2(3y^2 - 2y - 1)}$.

Part b

Students were asked to use a tangent line to the curve at $(2, -1)$ to approximate the y-coordinate of the point P on the curve that has x-coordinate 1.6.

Part c

Students were told that there is a point S on the curve for $x > 0$ and $y > 0$ where the tangent line to the curve is vertical. Students were then asked to find the y-coordinate of S.

Part d

Students were given the equation $2xy + \ln y = 8$ that defines a curve H, and were told that a particle moves along it so that $dx/dt = 3$ at the time where the particle is at the point $(4, 1)$ on the curve. Students were then asked to find the value of dy/dt at this time.

Comments on Student Responses and Scoring Guidelines

Part a

The first part could earn the student P1 and P2. The correct differentiation of the equation is

$$3y^2\frac{dy}{dx} - 2y\frac{dy}{dx} - \frac{dy}{dx} + \frac{x}{2} = 0.$$

Factoring out dy/dx and subtracting $x/2$ from both sides gives

$$(3y^2 - 2y - 1)\frac{dy}{dx} = -\frac{x}{2}. \tag{1}$$

P1 is earned for correct differentiation. Students could use alternate notations (y', dy, dx) and still earn P1. Students who dropped the "equals 0" and presented a differentiated expression earned P1 if they subsequently recovered in solving for dy/dx.

To earn P2, students must have earned P1, and students must perform the algebra to verify the given expression for dy/dx. Students presenting only Equation 1 earned P2 provided there were no subsequent errors.

Part b

This part of the question could earn the student points P3 and P4. Presenting the correct slope of the tangent line, $-1/4$, earns P3, while using the tangent line for the approximation of y at $x = 1.6$ earns P4. The response is not required to declare $-1/4$ as the value of the slope to earn P3—the response can simply use it (correctly) in the equation of the tangent line. However, if an incorrect nonzero slope is declared as the value of dy/dx and used correctly in the tangent line, P3 is not earned, but P4 is earned. On the other hand, if an incorrect slope is *not* declared as the value of dy/dx, and used in the tangent line, neither P3 nor P4 are earned. Simplification of the tangent is not required; presenting $-1-\frac{1}{4}(1.6-2)$ or $-\frac{1}{4}(1.6-2)-1$ earns both P3 and P4. Indeed, presenting $-1-\frac{1}{4}(1.6-2)$ earns P4 regardless of any subsequent arithmetic errors. However, ambiguous responses, such as $-1-\frac{1}{4}(1.6-2$, require subsequent simplification to resolve the ambiguity before earning P4.

Part c

Both P5 and P6 were available in this part. Setting $2(3y^2-2y-1)$ equal to 0 earns P5. Any solution that sets both the numerator and denominator of dy/dx equal to 0 earns P5. Earning P5 and declaring an answer of $y = 1$ earns P6. A response that includes a solution of $y = -1/3$ must rule this solution out and declare $y = 1$ as the answer to earn P6.

Part d

In the last part, points P7, P8, and P9 were available. An attempt at the implicit differentiation of $2xy + \ln y = 8$ with respect to t (with at most one error) earns P7. The following are each considered "one error."

- Writing $\frac{dy}{dt}$ as $\frac{dy}{dx}$ with no recovery, but still uses $\frac{dx}{dt}$.
- Writing the derivative of $2xy$ as $2\frac{dx}{dt}\frac{dy}{dt}$.
- Writing the derivative of $\ln y$ as $\frac{1}{dy/dt}$.

Students could use alternate notations, such as x', y', dx, and dy, provided they connect these notations to dy/dt, or they show the substitution of 3 into their ambiguous presentation of dx/dt.

An equation mathematically equivalent to

$$2y\frac{dx}{dt} + 2x\frac{dy}{dt} + \frac{1}{y}\frac{dy}{dt} = 0$$

involving dx/dt and dy/dt earns P8.

Earning P7 and P8, with no errors in differentiation, and declaring the value $-2/3$ earns P9.

An alternate solution, starting from $dy/dt = (dy/dx)(dx/dt)$, can earn P7, P8, and P9:

$$2y + 2x\frac{dy}{dx} + \frac{1}{y}\frac{dy}{dx} = 0$$

and thus,

$$\frac{dy}{dx} = \frac{-2y}{2x + \dfrac{1}{y}}.$$

At the point (4, 1), we get $dy/dx = -2/9$, and therefore $dy/dt = (-2/9)(3) = -2/3$.

<u>Observations and Recommendations for Teachers</u>

(1) Communication is important. Using correct notations for the derivatives in this problem aids in knowing what to do. Some students were "lost in the weeds" with their ambiguous notation. Students in part (d) who wrote y' lost track of what they meant by that: was that dy/dx or dy/dt? Teachers should demand use of correct notation in the presence of many variables.

(2) In many responses, attempts to evaluate the derivatives did not earn points due to arthmetic errors: addition errors, subtraction errors, parentheses errors, and order of operations. Although simplification is not required, writing down an exact evlaution in correct notation is required. However, simplification of an incorrect expression that resolves to the correct answer can earn points. The take-away here is that teachers should demand simplification of expressions in their classes. This aids in writing down correct forms of answers and aids in using simpler numerical expressions to finish the problem or solve a later part.

(3) In solving the equation $3y^2 - 2y - 1 = 0$, some students wrote this as $y(3y - 2) = 1$ and attempted to solve. Others appeared to use a method called "slide and divide" which transforms the quadratic into another with the same roots. Unfortunately, this method done incorrectly looks like gibberish and does not yield the correct answer. Still others used the quadratic formula and generally obtained the correct answer. Solving quadratics is an expected skill on the AP Exam, and is used often. Students should have no problem in finding roots of a quadratic.

(4) In part (a), some students took the given derivative and treated it like a differential equation, solving for y. However, with an ambiguous constant of integration, this does not yield the given equation of the curve. This approach did not earn P1 nor P2.

(5) In part (d), the student writing only $dy/dt = (dy/dx)(dx/dt)$ did earn any of the three points. The student differentiating with respect to x—completely ignoring t in the response—also did not earn any of the three points. Again, notation matters!