<u>Problem Overview:</u>

Two particles, *H* and *J*, are moving along the *x*-axis. At time t the position of particle *H* and the velocity of particle *J* are given by $x_H(t) = e^{t^2-4t}$ and $v_J(t) = 2t\left(t^2-1\right)^3$ respectively.

Part a:

Showing work that leads to the answer, students had to find the velocity of particle *H* at time $t=1$.

Part b:

Students were asked to find, during the time interval $0<t<5$, the open intervals when particles *H* and *J* were moving in opposite directions.

Part c:

Students were told that $v_J(2)>0$. The question was whether the speed of particle *J* was increasing, decreasing or neither at time $t=2$ and to provide a reason for the answer.

Part d:

Students were given that at time $t=0$ particle *J* is at position $x=7$. Then, work had to be shown leading to the position of particle *J* at time $t=2$.

<u>Comments on student responses and scoring guidelines:</u>

Part a worth 2 points

The velocity can be computed at $t=1$ using the derivative, $x_H{}'(t)$. **P1** was earned for showing evidence of the consideration of $x_H{}'(t)$. This could be accomplished by showing the actual derivative as in $(2t-4)e^{t^2-4t}$. Simply showing something like $x_H{}'(1)$ or $x_H{}'$ also earned **P1**. This provided support for the answer, $-2e^{-3}$.

With the presentation of the answer, this would have earned **P2** as well as **P1**. An unsupported answer earned **P2** but not **P1**. The presentation of $2t-4e^{t^2-4t}$, omitting parentheses, earned **P1**. This earned **P2** only if resolved to the correct answer $-2e^{-3}$. A presentation of $(2t-4)e^{t^2-4t}$ earned **P2** (the point was "banked") even if a subsequent attempt at simplification contained an error.

Unfortunately, some students renamed the velocity function, usually as $H'(t)$. Another not uncommon error was omitting parentheses when writing the expression $(2t-4)e^{t^2-4t}$.

Part b: worth 3 points

Signs of $x_H{}'(t)$ and $v_J(t)$ needed to be compared in order to answer this question about the relative directions of the motions of the particles. The consideration of the sign of either these functions earned **P3**. If at least one of these functions was analyzed properly regarding directions of motion over the interval $0<t<5$, **P4** was earned. It was often the case that a number line analysis of signs of velocity was presented without communicating what the number line indicated and did not earn **P4**.

Earning **P5** required the most work. This involved analysis of the signs and directions of motions of both particles as well as the answer, that the particles are moving in opposite directions for $1<t<2$. The response had to have earned **P4** in order to earn **P5**, so not many students earned **P5**.

Part c: worth 1 point

In order to have earned **P6**, a response had to indicate that the speed of particle J is increasing at time $t=2$ and appeal appropriately to the fact that both $v_J(2)$ and $v_J{}'(2)$ have the same sign. The values of $v_J(2)$ and $v_J{}'(2)$ did not need to appear in the response, but if they were presented, they and any work toward simplification had to be correct. A response could either import the analysis of $v_J(2)$ from part b or restart. Some responses attempted to calculate $v_J{}'(2)$ even though the sign was given. Some of these made errors in calculating $v_J{}'(t)$.

Part d: worth 3 points

There was more than one approach to this part of the question, and readers had to be careful in scoring. **P7** was earned for using $W(t)$ as the integrand of either a definite or indefinite integral, with or without the *dt*.

However, a missing *dt* can introduce an ambiguity as in $\int_0^2 2t\left(t^2-1\right)^3+7$, which has not yet earned **P7**.

Presenting more as in $=\frac{1}{4}\left(t^2-1\right)^4\Big|_0^2+7$ resolved the ambiguity and earned both **P7** and **P8**. On the other hand,

continuing with $=\left[\frac{1}{4}\left(t^2-1\right)^4+7t\right]_0^2$ made it clear that the integrand was incorrect and did not earn **P7** but did earn **P8**.

P8 was earned for the antiderivative of $W(t)$ with or without the constant of integration. Some students, unfortunately, had arithmetic difficulties with the coefficients in the antiderivative. Some responses expanded $\left(t^2-1\right)^3$ yielding the antiderivative $\frac{1}{4}t^8 - t^6 + \frac{3}{2}t^4 - t^2$, but many doing this had difficulty with the expansion. Some responses showed the *u*-substitution $u = t^2 - 1$ correctly and in the course of the work, made it to $\frac{1}{4}u^4$. Thus **P8** was earned. Some responses worked with the indefinite integral showing $\int v_J(t)\,dt = \frac{1}{4}\left(t^2-1\right)^4 + C$ which earned **P7** and **P8**. Correct work, resolving $+C$ and leading to the final answer, earned **P9** as well.

P9 was earned for the answer, simplified or not. **P8** must have been earned in order for a response to be eligible for **P9**. Showing $7+\frac{1}{4}\left((3)^4-(-1)^4\right)$ or equivalent earned **P9**, despite any errors in subsequent attempts at simplification. However, ambivalent responses such as $7+\frac{1}{4}((3)^4-(-1)^4$ had to be resolved before **P9** was earned.

Observations and recommendations for teachers:

(1) Given the position of a moving particle, finding the velocity at a certain time requires a calculation of the derivative of the position function. Far too many students failed to include parentheses when applying the chain rule in deriving $x_H{}'(t)$. If $2t-1e^{t^2-t}$ was presented rather than $\left(2t-1\right)e^{t^2-t}$, there was often an error in the final answer resulting in not earning **P2**. On the 2025 exam in this part of the question, $2t-1e^{t^2-t}$ earned **P1**. However, it is not always the case on the AP exam that such a notational error earns a point.

(2) In part b, directions of motions of the two particles under examination in this question certainly required examining the velocities of the particles. Thus, evidence of that for either particle earned **P3**. On the FRQ section of the exam, this is known as an entry point. In this case it means that there is evidence that the student is considering one of the functions necessary to answer this question.

(3) **P4** was earned for analyzing the direction of motion or the signs of velocity on $0<t<5$ for one of the particles. Too many students did not analyze the entire interval. Regardless of whether the interval $1<t<2$ which is the final answer was shown, work had to indicate that what is happening on the entire interval $0<t<5$ had been considered. It took the complete analysis of signs of both velocities, along with the final answer, to earn **P5**. Again, students often failed to show that signs had been considered across the entirety of $0<t<5$. In examining the characteristic(s) of a function on an interval, it is important to show both where the characteristic(s) are found and where they are not.

(4) **P6** is the only point that can be earned in part c of this question, and there is no entry point available. Both the answer and supporting work are required in order to earn **P6**. Once again, properties of two functions needed to be compared, this time at a point rather than over an interval. Students should know to compare the signs of velocity and acceleration at a point to determine if speed is increasing, decreasing or neither.

(5) In part c, although the sign of $v_J'(2)$ was given, some students launched into a computation incorrectly. Worse, some students tried to find the values of one or both of $v_J'(2)$ and $v_J(2)$. A number of such efforts made computational errors unnecessarily since all that was needed to be examined were their signs. When students try to find values of functions (which may not be needed) the values found must be correct. Students should not bother to show work that is not required for answering the question. If some such work is written and its irrelevance recognized later, just cross it out, as readers do not read crossed out work.

(6) In part d, the position of the particle *J* is requested at time $t = 2$, given an initial position of 7 at $t = 0$. This requires use of the change in position over the time interval $0 \le t \le 2$. The net change in position is given by the definite integral of the rate of change in position, the velocity v_J. **P7** is an entry point into this part of the question and is earned for showing $v_J = 2t\left(t_2 - 1\right)^3$ in either a definite or indefinite integral.

(7) **P8** is earned for the antiderivative of $v_J = 2t\left(t_2 - 1\right)^3$, at least in a form as correct as $k\left(t_2 - 1\right)^4$ where *k* is positive. If $k \ne \frac{1}{4}$, the response is not eligible to earn **P9**, the answer point. Adding the initial position to the net change in position is an important concept. This idea has appeared in other contexts on the AP Calculus Exam. See, for example, 2024AB/BC1 part c.