2023 AB-2 Chuck Garner, Rockdale Magnet School

Problem Overview

Students were told that a swimmer goes back and forth along a straight path in a pool that is 50 meters long over a duration of 90 seconds. Students were given the swimmer's velocity as

$$v(t) = 2.38e^{-0.02t} \sin\left(\frac{\pi}{56}t\right),$$

where t is in seconds and v is in meters per second.

Part a

Students were asked to find all times, $0 < t < 90$, where the swimmer changes direction, and to give a reason for their answer.

Part b

Students were asked to find the swimmer's acceleration at $t = 60$, to show their steup, and to show units of measure. Then students were asked to determine whether the swimmer is speeding up or slowing down at $t = 60$, and to give a reason for their answer.

Part c

Students were asked to find the distance from the swimmer's position at $t = 20$ to the swimmer's position at $t = 80$, and to show the setup for the calculations.

Part d

Students were asked to find the total distance swam over $0 < t < 90$, and to show the setup.

Comments on Student Responses and Scoring Guidelines

Part a

This part of the question could earn the student two points. To earn the first point, the student must have written evidence of considering the sign of $v(t)$. This could be shown by writing "$v(t) = 0$" or by writing "The swimmer changes direction when $v(t)$ changes sign" or some variation indicating that the student was looking for zeros of the velocity or a sign change of the velocity.

The second point was earned for reporting the answer $t = 56$. However, simply writing "$t = 56$" with no supporting work did not earn either of the two points. Any other values of t outside the interval $0 < t < 90$ where the velocity is zero were not read and did not affect the points students could earn.

Part b

This part of the question could earn the student three points. The first point was earned if the student reported the value $v'(60) = -0.036$. The student could not earn this point by writing "$a(60) = -0.036$" unless the connection $a(t) = v'(t)$ was explicitly shown.

The second point was earned for the correct units of meters per second per second (m/s^2 is perfectly acceptable) provided the student declared a value of $v'(60)$, even if the value was incorrect.

The third point was earned if the student's response was consistent with the negative velocity at $t = 60$ ($v(60) = -0.159$ or -0.16) and with their value of $v'(60)$ from part (a). The expected correct response was that the swimmer is speeding up because both the swimmer's velocity and acceleration are negative at $t = 60$. To earn this point, the student was not required to declare a value of $v(60)$, only that the sign is negative, or that it is the same sign with the declared negative value of $v'(60)$ from part (a). That is, the student writing "The swimmer is speeding up because $v(60)$ and $v'(60)$ have the same sign" earns this point, provided a negative value was declared in part (a). However, any response that used an incorrect sign or an incorrect value of $v(60)$ did not earn the third point.

If a student was in degree mode for this part of the problem, the student did not earn the first point but was eligible to earn the second and third points. However, in degree mode, there are two possible values of $v'(60)$, one is -0.000141 and the other is 0.039. Either of these values with the degree mode value of $v(60) = 0.042$ (or with an indication that $v(60) > 0$) and with a consistent response earns the the third point.

Part c

Two points are available to the student in part (c). To earn the first point, the student must have written the definite integral $\int_{20}^{80} v(t)\ dt$. The differential was not read for this point. Any errors in the limits of integration did not earn this point.

The second point was earned only for an answer of 23.384 or 23.383, but it must be attached to a definite or indefinite integral of $v(t)$. The degree mode answer of 2.408 was accepted, provided the student was consistent and also used degree mode in part (b).

Part d

In the last part of this problem, two points were again available. The first point was earned only for

$$\int_0^{90} |v(t)|\ dt \qquad \text{or} \qquad \int_0^{56} v(t)\ dt - \int_{56}^{90} v(t)\ dt$$

or the equivalent. Differentials were again not read for this point. Indefinite integrals or incorrect limits of integration did not earn this point.

The second point was earned by declaring the correct answer of 62.164, but it must be attached to a definite or indefinite integral of $v(t)$. The degree mode answers of 3.128 or 3.127 was accepted, again provided that the student was consistent in using degree mode through previous parts.

Observations and Recommendations for Teachers

(1) Students should know more than just looking for zeros of the derivative. They should understand what a sign change means. Many students in part (a) wrote some variation of "The swimmer changes direction because that is where the velocity is zero." This is not sufficient reasoning. Students should be given problems in class where the derivative is zero, but no sign change occurs in order to help them understand that more justification for a change in direction is required than a zero derivative.

(2) Students should always make sure to write the correct definite integrals, particularly on calculator problems. Writing an incorrect definite integral and then using the calculator to evaluate that incorrect definite integral does not earn the student any points. Even though the value presented to an incorrect integral may be the accurate value for that integral, the value and the integral does not answer the question. There are no consistency points in presenting an accurate answer to an incorrect definite integral.

(3) Students should understand the distinction between net distance and total distance. Many students put absolute value bars around $v(t)$ when not needed in part (c), or put absolute value bars around the entire integral in part (d). However, many students used no absolute value bars at all! This was especially unfortunate in part (d), where absence of absolute value bars around $v(t)$ did not earn the first point, and the corresponding integral of $v(t)$ is the wrong value and did not earn the second point.

(4) In a similar vein with Observation (3), students in parts (c) and (d) attempted to break-up the given intervals into subintervals over which they then calculated definite integrals over the subintervals. That is, some students attempted to split $\int_0^{90} |v(t)|\ dt$ into the integral of v from 0 to 56, and another integral of v from 56 to 90. While this is a correct approach, most of the students who did this got lost in signs and

absolute values, and did not earn the point. For example,

$$\left| \int_0^{56} v(t)\ dt + \int_{56}^{90} v(t)\ dt \right|$$

and

$$\int_0^{90} |v(t)|\ dt - \int_0^{56} |v(t)|\ dt$$

and

$$\int_0^{56} v(t)\ dt + \int_{56}^{90} v(t)\ dt$$

were all attempted by students. These expressions also did not earn the answer point. It is strongly suggested that students should not split up an integral that represents the total distance, unless an antiderivative is required.

(5) In part (a), many answers of $t = 0$ were reported. This answer is not in the interval, but in the presence of the correct answer $t = 56$, this was fine. However, some students presented only $t = 0$ as their answer, and did not earn the answer point. Students should be aware of the intervals in the problems.

(6) Students should use the names of functions. Students instead carried $2.38e^{-0.02t}\sin(\pi t/56)$ throughout instead of simply writing $v(t)$. There were many copy errors in all parts of this problem which prevented otherwise good calculus from being awarded points.