2022 AB/BC1 Marshall Ransom, Georgia Southern University

Problem Overview:

Students were told that $A(t) = 450\sqrt{\sin(0.62t)}$ gives the rate at which cars arrive at a toll plaza between the times 5 AM and 10 AM. A is measured in vehicles per hour and t is the number of hours after 5 AM. It was also stated that traffic is flowing smoothly at 5 AM with no vehicles waiting in line.

Part a:

Students were asked to write but not evaluate an integral expression that gives the total number of vehicles arriving at the toll plaza between 6 AM ($t = 1$) and 10 AM ($t = 5$).

Part b:

Students were asked to find the average value of the rate, in vehicles per hour, at which vehicles arrive at the toll plaza from 6 AM ($t = 1$) and 10 AM ($t = 5$).

Part c:

Students were asked if the rate at which vehicles arrive at the toll plaza at 6 AM ($t = 1$) is increasing or decreasing and to provide a reason for the answer.

Part d:

Students were told that a line forms whenever $A(t) \geq 400$ and that a is the time when a line first begins to form. The number of vehicles in line at time t, for $a \leq t \leq 4$ is given as $N(t) = \int_a^t \left(A(x) - 400\right) dx$. Students needed to find, to the nearest whole number, the greatest number of vehicles in line at the toll plaza in the time interval $a \leq t \leq 4$ and to justify this answer.

Comments on student responses and scoring guidelines:

<u>Part a</u> worth 1 point

Since $A(t)$ is the rate at which vehicles arrive and is always non-negative, the total number of vehicles arriving is given by $\int_1^5 A(t)\,dt$. That integral was worth 1 point. $\int_1^5 |A(t)|\,dt$ was also acceptable. Students were not penalized for a missing dt or for "misspelling" and using dx. Some students chose to write $450\sqrt{\sin(0.62t)}$ even though this function had been named $A(t)$ in the stem of the problem. This did lead to some copy errors which were penalized, such as writing $450\sqrt{\sin(0.62)t}$. Most readers saw a few students write $A(t) = \int_1^5 A(t)\,dt$ or $\int_6^{10} A(t)\,dt$. These responses were penalized, but if used again in subsequent parts of the problems ignored. If used again, the remainder of the work in subsequent responses was read for accuracy, requiring the correct values and work expected.

<u>Part b:</u> worth 2 points

Students could earn 1 point for using the average value formula and 1 point for the answer. The answer had to be to three decimal place accuracy. Thus $\frac{1}{5-1}\int_1^5 A(t)\,dt = 375.536$ or 375.537 earned the first point. Work such as $\frac{1}{5-1}\int_1^5 A(t)\,dt = 375.54$ or $\frac{1}{5-1}\int_1^5 A(t)\,dt = 375.536 \approx 375$ did not earn the answer point. (See **<u>Observations and recommendations for teachers</u>** (3) below). A number of students confused average value of this function with the average rate of change and provided something like $\frac{A(5)-A(1)}{5-1}$ which earned no points in part b.

It was in part b where readers first noticed some students in degree mode. Thus $\frac{1}{4}\int_1^5 A(t)\,dt = 79.416$. This response is penalized but did earn 1 of the two points in part b. Subsequent work by a degree mode student on this question was read for accurate degree mode answers for full credit, with the exception of part d.

<u>Part c:</u> worth 2 points

The expected response was for an appeal to the sign of $A'(1)$. The first point was earned for considering or trying to work with $A'(1)$ or something like $\left.\frac{d}{dt}A(t)\right|_{t=1}$. The second point was earned for the answer along with a reason. The value of $A'(1)$ was not requested. Thus "increasing because $A'(1) > 0$" earned both points. A number of students attempted to compute an expression for $A'(t)$. Errors in this computation were ignored unless they resulted in an incorrect value for $A'(1)$. Values presented for $A'(1)$ had to be correct to the nearest whole number or the number of decimal places presented up to three. Otherwise, the response was not eligible to earn the second point. Some students also pointed out that $A(1) > 0$ and reasoned as follows: "A is increasing at $t=1$ because $A'(1)>0$ and $A(1)>0$." This response did not earn the second point. The fact is that, in this case, $A(1)>0$ is not necessary in determining that $A(t)$ is increasing at $t=1$. A student in degree mode may have presented $A'(1)=23.404$, a correct degree mode value that was not penalized if in part b the student had already been penalized. A number of students used a discrete approach, showing some values of $A(t)$ getting larger. This will never be an acceptable justification on a calculus exam and is tantamount to showing a graph and saying that $A(t)$ is increasing at $t=1$ because the graph seems to go up.

Part d: worth 4 points

The first point was earned for trying to set the derivative of N equal to 0. This could be shown overtly as in $N'(t)=0$ or $A(t)=400$ or by writing $A(t)\geq 400$ and showing at least one of the values of a or b. Finding the values of both a and b earned the second point. Since values of a and b were not requested, it was only necessary for the response to show these accurate to one, two or three decimal place accuracy. The third point was earned for the answer: either one of the whole numbers 71 or 72 or the three decimal place accuracy of 71.254. The fourth point was earned for justifying that the answer given is an absolute maximum value for $N(t)$. Because $N(t)$ is being examined on the closed interval given by $a\leq t\leq 4$, values of $N(t)$ at the endpoints and the critical point could be shown in the presence of the answer and earn the fourth point. Since b is the only location of a critical point on $a\leq t\leq 4$, an alternative justification was also acceptable. This required stating that $A(t)\geq 400$ for $a\leq t\leq b$ as well as that $A(t)\leq 400$ for $b\leq t\leq 4$.

A degree mode student created the situation wherein $A(t)-400<0$ for all values of t in the interval. Since there are no critical points to examine, the degree mode student could earn at most the first point.

Observations and recommendations for teachers:

(1) $450\sqrt{\sin(0.62t)}$ had been named $A(t)$ in the stem of the problem. It is always wise to use this name rather than to copy the entire expression, thereby risking a copy error. Copy errors can lead to erroneous or impossible values, thus making students ineligible for earning some subsequent points.

(2) The accumulation of change can be calculated using a definite integral of the rate of change over an appropriate time interval (values of t). Using "clock" values such as 6 for $t=1$ hour past 5 AM and 10 for $t=5$ hours past 5 AM set up an integrand with imaginary values because sine under the radical can be negative on the "clock" interval from 6 to 10.

(3) Part b asked for the average value of the rate of change (in vehicles per hour) over the time interval given by 6 AM ($t=1$) to 10 AM ($t=5$). Since this average value is a requested answer it had to be presented to three decimal place accuracy. A number of students felt that one could not have a fractional value of a vehicle and rounded this answer to a whole number. In fact, using fractional values does make

sense in the context of this part of the question. For example, it certainly would make sense to conclude that 350.5 vehicles per hour arrive at the toll plaza. 0.5 vehicles per hour simply means that in a two hour period there is an average of one vehicle arriving.

(4) The average value of a function $f(x)$ on an interval [*a*, *b*] is given by $\frac{1}{b-a}\int_a^b f(x)\,dx$. This is *NOT* to be confused with the average rate of change of the function on the interval given by $\frac{f(b)-f(a)}{b-a}$. AB/BC1 is a calculator active question. Students are expected to be able to compute an estimate of the value of a definite integral, providing an answer correct to three decimal places. Also, it is apparent from scoring many questions that some students still need to be reminded to be in radian mode when working with trigonometric functions.

(5) Using discrete values of a function to try to determine increasing or decreasing always leaves out infinitely many values which may or may not also be increasing or decreasing. Furthermore, unless specifically required in a part of the question (and that hasn't happened on the AP Calculus Exam in years except for a sketch through a given slope field), **a student-provided graph is treated as scratch work. It is ignored by readers and not valid for use in an explanation or justification. To explain increasing or decreasing, calculus must be used, meaning an appeal to the sign of the derivative.**

(6) Part d is frequently on the AP Calculus Exam: on a closed interval, find the location and value of an absolute extremum and justify that answer. $N(t)$ was given as $\int_a^t \left(A(x)-400\right)dx$ with a given as the time when a line of vehicles first forms. The process involves finding values of $N(t)$ at the endpoints, for which no point was awarded. Points were awarded for applying calculus as in searching for any critical point(s) by setting $N'(t)=0$. This earned the first point in part d. A number of students earned this first point despite not being able to find an expression for $N'(t)$ using the fundamental theorem. Finding $\frac{d}{dx}\int_a^t \left(A(x)-400\right)dx$ is a very important idea in the curriculum. By simply listing the values of *N* at the endpoints and the one critical point $b=3.597$, students were using what is sometimes referred to as the "candidates test" which could justify the final conclusion, in this case the value rather than the location of the extremum.