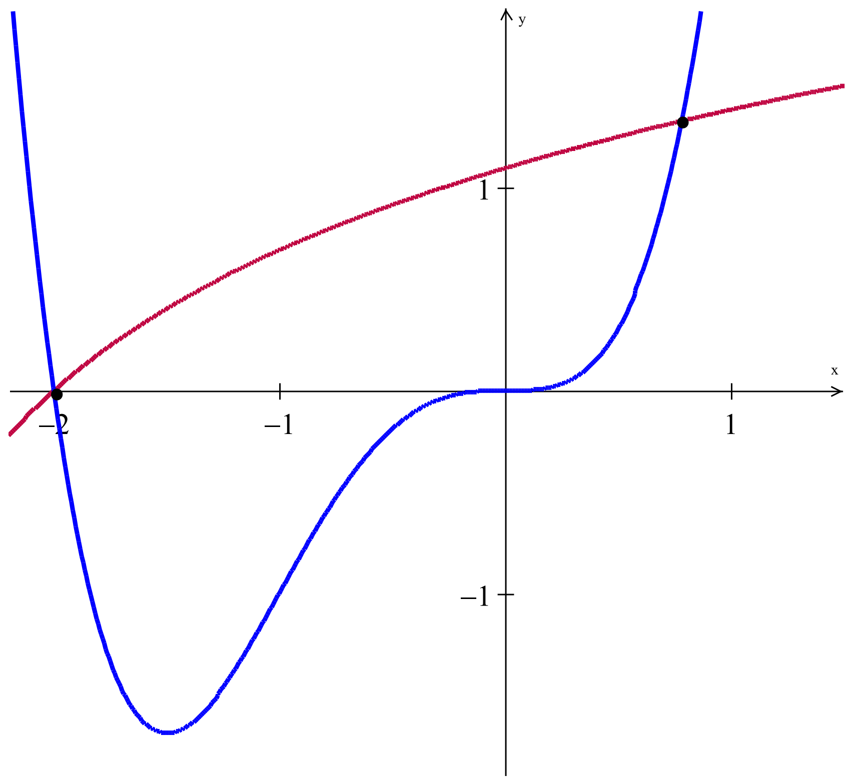
Problem Overview:



f and g are the functions defined by $f(x)=\ln(x+3)$ and $g(x)=x^4+2x^3$. The graphs of f and g are shown in the figure above and intersect at $x=-2$ and $x=B$ where $B>0$.

Part a:

Students were asked to find the area of the region enclosed by f and g.

Part b:

$h(x)$ is given as the vertical distance between the graphs of f and g. Students were asked to determine whether h is increasing or decreasing at -0.5 and to give a reason for the answer.

Part c:

The region enclosed by the graphs of f and g is said to be the base of a solid. Cross sections of the solid perpendicular to the x-axis are squares. Students were asked to find the volume of this solid.

Part d:

A vertical line in the *xy*-plane travels from left to right at the constant rate of 7 units per second along the base of the solid in part c. Students were asked to calculate the rate of change with respect to time of the area of the cross section above the vertical line at $x = -0.5$.

Comments on student responses and scoring guidelines:

Part a worth 3 points

The first point is earned for the integrand which may be presented as $f - g$, $g - f$, $|f - g|$ or equivalently. Some students used the explicit expressions, and some made the error resulting in $\ln(x+3) - x^4 + 2x^3$ (AND not showing $f - g$ in the work). A scoring consideration for students continuing to work with this special case was applied in all parts of this question. When this error was first presented, students lost a point. In part a, a consistent answer is negative and cannot earn the third point for the answer. The second point in part a is for the limits on the integral. If the second point was not earned because of an incorrect value of B (a common student error) but the lower limit was -2, the third point was earned with a consistent answer. If the second point was not earned because of using a lower limit of 0 but the value of B was correct, the response earned the third point with the consistent answer of 0.707 (or 0.708). Use of any other incorrect limits earned neither the second nor third points. A response that earned the second point could only earn the third point with a correct answer.

Part b: worth 2 points

The first point was earned for considering either $h'(-0.5)$ or $f'(x) - g'(x)$. The second point was earned for the answer with a reason as in "decreasing because $h'(-0.5) < 0$." However, if an incorrect value was reported for $h'(-0.5)$, this second point was not earned. Some students compared values of $f'(-0.5)$ and $g'(-0.5)$ verbally rather than computing $h'(-0.5)$ and often had difficulties communicating correctly.

The only response reporting "increasing" that could earn the second point was the special case in which there was consistent work with $\ln(x+3) - x^4 + 2x^3$.

Part c: worth 2 points

The first point was earned for an integrand of the form $k\left(f(x) - g(x)\right)^2$ or its equivalent with $k \neq 0$. If $k \neq 1$, the second point could not be earned. The response not earning the first point was ineligible for the second point except in the case of a presentation error such as missing or mismatched parentheses, which was common error among students using the explicit expressions for the functions. In this case, a correct answer earned the second point for the answer. Use of incorrect limits resulting in a consistent answer was eligible for the second point, provided the incorrect limits were the same as in part a. A consistent answer using $\ln(x+3) - x^4 + 2x^3$ is 252.187 (or 252.188) and earned the second point.

Part d: worth 2 points

The first point was earned for computing $\frac{d}{dt}A(x)$ in such forms as $\frac{dA}{dx}\frac{dx}{dt}$, $A'(-0.5)\bullet 7$ or as a few students showed $2\left(f(x)-g(x)\right)\left(f'(x)-g'(x)\right)\frac{dx}{dt}$. The second point was earned for the answer. An incorrect $A(x)$ imported from part c allowed the response to be eligible for both points (provided a consistent answer was reported). The student using $\ln(x+3)-x^4+2x^3$ earned the second point for the consistent answer, 20.287.

A number of students used h^2 or even another variable such as in s^2 to represent the cross section area. Most of these students using their own variable did not define it clearly in relation to f and g. Also, it was common for such students to fail to apply the chain rule correctly as in $A'(x)=2s\frac{ds}{dt}$. The common errors in this example are the chain rule error of a missing $\frac{ds}{dx}$ as well as not realizing that the given rate of 7 is $\frac{dx}{dt}$.

Observations and recommendations for teachers:

(1) A number of responses did not calculate the value of B correctly. Use of a calculator solve capability is an expected skill. There may be other ways to calculate B depending on the calculator used. This type of calculator skill should be practiced in classes preparing for the AP Exam.

(2) Regarding part b, the vertical distance between the curves is TOP $-$BOTTOM $= f-g$, which is therefore $h(x)$. Since this is a calculator active question, the simplest way to compute $h'(-0.5)$ is to use the numerical derivative capability of the calculator. There is certainly no reason to compute derivatives by hand. Use of the function names (expressions should have been stored in the calculator) is much more efficient than working by hand with the expressions.

(3) The search for a reason for whether something is increasing or decreasing should send students immediately to looking at the sign of the derivative. Students who tried to explain part b verbally, in terms of the relative values of $f'(-0.5)$ and $g'(-0.5)$, had great difficulty communicating properly.

(4) Parts c and d focused on the area of a square cross section of a solid. The area is s^2 where s is the length of a side of the square, here $f-g$. This has already been indicated as $h(x)$, but a number of students in part d introduced a new variable without defining it as $f-g$. When introducing a new variable such as s or Y_1, it is imperative to define this variable. Also, students should read the question carefully. For some reason, a number of students used a circle as the cross section.

(5) When computing the rate of change of $A(x)$, the area of the cross section, many students failed to apply the chain rule properly. This was particularly obvious for work using s or h as the length of the side of the square, failing to recognize that the length of the side is a function of x and that the given 7 is $\frac{dx}{dt}$. Computing the rate of change of a function involves a derivative with respect to time. This should be practiced in situations where the function being analyzed is defined in terms of a variable other than t.

(6) It is very important to use function names in written work. In this question, the explicit expressions for the functions should be stored in the calculator. Thus, use of $f - g$ in written work would not risk copy, arithmetic or misplaced parentheses errors. There is no need for the explicit expressions of these functions to be seen in student work. Likewise, there is no need to compute derivatives of these explicit expressions. The numerical capabilities of the calculator perform the necessary computations when searching for a value or a sign of the derivative.