CSI: Calculus/Statistics Insider

Official Newsletter of the Georgia Association of Advanced Placement Math Teachers

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This publication includes:

GA^{2}PMT
Georgia Association of Advanced Placement Math Teachers
Letter from the President

It is with great humility that I write to you as president of such an amazing group of educators. The Georgia Association of Advanced Placement Math Teachers is filled with wonderful teachers who have made tremendous impacts on my career as a teacher and the careers of so many more. I am humbled to follow in the footsteps of past presidents who worked so tirelessly to build an organization which serves us so well and in so many ways. In this newsletter, I would like to share of some of those ways we hope to enrich your skills and knowledge as well as some new ways.

First, I would like to take this opportunity to thank our past president, Debbie Kohler. She has worked so diligently for more than six years a member of our board of directors, and as our president for the past two years. Debbie has worked untold hours in planning for our annual conference each year. Her leadership has brought valuable resources and well-informed people into our organization. She has been a light to me with her selfless dedication to GAAPMT.

Second, I would like to announce the name of the featured speaker for the 2018 Georgia Mathematics Conference at Rock Eagle – Vicki Carter. Vicki has been involved with AP Calculus for over 40 years. She has worked has a College Board consultant, an Exam Leader at the AP reading, and just recently finished serving on the Test Development Committee. She is also a National Instructor for Texas Instruments' Teachers Teaching with Technology. Vicki will be sharing from her many years of experience and from her great familiarity with the new frameworks. She will be presenting for all our Calculus sessions on Friday morning. I am extremely excited that Vicki can share her time with us.

Fear not, statistics teachers! We will have morning sessions for statistics as well, featuring many of our wonderful and talented local teachers. Our hope is to have a nationally renowned speaker in the field of AP Statistics for our 2019 conference.

Third, I want to invite each of you who are on Facebook to visit our new page for the organization and join our board members who are already sharing great things. Many thanks to Bill Shillito for setting the page up. Thanks also to Rand Wise, David Custer, and Vicki Greenberg for adding to the page. I hope this space can be a productive way for all of us to share ideas, questions, and thoughts as we prepare our students for their AP mathematics exams.

Last, I wish all of you the best in the next few weeks as you enter the homestretch for your class. I know that you yourselves are working tirelessly to present your students with a great course in mathematics and also their best chance at succeeding on the exam. Please reach out to me at coachwilson@numail.org if there is any need you would like for our organization to try to meet.

In your service,
Dennis Wilson

Some valuable links for AP Calculus

AP Calculus from those in the know

Stephen Davis, the current Chief Reader for AP Calculus presented at the T^3 International conference in March. The slides for his presentation are available at the link below. An important part of this presentation includes an extension of last year's "Alphabet Soup" problem, free response question 6 on the AB exam. The extension starts on slide 46 an cover L'Hospital's rule. It is of interest because it shows how the topic will be scored if it appears on this years' exam. (I had a sneaky suspicion from his presentation that it would.)

http://www.ncaapmt.org/archive/crTalks/t3Panel-mar2018.pdf

Some of the cut scores from the exam were shared as well. The first score is that which the student receives. The second score, the cut score, is the score out of a total of 108 which the student must achieve to attain the reported score.

AB	**BC**
3 – 44/108	3 – 41/108
5 – 71/108	5 – 65/108

TI in Focus – AP Calculus

Recently, Texas Instruments posted some videos which could serve as a valuable resource for AP Calculus teachers. In these videos, Former Chief Reader Steve Kokoska discusses all nine free response questions from this past year. The videos discuss scoring, problem extensions, and a summary of topics covered. These videos are useful regardless of the technology used by your students. Tom Dick also present on video which teach how to use both the TI-84 and TI-Nspire on the problems.

https://education.ti.com/en/resources/ap-calculus

The John Neff Award

It's never too early to begin thinking about the John Neff Award. Dr. John Neff was a faculty member at the Georgia Institute of Technology and a former President of the Georgia Council for Teachers of Mathematics. He was a mentor who truly inspired teachers and was a delight to talk to. The Georgia Association of Advanced Placement Teachers created an award in his memory due to his legacy in mathematics, advanced placement mathematics and education. We encourage you to nominate teachers who have taught AP Calculus and/or AP Statistics for at least five years, have demonstrated excellent teaching in their classrooms, and demonstrate a willingness to mentor others. This person also contributes to the teaching of mathematics beyond their classroom. Nominations will be taken in the spring and early fall for this honor. You need to submit a brief email that states who you would like to

nominate, their qualifications and why you believe they deserve this award. If you would like to send a nomination now, please send it to coachwilson@numail.org,

2018 College Board Summer Institutes in Georgia

Click on name of school to register.

Location	Calculus AB	Calculus BC	Statistics
Kennesaw State University	July 9	July 16	July 9
The Marist School	June 11 June 25	June 18	
University of Georgia	July 9	June 18	July 9
UT – Chattanooga	June 18		June 25
Walton High School	June 25		June 25
Woodward Academy	June 4 July 16	June 4	June 4 July 16

Link to more information about summer institutes:
http://apcentral.collegeboard.com/InstitutesAndWorkshops

Link to information about AP Calculus AB/BC 2016 curriculum changes:
https://advancesinap.collegeboard.org/stem/calculus

Link to receive newsletter: http://goo.gl/forms/MbQUWbvQ1L

AP Calculus

2017 QUESTION AB/BC4

Please view the questions here:

https://apcentral.collegeboard.org/courses/ap-calculus-ab/exam?course=ap-calculus-ab

2017 AB4 Marshall Ransom, Georgia Southern University

Problem Overview:

A boiled potato is removed from cooking and left to cool at time $t = 0$. At $t = 0$ the internal temperature of the potato is 91° Celsius ($^\circ$C). At all times $t > 0$, this temperature is greater than 27°. At times t in minutes, the internal temperature of the potato can be modeled by a function H that satisfies the differential equation $\frac{dH}{dt} = -\frac{1}{4}(H - 27)$ where $H(t)$ is measured in degrees Celsius and $H(0) = 91$.

Part a:

Students were asked to write an equation of the line tangent to the graph of H at the point where $t = 0$ and use this equation to approximate the internal temperature of the potato at time $t = 3$.

Part b:

Using $\frac{d^2H}{dt^2}$, students were asked to determine whether the answer in part (a) is an underestimate or an overestimate of the actual internal temperature of the potato at time $t = 3$.

Part c:

An alternative model for the internal temperature of the potato was given by a function G satisfying the differential equation $\frac{dG}{dt} = -(G - 27)^{2/3}$ with $G(t)$ measured in degrees Celsius and $G(0) = 91$.
Students were asked to find an expression for $G(t)$ and, using this model, find the internal temperature at $t = 3$.

Comments on student responses and scoring guidelines:

Part a:

Three points were possible for students to earn. The first point was for the slope of the tangent line. This could be expressed with or without correct units as in $-\frac{1}{4}(91-27)$ or even $\frac{dy}{dx}=-16$. An incorrect slope could be used in an equation for the second point if that equation indicated a line passing through (0, 91). Thus, $y-91=-16x$ earned the first two points. The answer $y=16x+91$ earned only the equation point because the slope is incorrect, but the point (0, 91) is on this line. Using an incorrect slope could make it impossible for students to earn the third point, as in $\frac{dH}{dt}=-0.25(91-27)=-15 \rightarrow y=-15x+91 \rightarrow y=46$. This student earned one point for slope and one point for an equation, the arithmetic error coming off the third point. A common error was substituting 0 rather than 91 in the tangent line equation. An equation point could be earned using the slope $\frac{27}{4}$ obtained in this way, but the approximation was then 111.25 and did not earn the third point, 111.25 being greater than the initial temperature of the potato.

Part b:

$\frac{d^2H}{dt^2}=$ the equivalent of $\left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right)(H-27)$ or $\left(-\frac{1}{4}\right)\frac{dH}{dt}$ had to be calculated as well as be indicated as positive, and the answer "underestimate" had to be given. Common errors were referring only to "concave up" or discussing the second derivative at one point rather than on an interval. Students did not need to appeal to an interval, but they could not present a local argument showing $\frac{d^2H}{dt^2}>0$ at only one point. And only one point is what students earned with all this work.

Part c:

There were five points available. The first point was for separating the variables and the second for the antiderivatives. Many students launched straight into calculating the antiderivative of $-(G-27)^{2/3}$, thus ignoring the variable t and earning zero points in part (c). The third point was for both showing a correct constant of integration and using (substituting) the initial condition $(t,G)=(0,91)$. A student's value of C, either correct or incorrect, could be used to earn the fourth point by showing an equation involving both G and t. The fifth point was for both solving for $G(t)$ and showing the value of $G(3)$, and could only be earned if the value of C was correct.

Observations and recommendations for teachers:

1. Students have to show work on this test. Even a simple calculation for slope needs to be shown and/or connected to the information given that will lead to that slope. Part (a) in this problem was scored more charitably. A simple presentation of $y = -16x + 91$ earned both the slope and equation points. More work shown is sometimes needed, and has been, on the AP Calculus Exam. As simple as it seems, showing $\frac{dH}{dt} = -\frac{1}{4}(91-27)$ is an important display of knowledge because the basic calculus concept here is that the slope of a line tangent to the graph of H is given by $\frac{dH}{dt}$. It is clear from scoring hundreds of exams that some students do not make this connection. Teachers should require such work from their students.
2. Students should be aware of the context of these problems. Work resulting in an internal potato temperature of greater than 91 should be immediately suspect, and students should check back through their work if such a result arises from their work. This type of error will not be awarded a point.
3. When deciding whether a linear approximation is an over- or under- estimate, it should be clear from the concavity. However, the test often specifically asks for students to use the second derivative. It is the sign of the second derivative that determines the nature of the concavity. The word "concavity" can be ignored if the sign of the second derivative is correctly interpreted. Use of the proper concavity in determining the nature of the estimate is not sufficient to earn an explanation point in the absence of work showing the sign of the second derivative.
4. Many readers felt that students had difficulty in part (c) because no t showed in the given information. The first step in separating variables given $\frac{dy}{dt} = \text{expression}$ is to "multiply" both sides of the equation by dt. Sometimes this dt will be the only term showing on the right side of the equation. That led some students into more difficulties. There is an integrand of 1. There is something elementary, but important, going on here. The integrals $\int dt$, $\int dx$, $\int d\theta$, and $\int d(uv)$ should be quickly, automatically, calculated by students. The integral $\int d(\text{anything}) = \text{anything} + C$. Not recognizing this simple idea put many students in the situation of earning 0 of 5 points in part (c).
5. Scoring the solving of a separable differentiable equation requires only one correct use of $+C$ even though technically there is a C on both sides of the equation after calculating antiderivatives. A point is earned for this $+C$ and using (substituting) the initial condition in the resulting equation. This is a recent change in the scoring. In the past a correct $+C$ earned a point and another point was awarded for using the initial condition. Arithmetically calculating the value of C incorrectly comes off the final answer point.
6. Students should carefully read the question posed. Students were asked to find in part (c) an expression for $G(t)$, which means to solve for $G(t)$. In this case, one point was awarded for preliminary work showing an equation involving $G(t)$ and t. In words, it was asked of students to calculate $G(3)$. Students should not worry about what work will award them points. But they should carefully read the question in order to make certain that all that has been asked for has been provided.

Please go to GAAPMT.ORG to view all the 2016 AP Calculus AB/BC readers' report and analyses by readers at last year's exam.

AP Statistics
2017 QUESTION 5

Please view the questions here:
https://apcentral.collegeboard.org/courses/ap-calculus-ab/exam?course=statistics

2017 Question 5 Billy Esra , Bishop Hall Charter School

Question 5: INTENT OF THE QUESTION

"The primary goal of this question was to assess a student's ability to identify, set up, perform, and interpret the results of an appropriate hypothesis test to address a particular question. More specific goals were to assess a student's ability to (1) state appropriate hypotheses; (2) identify the appropriate statistical test procedure and check appropriate conditions for inference; (3) calculate the appropriate test statistic and *p*-value; and (4) draw an appropriate conclusion, with justification, in the context of the study." (https://secure-media.collegeboard.org/ap/pdf/ap17-sg-statistics.pdf).

SAMPLE SOLUTION and COMMENTS:

This question was a very straight forward Chi-Square Test of Independence question. Excluding the investigative task (question 6), question 5 had the lowest mean score (1.51). The inclusion of a segmented bar graph distracted some students from performing a chi-square test, but most students who attempted the question chose one of the chi-square tests to perform. Students had varying level of success on carrying out the test.

a. The student had to identify the correct hypothesis test by name or formula, state the correct pair of hypotheses, and check the correct conditions.
    - Students could identify the test as the chi-square test of independence (or association) to name the correct test. Without explicitly naming the correct test, they could also get credit for naming if they had the correct formula somewhere in their response. The formula could be just the symbolic formula or the formula with the correct numbers substituted. Using the formula instead of a name only worked if there were no mistakes.
    - Students who wrote Chi-Square (with no formula or errors in the formula), Chi-Square Test of Homogeneity, or Chi-Square Goodness of Fit were not awarded credit for the correct name.
    - At least one of the hypothesis needed to be in context and the hypothesis had to represent the population, not the sample.
    - The hypotheses should be about the population, not the sample.
    - For conditions of inference:
        – Expected counts were to be reported and checked. Students were expected to list all expected counts and note that they were all at least 5. Alternately, they could show that the smallest expected count was at least 5.
        – Students could not include any incorrect conditions, such as Normality.
        – Random sample condition was stated in the prompt so it did not have to be checked by the student.
b. The student had to correctly calculate the chi-square test statistic and calculate the correct *p*-value (or *p*-value range from the table).

- Students did not have to show any calculations to get credit for the test statistic and the *p*-value. But if they did the work had to be correct (even if the correct values were given).
- If students made an error along the way (for example using the wrong degrees of freedom), they would lose credit for one component but not both (as long as their answer followed correctly from their miscalculation).
- Students completely lost credit for this part of their response if they reported the wrong type of test statistic (like *t* or *z*) even if the *p*-value was correct.

c. The student had to write the correct conclusion in context about the alternative hypothesis and justify the conclusion by linking the p-value with a reasonable alpha level.
   - The conclusion could not be about the null hypothesis.
   - Students who found the wrong *p*-value could still get full credit for this part of their response if they made the correct conclusion based on their incorrect *p*-value.
   - Students should not use absolute language in the conclusion (for example, I conclude that there is an association …). There should be some level of uncertainty. There is evidence, or there is not evidence, of an association.

NOTES/COMMON MISTAKES

1. The segmented bar graph was a big distractor for statistically weak students (or perhaps students who had no experience with chi-square). If a student started writing about the segmented bar graph, they almost never recovered and almost universally earned a score of zero.
2. In years past, students did not always have to explicitly say chi-square for homogeneity or chi-square test for association/independence. In fact, they sometimes did not have to name the test at all. If they just typed $\chi^2 = \#$, that was sometimes enough to get credit for naming the test. But this year, students were expected to choose the correct chi-square test.
3. Many students incorrectly found the expected counts by assuming that expected number would be the same for each cell in the table instead of correctly using the formula for expected counts or expected value matrix in calculator.
4. Some students lost points for conditions by listing the conditions for other inference procedures like Normal, n>30, and np>10. Any incorrect condition checked would cost the students points. The word Normal anywhere in the response would cost them the conditions part of their response. Some students would draw a Normal curve instead of a chi-square curve. The curve alone would not cause them to lose points, but if the curve included a scale with 0 in the middle or was shaded on both sides the student would lose points since the distribution is not Normal.
5. Degrees of Freedom was not explicitly part of the rubric, but if students wrote the wrong degrees of freedom down, they would lose credit for the test statistic (even if it would be the correct answer if they had used the correct degrees of freedom).
6. Linking the p-value and significance level to the decision in context proved difficult for many students. Students could give the correct interpretation of the p-value or explain how the conclusion follows from the p-value and not mention alpha levels (or significance levels) at all.

TEACHER SUGGESTIONS

I have to admit that this is the second year in a row that I was assigned to a chi-square question at the reading. I read and scored the Choco- and Apple-Zuties question last year as well as this chi-square question this year. There were some major differences between the two rubrics which is leading me to

rethink how I teach chi-square. When I first realized that I would be reading this question, I was excited because I thought that students would be very successful with this chi-square question. Chi-square is often one of the last topics covered and students generally seem to do well with that topic in my classroom. I was very, very wrong (as I usually am about how students will respond to questions on the test). To me, chi-square questions tend to look a lot like chi-square questions. This question included a segmented bar graph to distract students. Students could use the graph to supplement their chi-square test conclusion, but the question could be answered without referring to the graph at all.

As an AP Statistics teacher, I plan to do a better job at working with students on choosing the appropriate test after finishing up the inference unit. Perhaps I'm too quick to begin review, at large, and could help them with choosing the right test for the right situation. Students should have recognized the categorical variables and immediately thought chi-square when asked "provide convincing statistical evidence." Without performing a chi-square test here, students generally would score a zero on the question.

Perhaps students need a more formulaic method for organizing inference questions. I've started using a "four-corners" approach where students draw two intersecting lines to organize what goes where. I heard about this at the Best Practices night at the reading a few years ago, and I've started using it with students. So far, anecdotally, it appears to be helping them remember all of the components of an inference procedure answer. It was clear when reading exams that students who were taught some kind of organizational strategy for inference tended to do better on this questions than the students whose responses were more disorganized.

To much consternation of statistics teachers, the rubrics for inference procedure questions have not required students to use formulas. Just reporting the necessary statistics has been enough to earn credit for calculations. But if students do use formulas, they must use the correct formula and fill it in correctly. I would say that many students lost points for trying to write the formula or trying to fill in the formula. Generally speaking, the more that a student writes in their response, the more likely they were to include something that would cause their score to decrease. Reporting only the statistics themselves seems to be a consistent rubric element. Not showing work seemed to work to the advantage of many students. Alternately, showing work correctly (while not required) could get students points for another part of the rubric. For example, if a student wrote the formula correctly and filled in the expected counts into the formula, they could get credit for listing the expected counts even if they had not listed them elsewhere. Also, as mentioned previously writing the correct formula or correctly filling in the counts into the formula, could get students credit for the name of the test.

I think that is important for students to understand why they are checking the conditions for inference. In this case, the expected counts are what we would expect if the two variables were in fact independent. We are therefore testing if the observed counts are far enough from those expected counts, that we have evidence of an association. Students who were just listing conditions often included something that did not make sense for this type of question. Understanding why conditions must be met may help them when deciding which conditions to use for each problem. Understanding why you check for Normality of the sampling distribution for other inference procedures might sway students away from writing it as a condition for chi-square (especially since chi-square is always right skewed).

I loved this question. It was a very straight forward application of the chi-square test for independence, but I really wanted students to perform better than they did, overall. Chi-square questions show up in the AP Statistics FRQ questions almost every year. Though it is always possible that next year's test will not have a Chi-square question, I would make sure that you have built in time in the pacing of your course to cover it well. Chi-square questions could be an easy way for your students to distinguish themselves on the test.

GA²PMT Membership Request

The benefits of belonging to this organization can make a difference in your students' scores on the AP Exams.

Becoming a member of a professional organization is one of the indicators listed in the Georgia Teacher Keys Effectiveness System (TKES).

Performance Standard #9- Professionalism and Communication: The teacher exhibits a commitment to professional ethics and the school's mission, participates in professional growth opportunities to support student learning, and contributes to the profession.

To join GA²PMT, you may enter your information online using the following web address: **http://goo.gl/forms/MbQUWbvQ1L** or simply scan the QR code.
You may also complete the information below and mail it in with your membership dues, which are $10 per year.

GA²PMT Membership Form
(Oct. 1, 2018 – Sept. 30, 2019)

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