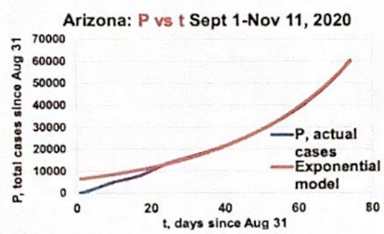
Modeling COVID-19 with Differential Equations

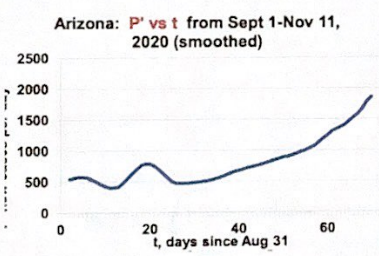
The graphs were created from data taken from an app sourced by the Covid-19 Data Repository at Johns Hopkins: https://isaac-flath.shinyapps.io/coronavirus2/.

1. The graph below was taken from the second surge in Arizona from Sept. 1 – Nov. 11, 2020. The blue curve represents the total number of cases vs. time and the red curve is the function that has been used to model the data. What type of function does the model appear to be? Exponential

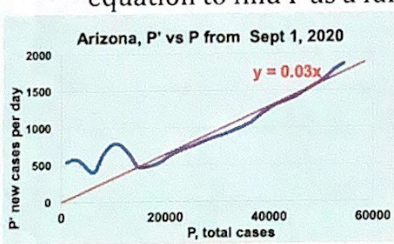


2. The graph below shows the new cases per day in Arizona over time. Does the shape of this graph confirm your hypothesis from #1 about what type of function models the total number of cases for this time period (ignoring the first 20 days)? Why?

Yes, because the derivative of an exponential function is an exponential function.



3. The graph below shows the new cases per day (P' or dP/dt) vs. the total number of cases (P). This graph is called a Phase Plane. Based on the information on this graph, write an equation for P' as a function of P. The number of cases on Sept. 1 in Arizona was 6,850. Solve this differential equation to find P as a function of t.



$$\frac{dP}{dt} = .03P$$

$$\int \frac{dP}{P} = \int .03\,dt$$

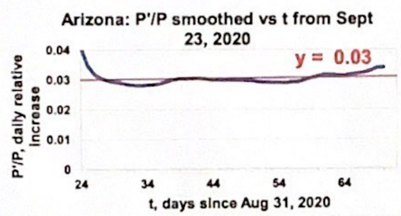
$$\ln P = .03t + C$$

$$\ln 6850 = C$$

$$P = \left(e^{.03t}\right)e^{\ln 6850}$$

$$P = 6850\,e^{.03t}$$

4. The graph below shows the daily relative increase in new cases (P'/P) vs. the total number of cases (P). Write an equation for P'/P as a function of P. Solve this differential equation.

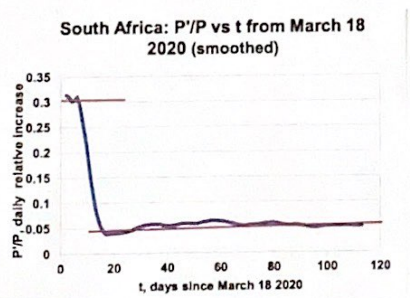


$$\frac{\frac{dP}{dt}}{P} = .03$$

$$P = 6850e^{.03t}$$

5. You should have gotten the same function for both #4 and 5. This function was a good fit for the data for these few months. Is this function a reasonable model in the long run? (This exponential model of population growth is credited to Thomas Malthus around 1800 and is known as the Malthusian model.) No, because this function increases w/o bound as t increases.

6. The graph below shows the relative daily increase is new cases (P'/P) vs. time for the first 4 months of the pandemic in South Africa starting on March 18. On March 23, South Africa imposed a strict lockdown. According to the graph, what was the effect of this lockdown?

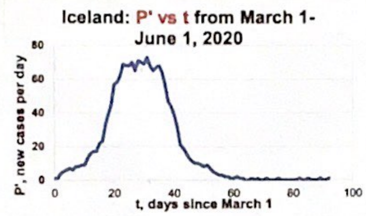


Reduced the daily relative increase in new cases from 30% to 5%.

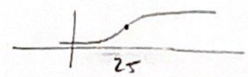
7. If there were 1600 cases in South Africa on March 23, what function would be a good model for the total number of cases over time March 23 – July 11? Do you think this will be a good model in the long run? No, see #5.

$$P = 1600e^{.05t}$$

8. The graph below represents the daily new cases in Iceland from March 1 – June 1. Based on this graph, do you think the graph of the total cases (P) vs. time for this time period will be exponential? Why or why not? If not exponential, what do you think P vs. t will look like for Iceland? Why do you think Iceland's Covid data looks like this?

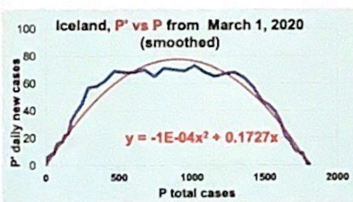


No, it will not be exponential because the daily number of new cases reaches a max after day 25 the decreses to 0.



Iceland is an island. It is also run by a woman!! :)

9. Below is the phase plane of the daily new cases in Iceland (P') vs. the total cases (P). A quadratic model has been fit to this data. Factor the quadratic model by factoring out 0.1727x to put it in the form of rx(1-x). Then write an equation to represent P' as a function of P. This model was first devised by Pierre Francois Verhulst in 1838 and is known as the Logistic Model.



$$\frac{dP}{dt} = .1727P\left(1 - \frac{P}{1727}\right)$$

10. Looking at the quadratic model in #9, what is the vertex of the parabola? What does this point tell you about the total number of cases over the long run? $P = \frac{-.1727}{-2 \cdot .0001} = \frac{1727}{2} =$

P will approach 1727 as $t \to \infty$. Because of symmetry.

11. Looking at the graph in #8 and your answer to #10, what point do you know exists on the graph of P vs. t? $\left(25, \frac{1727}{2}\right)$

12. Using the point you found in #11, solve the differential equation in # 9.

see next page

13. On TINspire, graph the Covid Data with the function P(t) you found in #12.

$$\frac{dP}{dt} = .1727P\left(1 - \frac{P}{1727}\right)$$

$$\frac{dP}{P\left(1 - \frac{P}{1727}\right)} = .1727\,dt$$

$$\left[\frac{1}{P\left(1 - \frac{P}{1727}\right)} = \frac{A}{P} + \frac{B}{1 - \frac{P}{1727}}\right] P\left(1 - \frac{P}{1727}\right)$$

$$1 = A\left(1 - \frac{P}{1727}\right) + BP$$

$$1 = A - A\frac{P}{1727} + BP$$

so: $A = 1$

$$\frac{-A}{1727} + B = 0$$

$$B = \frac{1}{1727}$$

$$\int \left[\frac{1}{P} + \frac{\frac{1}{1727}}{1 - \frac{P}{1727}}\right] dP = \int .1727\,dt$$

$$\ln P - \ln\left(1 - \frac{P}{1727}\right) = .1727t + C$$

$$\ln \frac{P}{1 - \frac{P}{1727}} = .1727t + C$$

$$\frac{P}{1 - \frac{P}{1727}} = e^{.1727t + C}$$

cont'd

$$P = e^{.1727t+c} - e^{.1727t+c}\frac{P}{1727}$$

$$P\left(1+\frac{e^{.1727t+c}}{1727}\right) = e^{.1727t+c}$$

$$P = \frac{e^{.1727t+c}}{1+\frac{e^{.1727t+c}}{1727}} \cdot \frac{1727e^{-(.1727t+c)}}{1727e^{-(.1727t+c)}}$$

$$P = \frac{1727}{1727e^{-.1727t+c}+1}$$

$$\frac{1727}{2} = \frac{1727}{1727e^{-.1727(25)+c}+1}$$

$$\frac{1}{2} = \frac{1}{1727e^{-.1727\cdot 25+c}+1}$$

$$e^{-.1727\cdot 25+c} = \frac{1}{1727}$$

$$-.1727(25)+c = \ln\left(\frac{1}{1727}\right)$$

$$c = \ln\frac{1}{1727} + .1727(25)$$

$$c = -3.13664$$

Better Model using $\left(29, \frac{1727}{2}\right)$

$$P = \frac{1727}{1727e^{-.1727t-2.44584}+1}$$

Logistic Model from Calculator: $P = \frac{1816.5}{1+146.258e^{-.1706t}} + 12.8147$