

2021

AP®

 CollegeBoard

AP® Calculus AB/ Calculus BC

Sample Student Responses and Scoring Commentary

DRAFT

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r (centimeters)	0	1	2	2.5	4
$f(r)$ (milligrams per square centimeter)	1	2	6	10	18

1. The density of a bacteria population in a circular petri dish at a distance r centimeters from the center of the dish is given by an increasing, differentiable function f , where $f(r)$ is measured in milligrams per square centimeter. Values of $f(r)$ for selected values of r are given in the table above.
- (a) Use the data in the table to estimate $f'(2.25)$. Using correct units, interpret the meaning of your answer in the context of this problem.
- (b) The total mass, in milligrams, of bacteria in the petri dish is given by the integral expression

$$2\pi \int_0^4 r f(r) dr.$$
 Approximate the value of $2\pi \int_0^4 r f(r) dr$ using a right Riemann sum with the four subintervals indicated by the data in the table.
- (c) Is the approximation found in part (b) an overestimate or underestimate of the total mass of bacteria in the petri dish? Explain your reasoning.
- (d) The density of bacteria in the petri dish, for $1 \leq r \leq 4$, is modeled by the function g defined by

$$g(r) = 2 - 16(\cos(1.57\sqrt{r}))^3.$$
 For what value of k , $1 < k < 4$, is $g(k)$ equal to the average value of $g(r)$ on the interval $1 \leq r \leq 4$?

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

Part A (AB or BC): Graphing calculator required**Question 1****9 points****General Scoring Notes**

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Scoring guidelines and notes contain examples of the most common approaches seen in student responses. These guidelines can be applied to alternate approaches to ensure that these alternate approaches are scored appropriately.

r (centimeters)	0	1	2	2.5	4
$f(r)$ (milligrams per square centimeter)	1	2	6	10	18

The density of a bacteria population in a circular petri dish at a distance r centimeters from the center of the dish is given by an increasing, differentiable function f , where $f(r)$ is measured in milligrams per square centimeter. Values of $f(r)$ for selected values of r are given in the table above.

Model Solution**Scoring**

- (a) Use the data in the table to estimate $f'(2.25)$. Using correct units, interpret the meaning of your answer in the context of this problem.

$$f'(2.25) \approx \frac{f(2.5) - f(2)}{2.5 - 2} = \frac{10 - 6}{0.5} = 8$$

Estimate

1 point

At a distance of $r = 2.25$ centimeters from the center of the petri dish, the density of the bacteria population is increasing at a rate of 8 milligrams per square centimeter per centimeter.

Interpretation with units

1 point

Scoring notes:

- To earn the first point, the response must provide both a difference and a quotient and must explicitly use values of f from the table.
- Simplification of the numerical value is not required to earn the first point. If the numerical value is simplified, it must be correct.
- The interpretation requires all of the following: distance $r = 2.25$, density of bacteria (population) is increasing or changing, at a rate of 8, and units of milligrams per square centimeter per centimeter.
- The second point (interpretation) cannot be earned without a nonzero presented value for $f'(2.25)$.
- To earn the second point, the interpretation must be consistent with the presented nonzero value for $f'(2.25)$. In particular, if the presented value for $f'(2.25)$ is negative, the interpretation must include “decreasing at a rate of $|f'(2.25)|$ ” or “changing at a rate of $f'(2.25)$.” The second point cannot be earned for an incorrect statement such as “the bacteria density is decreasing at a rate of $-8 \dots$ ” even for a presented $f'(2.25) = -8$.
- The units ($\text{mg/cm}^2/\text{cm}$) may be attached to the estimate of $f'(2.25)$ and if so, do not need to be repeated in the interpretation.
- If units attached to the estimate do not agree with units in the interpretation, read the units in the interpretation.

Total for part (a) 2 points

- (b) The total mass, in milligrams, of bacteria in the petri dish is given by the integral expression $2\pi \int_0^4 r f(r) dr$. Approximate the value of $2\pi \int_0^4 r f(r) dr$ using a right Riemann sum with the four subintervals indicated by the data in the table.

$2\pi \int_0^4 r f(r) dr \approx 2\pi(1 \cdot f(1) \cdot (1 - 0) + 2 \cdot f(2) \cdot (2 - 1) + 2.5 \cdot f(2.5) \cdot (2.5 - 2) + 4 \cdot f(4) \cdot (4 - 2.5))$	Right Riemann sum setup	1 point
$= 2\pi(1 \cdot 2 \cdot 1 + 2 \cdot 6 \cdot 1 + 2.5 \cdot 10 \cdot 0.5 + 4 \cdot 18 \cdot 1.5)$ $= 269\pi = 845.088$	Approximation	1 point

Scoring notes:

- The presence or absence of 2π has no bearing on earning the first point.
- The first point is earned for a sum of 4 products with at most one error in any single value among the four products. Multiplication by 1 in any term does not need to be shown, but all other products must be explicitly shown.
- A response of $1 \cdot f(1) \cdot (1 - 0) + 2 \cdot f(2) \cdot (2 - 1) + 2.5 \cdot f(2.5) \cdot (2.5 - 2) + 4 \cdot f(4) \cdot (4 - 2.5)$ earns the first point but not the second.
- A response with any error in the Riemann sum is not eligible for the second point.
- A response that provides a completely correct left Riemann sum for $2\pi \int_0^4 r f(r) dr$ and approximation (91π) earns one of the two points. A response that has any error in a left Riemann sum or evaluation for $2\pi \int_0^4 r f(r) dr$ earns no points.
- A response that provides a completely correct right Riemann sum for $2\pi \int_0^4 f(r) dr$ and approximation (80π) earns one of the two points. A response that has any error in a right Riemann sum or evaluation for $2\pi \int_0^4 f(r) dr$ earns no points.
- Simplification of the numerical value is not required to earn the second point. If a numerical value is given, it must be correct to three decimal places.

Total for part (b) 2 points

- (c) Is the approximation found in part (b) an overestimate or underestimate of the total mass of bacteria in the petri dish? Explain your reasoning.

$$\frac{d}{dr}(rf(r)) = f(r) + rf'(r)$$

Product rule
expression for

1 point

$$\frac{d}{dr}(rf(r))$$

Because f is nonnegative and increasing, $\frac{d}{dr}(rf(r)) > 0$ on the interval $0 \leq r \leq 4$. Thus, the integrand $rf(r)$ is strictly increasing.

Answer with explanation

1 point

Therefore, the right Riemann sum approximation of $2\pi \int_0^4 rf(r) dr$ is an overestimate.

Scoring notes:

- To earn the second point, a response must explain that $rf(r)$ is increasing, and therefore the right Riemann sum is an overestimate. The second point can be earned without having earned the first point.
- A response that attempts to explain based on a left Riemann sum for $2\pi \int_0^4 rf(r) dr$ from part (b) earns no points.
- A response that attempts to explain based on a right Riemann sum for $2\pi \int_0^4 f(r) dr$ from part (b) earns no points.

Total for part (c) 2 points

- (d) The density of bacteria in the petri dish, for $1 \leq r \leq 4$, is modeled by the function g defined by $g(r) = 2 - 16(\cos(1.57\sqrt{r}))^3$. For what value of k , $1 < k < 4$, is $g(k)$ equal to the average value of $g(r)$ on the interval $1 \leq r \leq 4$?

Average value = $g_{\text{avg}} = \frac{1}{4-1} \int_1^4 g(r) dr$	Definite integral	1 point
$\frac{1}{4-1} \int_1^4 g(r) dr = 9.875795$	Average value	1 point
$g(k) = g_{\text{avg}} \Rightarrow k = 2.497$	Answer	1 point

Scoring notes:

- The first point is earned for a definite integral, with or without $\frac{1}{4-1}$ or $\frac{1}{3}$.
- A response that presents a definite integral with incorrect limits but a correct integrand earns the first point.
- Presentation of the numerical value 9.875795 is not required to earn the second point. This point can be earned by the average value setup: $\frac{1}{3} \int_1^4 g(r) dr$.
- Once earned for the average value setup, the second point cannot be lost. Subsequent errors will result in not earning the third point.
- The third point is earned only for the value $k = 2.497$.
- The third point cannot be earned without the second.
- Special case: A response that does not provide the average value setup but presents an average value of -13.955 is using degree mode on their calculator. This response would not earn the second point, but could earn the third point for an answer of $k = 2.5$ (or 2.499).

Total for part (d)	3 points
Total for question 1	9 points

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Answer QUESTION 1 parts (a) and (b) on this page.

r (centimeters)	0	1	2	2.5	4
$f(r)$ (milligrams per square centimeter)	1	2	6	10	18

Response for question 1(a)

$$f'(2.25) \approx \frac{f(2.5) - f(2)}{2.5 - 2} = \frac{10 - 6}{0.5} = \frac{4}{0.5} = 8 \text{ milligrams per square centimeter per centimeter}$$

The density of bacteria changes at a rate of approximately 8 milligrams per square centimeter per centimeter at distance $r = 2.25$ centimeters from the center of the dish

Response for question 1(b)

$$2\pi \int_0^4 r \cdot f(r) dr \approx 2\pi (2 \cdot 1 + 6 \cdot 1.2 + 10 \cdot 0.5 \cdot 2.5 + 18 \cdot 1.5 \cdot 4) \\ = 2\pi (2 + 12 + 12.5 + 108) = 2\pi (134.5) = 269\pi \text{ milligrams}$$

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Answer QUESTION 1 parts (c) and (d) on this page.

Response for question 1(c)

As a rule, Right riemann Sums are always an over estimate for functions with positive slope, and underestimates for functions with negative slope. The slope of $r \cdot f(r)$ is equal to $r \cdot f(r) + r \cdot f'(r)$. Since r , r' , $f(r)$, and $f'(r)$ are always positive on the interval $[0,4]$, $r \cdot f(r)$ always has a positive slope on that interval. Since it's a positive sloped function, the right riemann sum for $r \cdot f(r)$ from 0 to 4 is an over estimate.

Response for question 1(d)

$$\text{Avg of } g(r) = \frac{1}{4-1} \int_1^4 (2 - 16(\cos(1.57\sqrt{r}))^3) dr = \frac{29.627}{3} = 9.876$$
$$g(k) = 2 - 16(\cos(1.57\sqrt{k}))^3 = 9.876$$
$$k = 2.497$$



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Answer QUESTION 1 parts (a) and (b) on this page.

r (centimeters)	0	1	2	2.5	4
$f(r)$ (milligrams per square centimeter)	1	2	6	10	18

Response for question 1(a)

$$f'(2.25) \approx \frac{f(2.5) - f(2)}{2.5 - 2} = \frac{10 - 6}{0.5} = 8$$

$$f'(2.25) \approx 8 \text{ mg/cm}^2/\text{cm}$$

8 milligrams per square centimeter per centimeter is the rate at which the density of bacteria is changing moving from the center to the edge of the dish when $r = 2.25$ centimeters from the center.

Response for question 1(b)

$$M = 2\pi \int_0^4 r f(r) dr$$

$$\approx 2\pi \sum r f(r) \Delta r = 2\pi \left(1(2)(1-0) + 2(6)(2-1) + 2.5(10)(2.5-2) + 4(18)(4-2.5) \right)$$

$$M \approx 845.088 \text{ mg} \quad (\text{or } 269\pi \text{ mg})$$

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Answer QUESTION 1 parts (c) and (d) on this page.

Response for question 1(c)



The approximation is an overestimation, because $\theta(r)$ is an increasing function, and $r > 0$, so $\theta(r)$ is an increasing function, and right Riemann sums always overapproximate increasing functions.

Response for question 1(d)

$$g(r) = 2 - 16(\cos(1.57\sqrt{r}))^3$$

$$g_a = \frac{1}{4} \int_1^4 g(r) dr = g(k)$$

$$\frac{1}{3} \int_1^4 (2 - 16(\cos(1.57\sqrt{r}))^3) dr = 2 - 16(\cos(1.57\sqrt{k}))^3$$

$$9.876 = 2 - 16(\cos(1.57\sqrt{k}))^3$$

$$k = 2.797 \text{ km}$$



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Answer QUESTION 1 parts (a) and (b) on this page.

r (centimeters)	0	1	2	2.5	4
$f(r)$ (milligrams per square centimeter)	1	2	6	10	18

Response for question 1(a)

$$f'(2.25) \approx \frac{f(2.5) - f(2)}{2.5 - 2} = \frac{10 - 6}{.5} = 8 \text{ milligrams per square cm per cm, which is the rate of change of the population density of a bacteria population at } r = 2.25 \text{ cm away from the center of a petri dish}$$

Response for question 1(b)

$$\begin{aligned} & \cancel{2\pi \int_0^4 r f(r) dr} \\ & 2\pi \left[1(f(1)) + 1(f(2)) + .5(f(2.5)) + 1.5(f(4)) \right] \\ & = 2\pi (2 + 12 + 12.5 + 108) \\ & = 845.088 \text{ milligrams} \end{aligned}$$

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Answer QUESTION 1 parts (c) and (d) on this page.

Response for question 1(c)

The approximation in part (b)
is an overestimate, as $f(r)$ is
increasing on the interval $(0, 4)$,
and thus the right Riemann
sum only accounts for the larger
of two values.

Response for question 1(d)

$$\text{average value} = \int_1^4 g(r) dr \left(\frac{1}{4-1} \right) = \frac{1}{3}(29.6273\ldots) \\ = 9.876$$

$$g(k) = 9.876 = 2 - 16(\cos(1.57\sqrt{k}))^3$$

$$k = 2.497 \text{ cm}$$



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Answer QUESTION 1 parts (a) and (b) on this page.

r (centimeters)	0	1	2	2.5	4
$f(r)$ (milligrams per square centimeter)	1	2	6	10	18

Response for question 1(a)

$$f'(2.25) \approx \frac{f(2.5) - f(2)}{2.5 - 2} = \frac{10 - 6}{0.5} = 8 \text{ milligrams/cm}^3$$

The rate of change of $f(r)$ at distance $r = 2.25$ centimeters
is approximately 8 milligrams per cubic centimeter

Response for question 1(b)

$$2\pi \int_0^4 r f(r) dr \approx 2\pi [1(1)f(1) + 1(2)f(2) + 0.5(2.5)f(2.5) + 1.5(4)f(4)] \\ \approx 2\pi [2 + 12 + 12.5 + 108] = 269\pi$$

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Answer QUESTION 1 parts (c) and (d) on this page.

Response for question 1(c)

for $[0, 4]$, $f(r)$ is increasing (as is r)

so $r f(r)$ is increasing.

As a right Riemann sum was used
to take the integral of an increasing
function, it was an overestimate

Response for question 1(d)

$$\text{Avg. value of } g(r) \text{ on } [1, 4] = \frac{1}{4-1} \int_1^4 (2 - 16(\cos(1.57\sqrt{r})))^3 dr = 9.87579487$$

$$2 - 16(\cos(1.57\sqrt{K}))^3 = 9.875794868$$

at $K = 2.497$ centimeters

0029172



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Answer QUESTION 1 parts (a) and (b) on this page.

r (centimeters)	0	1	2	2.5	4
$f(r)$ (milligrams per square centimeter)	1	2	6	10	18

Response for question 1(a)

$$f'(2.25) \approx \frac{f(2.5) - f(2)}{2.5 - 2} = \frac{10 - 6}{2.5 - 2} = \frac{4}{0.5} = 8$$

$f'(2.25) \approx 8$, meaning that where $r = 2.25$ centimeters, the density of bacteria in the dish is increasing at a rate of approximately 8 milligrams per cm^2 per cm as the radius increases.

Response for question 1(b)

$$2\pi \int_0^4 r f(r) dr \approx 2\pi [1(1.2) + 1(2.6) + 0.5(2.5 \cdot 10) + 1.5(4 \cdot 18)] \\ = 245\pi = 769.690$$

$$2\pi \int_0^4 r f(r) dr \approx 769.690$$

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Answer QUESTION 1 parts (c) and (d) on this page.

Response for question 1(c)

The approximation found in part (b) is an overestimate of the total mass of bacteria in the petri dish because $f(r)$ is increasing for all r , so a right Riemann sum uses the highest value on each subinterval and thus overestimates the value.

Response for question 1(d)

Average value of $g(r)$ on $1 \leq r \leq 4$

$$= \frac{1}{3} \int_1^4 g(r) dr = 9.8758$$

$g(k) = 9.8758$: Find r -value where $g(r)$ and $g = 9.8758$ intersect on $1 < r < 4$

$$k = 2.497$$



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Answer QUESTION 1 parts (a) and (b) on this page.

r (centimeters)	0	1	2	2.5	4
$f(r)$ (milligrams per square centimeter)	1	2	6	10	18

Response for question 1(a)

$$f'(2.25) \approx \frac{f(2.5) - f(2)}{2.5 - 2} = \frac{10 - 6}{2.5 - 2} = 8 \text{ milligrams per square centimeter per centimeter}$$

The density of the bacteria population is increasing at a rate of 8 milligrams per square centimeter per centimeter when $r=2.25$.

Response for question 1(b)

$$2\pi \int_0^4 rf(r) \approx 2\pi [1(2) + 1(6) + 0.5(10) + 1.5(18)] = 261.327$$

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Answer QUESTION 1 parts (c) and (d) on this page.

Response for question 1(c)

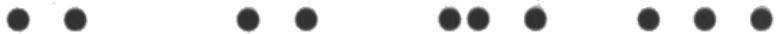
The approximation found in part b is an underestimate because $f(r)$ is increasing on $0 \leq r \leq 4$.

Response for question 1(d)

$$\text{avg. value of } g(r) = \frac{1}{4-1} \int_1^4 g(r) dr = 9.875794868$$

$$9.875 = 2 - 16(\cos(1.57\sqrt{K}))^3$$

$$K = 2.497$$



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Answer QUESTION 1 parts (a) and (b) on this page.

r (centimeters)	0	1	2	2.5	4
$f(r)$ (milligrams per square centimeter)	1	2	6	10	18

Response for question 1(a)

$$f'(2.25) \approx \frac{f(2.5) - f(2)}{2.5 - 2} = \frac{10 - 6}{0.5} = \frac{4}{0.5} = 8 \text{ mg/cm}^3$$

The rate of change of ^{the amount of} bacteria in a circular petri dish
at 2.25 cm is 8 mg/cm^3 .

Response for question 1(b)

$$2\pi \int_0^4 r f(r) dr \\ = 2\pi (40) = 80\pi \text{ mg}$$

Right Riemann Sum

$$1(2) + 1(4) + 0.5(10) + 1.5(18) = 40$$

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Answer QUESTION 1 parts (c) and (d) on this page.

Response for question 1(c)



The approximation is an overestimate because the curve $f(r)$ is increasing and concave up, and a right Riemann sum was used.

Response for question 1(d)

$$\frac{1}{4-1} \int_1^4 g(r) dr = \frac{1}{3} \int_1^4 2-14(\cos 1.57\pi r)^3 dr \\ = 9.876$$

$$g(k) = 2-14(\cos(1.57\pi k))^3 = 9.876 \\ k=2.497$$



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Answer QUESTION 1 parts (a) and (b) on this page.

r (centimeters)	0	1	2	2.5	4
$f(r)$ (milligrams per square centimeter)	1	2	6	10	18

Response for question 1(a)

$$f(2.25) = \frac{f(2.5) - f(2)}{2.5 - 2} = \frac{10 - 6}{2.5 - 2} = 8$$

$$f'(2.25) = 8 \text{ mg/cm}^2 \quad \text{rate at which density is increasing at } 2.25 \text{ cm.}$$

Response for question 1(b)

$$\begin{aligned} & 2\pi \int_0^4 r f(r) dr \\ & 2\pi \left((1)(2) + (1)(6) + .5(10) + 1.5(18) \right) \\ & 2\pi(40) = \boxed{80\pi} \end{aligned}$$

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Answer QUESTION 1 parts (c) and (d) on this page.

Response for question 1(c)

overestimate because the function
is increasing and right riemann
sum was used.

Response for question 1(d)

$$\frac{1}{4-1} \int_0^4 g(r) = g(k)$$

$$\frac{1}{3}(29.627) = 2 - 16(\cos(1.57\sqrt{k}))^3$$
$$9.87566 = 2 - 16(\cos(1.57)\sqrt{k})^3$$
$$\boxed{k = 2.497}$$



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Answer QUESTION 1 parts (a) and (b) on this page.

r (centimeters)	0	1	2	2.5	4
$f(r)$ (milligrams per square centimeter)	1	2	6	10	18

Response for question 1(a)

$$f'(2.25) = \frac{f(2.5) - f(2)}{2.5 - 2} \rightarrow \frac{10 - 6}{2.5 - 2} = \frac{4}{0.5} = 8$$

$$f'(2.25) = 8 \text{ mg per cm}^2 \text{ per cm}^2$$

At $r=2.25$, the rate of the density of the bacteria population
is increasing at a rate of $8 \text{ mg per cm}^2 \text{ per cm}^2$.

Response for question 1(b)

$$2\pi \cdot [1(2) + 1(6) + 0.5(10) + 1.5(18)]$$

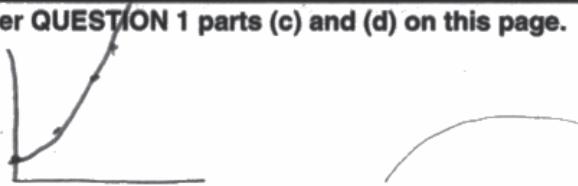
$$2\pi \cdot (2 + 6 + 5 + 27)$$

$$2\pi(40) = \boxed{80\pi = 251.327}$$

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Answer QUESTION 1 parts (c) and (d) on this page.

Response for question 1(c)



The right Riemann sum approximation is an overestimate due to the fact that the total mass of the bacteria is increasing since its represented by $f(r)$ which is the function of the bacteria's density which is an increasing differentiable function.

Response for question 1(d)

$$g(r) = 2 - 16(\cos(1.57\sqrt{r}))^3$$

$$\frac{1}{4-1} \int_1^4 g(r) dr = 4,875794869$$

$$g'(k) = 9,875794869$$

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Answer QUESTION 1 parts (a) and (b) on this page.

r (centimeters)	0	1	2	2.5	4
$f(r)$ (milligrams per square centimeter)	1	2	6	10	18

Response for question 1(a)

$$\frac{10-6}{2.5-2} = 8 \text{ milligrams per centimeter}^3$$

Response for question 1(b)

$$1(2) + 1(6) + 0.5(18) = 40$$

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Answer QUESTION 1 parts (c) and (d) on this page.

Response for question 1(c)

Since $f(r)$ is always positive, the approximation in part (b) is an overestimate.

Response for question 1(d)

$$0 < \frac{1}{4-1} \int_1^4 g(r) dr = g(k)$$

$$g(k) = 9.8758$$

$$k = 2.497$$



Question 1**Sample Identifier: A****Score: 9**

- The response earned 9 points: 2 points in part (a), 2 points in part (b), 2 points in part (c), and 3 points in part (d).
- In part (a) the difference quotient of $\frac{10 - 6}{0.5}$ in the first line would have earned the first point with no simplification. In this case, correct simplification to 8 in the first line earned the first point. The response earned the second point for an interpretation of the density of the bacteria changing at 8 milligrams per square centimeter per centimeter at $r = 2.25$.
- In part (b) the response earned the first point for the sum of products expression $2\pi \cdot (2 \cdot 1 \cdot 1 + 6 \cdot 1 \cdot 2 + 10 \cdot 0.5 \cdot 2.5 + 18 \cdot 1.5 \cdot 4)$ in the first line on the right. This sum of products expression would also have earned the second point with no simplification. In this case, correct simplification to 269π in the second line earned the second point.
- In part (c) the response earned the first point for the product rule expression of $r' \cdot f(r) + r \cdot f'(r)$ for $\frac{d}{dr}(rf(r))$ in the fourth line. The response earned the second point for the conclusion that $rf(r)$ has a positive slope since r , r' , $f(r)$, and $f'(r)$ are positive on the interval and therefore the estimate is an overestimate.
- In part (d) the response earned the first and second points for the definite integral $\frac{1}{4-1} \int_1^4 (2 - 16(\cos(1.57\sqrt{r})))^3 dr$ giving the average value in the first line. The response earned the third point for the correct value of $k = 2.497$ in the third line.

Sample Identifier: B**Score: 8**

- The response earned 8 points: 2 points in part (a), 2 points in part (b), 1 point in part (c), and 3 points in part (d).
- In part (a) the difference quotient of $\frac{10 - 6}{0.5}$ in the first line would have earned the first point with no simplification. In this case, correct simplification to 8 in the first line earned the first point. The response earned the second point for an interpretation of 8 milligrams per square centimeter per centimeter being the rate at which the density of the bacteria is changing when $r = 2.25$.
- In part (b) the response earned the first point for the sum of products expression $2\pi(1(2)(1 - 0) + 2(6)(2 - 1) + 2.5(10)(2.5 - 2) + 4(18)(4 - 2.5))$ in the second line. This sum of products expression also would have earned the second point with no simplification. In this case, correct simplification to 845.088 in the third line earned the second point.
- In part (c) the response did not earn the first point because there is no product rule expression for $\frac{d}{dr}(rf(r))$. The response earned the second point for the claim that $rf(r)$ is an increasing function in the second line and therefore the approximation is an overestimate.
- In part (d) the response earned the first and second points for the definite integral $\frac{1}{4-1} \int_1^4 g(r) dr$ giving the average value in the second line. The response earned the third point for the correct value of $k = 2.497$ in the fifth line.

Question 1 (continued)**Sample Identifier: C****Score: 7**

- The response earned 7 points: 2 points in part (a), 2 points in part (b), no points in part (c), and 3 points in part (d).
- In part (a) the difference quotient of $\frac{10 - 6}{.5}$ in the first line would have earned the first point with no simplification. In this case, correct simplification to 8 in the first line earned the first point. The response earned the second point for an interpretation of “8 milligrams per square cm per cm, which is the rate of change of the population density of a bacteria population at $r = 2.25$ cm away from the center.”
- In part (b) the response earned the first point for the sum of products expression $2\pi(1((1) \cdot f(1)) + 1((2)(f(2))) + .5(2.5f(2.5)) + 1.5(4(f(4)))$ in the third line. The response earned the second point with correct simplification to 845.088 in the fourth line.
- In part (c) the response did not earn the first point because there is no product rule expression for $\frac{d}{dr}(rf(r))$. The response did not earn the second point because there is no claim that $rf(r)$ is increasing.
- In part (d) the response earned the first and second points for the definite integral $\int_1^4 g(r) dr \left(\frac{1}{4-1}\right)$ giving the average value in the first line. The response earned the third point for the correct value of $k = 2.497$ in the fourth line.

Sample Identifier: D**Score: 7**

- The response earned 7 points: 1 point in part (a), 2 points in part (b), 1 point in part (c), and 3 points in part (d).
- In part (a) the difference quotient of $\frac{10 - 6}{0.5}$ in the first line would have earned the first point with no simplification. In this case, correct simplification to 8 in the first line earned the first point. The response did not earn the second point because the density of the bacteria is not referenced.
- In part (b) the response earned the first point for the sum of products expression $2\pi[1(1)f(1) + 1(2)f(2) + 0.5(2.5)f(2.5) + 1.5(4)f(4)]$ in the first line. The sum of products expression $2\pi[2 + 12 + 12.5 + 108]$ in the second line would have earned the second point with no simplification. In this case, simplification to 269π in the second line earned the second point.
- In part (c) the response did not earn the first point because there is no product rule expression for $\frac{d}{dr}(rf(r))$. The response earned the second point for the claim that $rf(r)$ is increasing in the second line and therefore the right Riemann sum is an overestimate in the fifth line.
- In part (d) the response earned the first and second points for the definite integral $\frac{1}{4-1} \int_1^4 (2 - 16(\cos(1.57\sqrt{r}))^3) dr$ giving the average value in the first line. The response earned the third point for the correct value of $k = 2.497$ in the third line.

Question 1 (continued)**Sample Identifier: E****Score: 6**

- The response earned 6 points: 2 points in part (a), 1 point in part (b), no points in part (c), and 3 points in part (d).
- In part (a) the difference quotient of $\frac{10 - 6}{2.5 - 2}$ in the first line would have earned the first point with no simplification. In this case, correct simplification to 8 in the first line earned the first point. The response earned the second point for an interpretation of the density of bacteria in the dish increasing at a rate of 8 milligrams per cm^2 per cm at $r = 2.25$ centimeters.
- In part (b) the response earned the first point for the sum of products expression $2\pi[1(1 \cdot 2) + 1(2 \cdot 6) + 0.5(2.5 \cdot 10) + 1.5(4 \cdot 18)]$ in the first line on the right. The response did not earn the second point because the simplification to 769.690 is incorrect.
- In part (c) the response did not earn the first point because there is no product rule expression for $\frac{d}{dr}(rf(r))$. The response did not earn the second point because there is no claim that $rf(r)$ is increasing.
- In part (d) the response earned the first and second points for the definite integral $\frac{1}{3} \int_1^4 g(r) dr$ giving the average value in the second line. The response earned the third point for the correct value of $k = 2.497$ in the fifth line.

Sample Identifier: F**Score: 6**

- The response earned 6 points: 2 points in part (a), 1 point in part (b), no points in part (c), and 3 points in part (d).
- In part (a) the difference quotient of $\frac{10 - 6}{2.5 - 2}$ in the first line would have earned the first point with no simplification. In this case, correct simplification to 8 in the first line earned the first point. The response earned the second point for an interpretation of density increasing at a rate of 8 milligrams per square centimeter per centimeter when $r = 2.25$.
- In part (b) the response earned one of the two points for providing a completely correct right Riemann sum for $2\pi \int_0^4 f(r) dr$. The sum of products expression $2\pi[1(2) + 1(6) + 0.5(10) + 1.5(18)]$ in the middle of the first line would have earned one of the two points with no simplification. In this case, correct simplification to 251.327 earned one of the two points.
- In part (c) the response did not earn either point because the response attempted to explain a right Riemann sum for $2\pi \int_0^4 f(r) dr$.
- In part (d) the response earned the first and second points for the definite integral $\frac{1}{4 - 1} \int_1^4 g(r) dr$ giving the average value in the first line. The response earned the third point for the correct value of $k = 2.497$ in the third line.

Question 1 (continued)**Sample Identifier: G****Score: 5**

- The response earned 5 points: 1 point in part (a), 1 point in part (b), no points in part (c), and 3 points in part (d).
- In part (a) the difference quotient of $\frac{10 - 6}{2.5 - 2}$ in the first line would have earned the first point with no simplification. In this case, correct simplification to 8 in the first line earned the first point. The response did not earn the second point because the response references the amount of bacteria rather than the density of bacteria.
- In part (b) the response earned one of the two points for providing a completely correct right Riemann sum for $2\pi \int_0^4 f(r) dr$, with the sum of products expression $1(2) + 1(6) + 0.5(10) + 1.5(18)$ in the second line on the right and the numerical value of 80π in the second line on the left earning one of the two points.
- In part (c) the response did not earn either point because the response attempted to explain a right Riemann sum for $2\pi \int_0^4 f(r) dr$.
- In part (d) the response earned the first and second points for the definite integral $\frac{1}{4-1} \int_1^4 g(r) dr$ giving the average value in the first line. The response earned the third point for the correct value of $k = 2.497$ in the fourth line.

Sample Identifier: H**Score: 5**

- The response earned 5 points: 1 point in part (a), 1 point in part (b), no points in part (c), and 3 points in part (d).
- In part (a) the difference quotient of $\frac{10 - 6}{2.5 - 2}$ in the first line would have earned the first point with no simplification. In this case, correct simplification to 8 in the first line earned the first point. The response did not earn the second point because the units on the interpretation are incorrect.
- In part (b) the response earned one of the two points for providing a completely correct right Riemann sum for $2\pi \int_0^4 f(r) dr$. The sum of products expression $2\pi((1)(2) + (1)(6) + .5(10) + 1.5(18)))$ in the second line would have earned one of the two points with no simplification. In this case, correct simplification to 80π in the third line earned one of the two points.
- In part (c) the response did not earn the first point because there is no product rule expression for $\frac{d}{dr}(rf(r))$. Reference to “the function” being an increasing function does not clearly indicate that “the function” refers to $rf(r)$ and therefore the second point is not earned.
- In part (d) the response earned the first and second points for the definite integral $\frac{1}{4-1} \int_1^4 g(r) dr$ giving the average value in the first line. The response earned the third point for the correct value of $k = 2.497$ in the fourth line.

Question 1 (continued)**Sample Identifier: I****Score: 4**

- The response earned 4 points: 1 point in part (a), 1 point in part (b), no points in part (c), and 2 points in part (d).
- In part (a) the difference quotient of $\frac{10 - 6}{2.5 - 2}$ in the first line would have earned the first point with no simplification. In this case, correct simplification to 8 in the first line earned the first point. The response did not earn the second point because the response states the rate of the density of the bacteria population is increasing at a rate and because the units in the interpretation are incorrect.
- In part (b) the response earned one of the two points for providing a completely correct right Riemann sum for $2\pi \int_0^4 f(r) dr$. The sum of products expression $2\pi \cdot [1(2) + 1(6) + 0.5(10) + 1.5(18)]$ in the first line would have earned one of the two points with no simplification. In this case, correct simplification to 251.327 in the third line earned one of the two points.
- In part (c) the response did not earn either point because the response attempted to explain a right Riemann sum for $2\pi \int_0^4 f(r) dr$.
- In part (d) the response earned the first and second points for the definite integral $\frac{1}{4-1} \int_1^4 g(r) dr$ giving the average value in the second line. The response did not earn the third point because no value is given for k .

Sample Identifier: J**Score: 4**

- The response earned 4 points: 1 point in part (a), no points in part (b), no points in part (c), and 3 points in part (d).
- In part (a) the difference quotient of $\frac{10 - 6}{2.5 - 2}$ in the first line would have earned the first point with no simplification. In this case, correct simplification to 8 in the first line earned the first point. The response did not earn the second point because the interpretation does not state that the density of bacteria is increasing when $r = 2.25$.
- In part (b) the response did not earn the first point because there is no sum of four products. The response contains an error in the Riemann sum and therefore is not eligible for the second point.
- In part (c) the response did not earn the first point because there is no product rule expression for $\frac{d}{dr}(rf(r))$. The response did not earn the second point because there is no claim that $rf(r)$ is increasing.
- In part (d) the response earned the first and second points for the definite integral $\frac{1}{4-1} \int_1^4 g(r) dr$ giving the average value in the first line. The response earned the third point for the correct value of $k = 2.497$ in the third line.

2. A particle, P , is moving along the x -axis. The velocity of particle P at time t is given by $v_P(t) = \sin\left(t^{1.5}\right)$ for $0 \leq t \leq \pi$. At time $t = 0$, particle P is at position $x = 5$.

A second particle, Q , also moves along the x -axis. The velocity of particle Q at time t is given by $v_Q(t) = (t - 1.8) \cdot 1.25^t$ for $0 \leq t \leq \pi$. At time $t = 0$, particle Q is at position $x = 10$.

- Find the positions of particles P and Q at time $t = 1$.
- Are particles P and Q moving toward each other or away from each other at time $t = 1$? Explain your reasoning.
- Find the acceleration of particle Q at time $t = 1$. Is the speed of particle Q increasing or decreasing at time $t = 1$? Explain your reasoning.
- Find the total distance traveled by particle P over the time interval $0 \leq t \leq \pi$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

Part A (AB): Graphing calculator required**Question 2****9 points****General Scoring Notes**

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Scoring guidelines and notes contain examples of the most common approaches seen in student responses. These guidelines can be applied to alternate approaches to ensure that these alternate approaches are scored appropriately.

A particle, P , is moving along the x -axis. The velocity of particle P at time t is given by $v_P(t) = \sin(t^{1.5})$ for $0 \leq t \leq \pi$. At time $t = 0$, particle P is at position $x = 5$.

A second particle, Q , also moves along the x -axis. The velocity of particle Q at time t is given by $v_Q(t) = (t - 1.8) \cdot 1.25^t$ for $0 \leq t \leq \pi$. At time $t = 0$, particle Q is at position $x = 10$.

Model Solution**Scoring**

- (a) Find the positions of particles P and Q at time $t = 1$.

$$x_P(1) = 5 + \int_0^1 v_P(t) dt = 5.370660$$

At time $t = 1$, the position of particle P is $x = 5.371$ (or 5.370).

One definite integral **1 point**

One position **1 point**

The other position **1 point**

$$x_Q(1) = 10 + \int_0^1 v_Q(t) dt = 8.564355$$

At time $t = 1$, the position of particle Q is $x = 8.564$.

Scoring notes:

- The first point is earned for the explicit presentation of at least one definite integral, either $\int_0^1 v_P(t) dt$ or $\int_0^1 v_Q(t) dt$.
- The first point must be earned to be eligible for the second and third points.
- The second point is earned for adding the initial condition to at least one of the definite integrals and finding the correct position.
- Writing $\int_0^1 v_P(t) + 5 = 5.370660$ does not earn a position point, because the missing dt makes this statement unclear or false. However, $5 + \int_0^1 v_P(t) dt = 5.370660$ does earn the position point because it is not ambiguous. Similarly, for the position of Q .
- Read unlabeled answers presented left to right, or top to bottom, as $x_P(1)$ and $x_Q(1)$, respectively.
- Special case 1: A response of $x_P(1) = 5 + \int_0^a v_P(t) dt = 5.370660$ AND $x_Q(1) = 10 + \int_0^a v_Q(t) dt = 8.564355$ for $a \neq 1$ earns one point.
- Special case 2: A response of $x_P(1) = 5 + \int v_P(t) dt = 5.370660$ AND $x_Q(1) = 10 + \int v_Q(t) dt = 8.564355$ or the equivalent, never providing the definite integrals, earns one point.
- Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is generally eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode). In degree mode, $x_P(1)$ is 5.007 (or 5.006).

Total for part (a) 3 points

- (b) Are particles P and Q moving toward each other or away from each other at time $t = 1$? Explain your reasoning.

$$v_P(1) = \sin(1^{1.5}) = 0.841471 > 0$$

At time $t = 1$, particle P is moving to the right.

$$v_Q(1) = (1 - 1.8) \cdot 1.25^1 = -1 < 0$$

At time $t = 1$, particle Q is moving to the left.

At time $t = 1$, $x_P(1) < x_Q(1)$, so particle P is to the left of particle Q .

Thus, at time $t = 1$, particles P and Q are moving toward each other.

Direction of motion for one particle

1 point

Answer with explanation

1 point

Scoring notes:

- The first point is earned for using the sign of $v_P(1)$ or $v_Q(1)$ to determine the direction of motion for one of the particles. This point cannot be earned without reference to the sign of $v_P(1)$ or $v_Q(1)$.
- It is not necessary to present an explicit value for $v_P(1)$, or $v_Q(1)$, but if a value is presented, it must be correct as far as reported, up to three places after the decimal.
- Read with imported incorrect position values from part (a).
- If one or both position values were not found in part (a), but are found in part (b), the points for part (a) are not earned retroactively.
- To earn the second point, the explanation must be based on the signs of $v_P(1)$ and $v_Q(1)$ and the relative positions of P and Q at $t = 1$. References to other values of time, such as $t = 0$, are not sufficient.
- Degree mode: $v_P(1) = 0.017$ (See degree mode statement in part (a).)

Total for part (b) 2 points

- (c) Find the acceleration of particle Q at time $t = 1$. Is the speed of particle Q increasing or decreasing at time $t = 1$? Explain your reasoning.

$$a_Q(1) = v'_Q(1) = 1.026856$$

The acceleration of particle Q is 1.027 (or 1.026) at time $t = 1$.

$$v_Q(1) = -1 < 0 \text{ and } a_Q(1) > 0$$

The speed of particle Q is decreasing at time $t = 1$ because the velocity and acceleration have opposite signs.

Setup and acceleration

1 point

Speed decreasing with reason

1 point

Scoring notes:

- To earn the first point, the acceleration must be explicitly connected to v'_Q (e.g., $v'_Q(1) = 1.026856$).
- The first point is not earned for an unsupported value of 1.027 (or 1.026). The setup, $v'_Q(1)$, must be shown. Presenting only $a_Q(1) = 1.027$ (or 1.026) without indication that $v'_Q = a_Q$ is not enough to earn the first point.
- A response does not need to present a value for $v_Q(1)$; the sign is sufficient.
- To earn the second point, a response must compare the signs of a_Q and v_Q at $t = 1$. Considering only one sign is not sufficient.
- After the first point has been earned, a response declaring only “velocity and acceleration are of opposite signs at $t = 1$ so the particle is slowing down” (or equivalent) earns the second point.
- The second point may be earned without the first, as long as the response does not present an incorrect value or sign for $v_Q(1)$ and concludes the particle is slowing down because velocity and acceleration have opposite signs at $t = 1$.

Total for part (c) 2 points

- (d) Find the total distance traveled by particle P over the time interval $0 \leq t \leq \pi$.

$$\int_0^\pi |v_P(t)| dt = 1.93148$$

Over the time interval $0 \leq t \leq \pi$, the total distance traveled by particle P is 1.931.

Definite integral

1 point

Answer

1 point

Scoring notes:

- The first point is earned for $\int_0^\pi |v_P(t)| dt$.
- The first point can also be earned for a sum (or difference) of definite integrals, such as $\int_0^{2.145029} v_P(t) dt - \int_{2.145029}^\pi v_P(t) dt$, provided the response has indicated $v_P(2.145029) = 0$.
- The second point can only be earned for the correct answer.
- The unsupported value 1.931 earns no points.
- A response reporting the distance traveled by particle Q as $\int_0^\pi |v_Q(t)| dt = 3.506$ earns the first point and is not eligible for the second point.
- In degree mode, the total distance traveled is 0.122. (See degree mode statement in part (a).) In the degree mode case, the response must present $\int_0^\pi |v_P(t)| dt$ in order to earn the first point because $\int_0^\pi |v_P(t)| dt = \int_0^\pi v_P(t) dt$.

Total for part (d) 2 points

Total for question 2 9 points

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Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

$$\text{position of } P = X_p = 5 + \int_0^1 \sin(t^{1.5}) dt = 5 + 0.37066$$

$$\text{Position of } Q = X_Q = 10 + \int_0^1 v_Q(t) dt = 8.56435$$

At time $t=1$, the position of particle **P** is 5.37066 and the position of particle **Q** is 8.56435

Response for question 2(b)

$$v_Q(1) = -1 \quad X_Q(1) = 5.37066$$

$$v_p(1) = 0.84147 \quad X_p(1) = 8.56435$$

At time $t=1$ the 2 particles are moving closer to each other because $X_p(1) = 5.37066$ and $X_Q(1) = 8.56435$, which means that particle **Q** is to the right of particle **P** at $t=1$, and since particle **Q** has a negative velocity it is moving left and particle **P** has a positive velocity it is moving right, so the particles are moving toward each other at time $t=1$.

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Answer QUESTION 2 parts (c) and (d) on this page.

Response for question 2(c)

$a_Q = \text{acceleration of particle } Q$

$$a_Q = v_Q'(t) \quad v_Q'(1) = 1.02686.$$

$v_Q(1) = -1$. Since the a_Q is 1.02686 and it is positive and a_Q is negative, the speed of particle Q at time $t=1$ must be decreasing.

Response for question 2(d)

Total distance particle P travelled

$$\text{from } 0 \leq t \leq \pi = \int_0^\pi |v_p(t)| dt = 1.93148$$

The total distance particle P travelled from $0 \leq t \leq \pi$ is 1.93148.

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Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

$$s_p(1) = s_p(0) + \int_0^1 s'_p(t) dt = 5 + 0.371 = 5.371$$

$$s_q(1) = s_q(0) + \int_0^1 s'_q(t) dt = 10 - 1.436 = 8.564$$

Response for question 2(b)

Particles P and Q are moving toward each other at $t=1$ because $v_p(1)$ is positive while $v_q(1)$ is negative, so Particle P is moving to the right while Particle Q is moving to the left.

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Answer QUESTION 2 parts (c) and (d) on this page.

Response for question 2(c)

~~Velocity~~ Acceleration

$$v_Q'(1) = a_Q(1) = 1.027$$

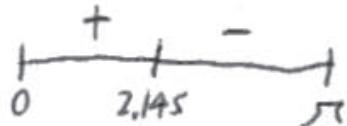
Particle Q is slowing down because its velocity is ~~positive~~ negative while its acceleration is positive,

Response for question 2(d)

~~Velocity~~

$$v_p(t) = \sin(\pi t^{1.5}) = 0$$

$$t = 2.145$$



$$\int_0^{2.145} \sin(t^{1.5}) dt - \int_{2.145}^{\pi} \sin(t^{1.5}) dt = 1.931$$

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Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

$$5 + \int_0^1 v_p(t) dt = 5.371$$

At time $t=1$, particle P is located at $x=5.371$.

$$10 + \int_0^1 v_Q(t) dt = 8.5643$$

At time $t=1$, particle Q is located at $x=8.564$.

Response for question 2(b)

$$v_p(1) = 0.8414$$

$$v_Q(1) = -1$$

Particles P and Q are moving toward each other at time $t=1$ because the position of each particle places particle P to the left of Particle Q (particle P is located at $x=5.371$ and Q at $x=8.564$); because $v_p(1) > 0$, particle P is moving ^{right} towards particle Q while $v_Q(1) < 0$, so particle Q is moving left toward particle P.

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Answer QUESTION 2 parts (c) and (d) on this page.

Response for question 2(c)

$$v_Q'(t) = a_Q(t)$$

$$a_Q(t) \Rightarrow a_Q(1) = 1.0268$$

$$a_Q(1) > 0 \quad v_Q(1)$$

$$a_Q(1) > 0 \quad v_Q(1) < 0$$

Because the acceleration and velocity of particle Q at time $t=1$ have opposite signs, the speed of the particle is decreasing.

Response for question 2(d)

$$5 + \int_0^{\pi} |v_p(t)| dt \approx 0.515 + 5$$



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Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

$$x_p(1) = 5 + \int_0^1 (\sin(t^{1.5})) dx = 5 + 0.371 = 5.371 \text{ units}$$

$$x_q(1) = 10 + \int_0^1 (1.25^b(t-1.8)) dx = 10 + (-1.436) = 8.564 \text{ units}$$

Response for question 2(b)

$$v_p(1) = 0.841 \quad a_p(1) = 0.810$$

$$v_q(1) = -1 \quad a_q(1) = 1.027$$

At $t=1$, the particles are moving away from each other as their velocities are going in different directions at $t=1$.

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Answer QUESTION 2 parts (c) and (d) on this page.

Response for question 2(c)

$$\alpha_Q(t) = \frac{v'(t)}{Q} = 1.25^t \ln 1.25(t-1.8) + 1.25^t$$

$$\alpha_Q(1) = 1.25^{(1)} \ln 1.25(1-1.8) + 1.25^1 = 1.027 \text{ units/t}^2$$

At $t=1$, the speed of Q is decreasing b/c $\alpha_Q(t)$ which is $v'_Q(t)$ is positive while $v_Q(1)$ is negative.

Response for question 2(d)

$$\int_0^n |\sin(t^{1.5})| dx = 1.931$$

total distance travelled = 1.931



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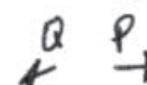
Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

$$P \cdot 5 + \int_0^1 v_P(t) dt = 5.3706028$$

$$Q \cdot 10 + \int_0^1 v_Q(t) dt = 8.584354524$$

Response for question 2(b)

P & Q are moving away from each other
At $t=1$ bc Q's $v(t)$ at $t=1$ is negative
meaning it's moving to the left while at
 $t=1$ for P it's $v(t)$ is positive meaning
it's moving to the right 
they are moving away from each other

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Answer QUESTION 2 parts (c) and (d) on this page.

Response for question 2(c)

$$q(1) = U'(1) = 1.026856478$$

Particle Q's Speed is decreasing (slowing down)
bc Q's $q(1) + U(1)$ have different signs

$$q(1) > 0 \quad U(1) < 0$$

Response for question 2(d)

$$\int_0^{T_1} |U_p(t)| dt = 1.931480748$$

2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

Particle P:

$$V_p(t) = \sin(t^{1.5})$$

$$X_p(t) = 5 + \int_0^t \sin(t^{1.5}) dt = 5.371$$

Particle Q:

$$V_q(t) = (t - 1.8) \cdot 1.25^+$$

$$X_q(t) = 10 + \int_0^t (t - 1.8) \cdot 1.25^+ dt$$

$$X_q(1) = 8.564$$

Response for question 2(b)

$$V_p(1) = \sin(1^{1.5})$$

$$V_q(1) = (1 - 1.8)(1.25')$$

$$V_p(1) = -0.8415$$

$$V_q(1) = -1$$

The particles are moving towards each other since their velocitys are different signs and they are travelling in opposite directions.

2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 parts (c) and (d) on this page.

Response for question 2(c)

$$V_Q(t) = (t - 1.8) \cdot 1.25^+ = 1.25^+(t - 1.8)$$

$$A_Q(t) = +1.25^{+1} - +2.25^{+1} 1.25^+ - 2.25^+$$

$$A_Q(1) = 1.25^0 - 1(2.25)^0$$

$$A_Q(1) = 1.25 - 1 = \boxed{0.25}$$

- it is decreasing since
the signs of acceleration
and velocity are different.

Response for question 2(d)

$$\text{total distance} = \int_0^{\pi} |V_p(t)| dt \\ = 1.931$$

2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

position of particle p:

$$5 + \int_0^1 v_p(t) dt = 5.371$$

position of particle Q:

$$10 + \int_0^1 v_q(t) dt = 8.564$$

Response for question 2(b)

The particles are moving away from each other at $t=1$ because v_p is positive and v_q is negative.

• 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 parts (c) and (d) on this page.

Response for question 2(c)

$$v_a'(1) = a_q(1) = 1.027$$

The speed is increasing because
the acceleration is positive

Response for question 2(d)

$$\int_0^{\pi} |v_p(t)| dt = 1.931$$

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2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

Position of P and Q at $t=1$

P:

$$5 + \int_0^1 \sin(t^{1.5}) dt$$

$$\approx 5.371$$

↓
Particle P

Q:

$$10 + \int_0^1 (t-1.5) \cdot 1.25^t dt$$

$$\approx 8.564$$

↓
Particle Q

Response for question 2(b)

P and Q are moving towards each other at $t=1$ because their velocity at $t=1$ is both positive.

• 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 parts (c) and (d) on this page.

Response for question 2(c)

$$V_Q(t) = (t - 1.5) \cdot 1.25^b$$

$$\ominus t=1$$

$$= 1.027$$

the particle Q is increasing at $t=1$ because the slope of the velocity is positive

Response for question 2(d)

total distance travelled

$$\int_0^\pi |\sin(t^{1.5})| dt$$

$$= 1.931$$

2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

$$P: v(t) = \sin(t^{1.5})$$

$$Q: v(t) = (t - 1.8) \cdot 1.25^t$$

$$\int_0^1 v_p(t) dt = \boxed{0.37066}$$

$$\int_0^1 v_Q(t) dt = \boxed{-1.4356}$$

Response for question 2(b)

Particles P & Q are moving away from each other at time $t=1$ because one velocity (P) is positive and one velocity (Q) is negative.

• 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 parts (c) and (d) on this page.

Response for question 2(c)

$$v(t) = \sin(t^{1.5})$$

~~$$\text{at } t=1 \quad v'(t) = a(t)$$~~

$$a(1) = 1.0269$$

The speed of particle Q ~~then~~ is decreasing at time $t=1$ because the velocity is negative and the acceleration is positive, which are different signs. $v(1) < 0$ and $a(1) > 0$.

Response for question 2(d)

$$\int_0^{\pi} |v_p(t)| dt$$

$$\int_0^{\pi} |\sin(t^{1.5})| dt$$

1.9315 = total distance

on interval $0 \leq t \leq \pi$



2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

P at time = 1

$$P(0) + \int_0^1 v_p(t) dt$$
$$5 + \int_0^1 v_p(t) dt = 5.571$$

Q at time = 1

$$Q(0) + \int_0^1 v_q(t) dt$$
$$10 + \int_0^1 v_q(t) dt = 8.564$$

Response for question 2(b)

V_P(1) = -3m/sV_Q(1) = -1

They are moving away. Particle Q and P are both moving to the right at t=1, so they are not moving closer, but rather moving away, both have a positive sign for their position when t=1.

Page 6

Use a pencil or pen with black or dark blue ink only. Do NOT write your name. Do NOT write outside the box.

• 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 parts (c) and (d) on this page.

Response for question 2(c)

$$V_Q(4) = 1.027$$

$$V_A(1) = -1$$

Speed is decreasing b/c velocity and acceleration
of particle Q at t=1 have different signs.

Response for question 2(d)

$$\int_0^{\pi} V_P(t) dt = .515$$



Question 2**Sample Identifier: A****Score: 9**

- The response earned 9 points: 3 points in part (a), 2 points in part (b), 2 points in part (c), and 2 points in part (d).
- In part (a), the response earned the first point for $\int_0^1 \sin(t^{1.5}) dt$ on line 1. The response earned the second point for $5 + \int_0^1 \sin(t^{1.5}) dt = 5 + 0.37066$ on line 1. Note that the numerical expression $5 + 0.37066$ need not be simplified. The response earned the third point for $10 + \int_0^1 v_Q(t) dt = 8.56435$ on line 2. Lines 3-5 summarize the results and contain correct information. Note the presented decimals are accurate to three decimal places, rounded or truncated.
- In part (b), the response earned the first point by stating that “since particle Q has a negative velocity it is moving left” on lines 7 and 8. The response earned the second point by stating that “ Q is to the right of particle P at $t = 1$,” “particle Q has a negative velocity it is moving left,” “particle P has a positive velocity it is moving right, so the particles are moving toward each other at time $t = 1$ ” on lines 6-10.
- In part (c), the response earned the first point on line 2 for $a_Q = v_Q'(t)$ and $v_Q'(1) = 1.02686$. Note that without $a_Q = v_Q'(t)$, the response would still have earned the first point for $v_Q'(1) = 1.02686$. The response earned the second point for comparing the signs of $a_Q(1)$ (positive) and $v_Q(1)$ (negative) and concluding that the speed of particle Q at time $t = 1$ must be decreasing on lines 3-6.
- In part (d), the response earned the first point for $\int_0^\pi |v_P(t)| dt$ on line 2. The response earned the second point for $\int_0^\pi |v_P(t)| dt = 1.93148$ on line 2. Lines 3 and 4 summarize the result and contain correct information. Note the presented decimal is accurate to three decimal places, rounded or truncated.

Question 2 (continued)**Sample Identifier: B****Score: 8**

- The response earned 8 points: 3 points in part (a), 1 point in part (b), 2 points in part (c), and 2 points in part (d).
- In part (a), the response earned the first point for $\int_0^1 \sin(t^{1.5}) dt$ on line 1. The response earned the second point for $S_P(0) + \int_0^1 \sin(t^{1.5}) dt = 5 + 0.371$ on line 1. Note that the response correctly simplified the numerical expression $5 + 0.371$ on line 1, however this was not needed. The response earned the third point for $S_Q(0) + \int_0^1 (t - 1.8) \cdot 1.25^t dt = 10 - 1.436$ on line 2. Note that the response correctly simplified the numerical expression $10 - 1.436$ on line 2, however this was not needed.
- In part (b), the response earned the first point on lines 2-4 for stating that $v_P(1)$ is positive, so particle P is moving to the right. The second point was not earned because the relative positions of P and Q at $t = 1$ were not referenced.
- In part (c), the response earned the first point for connecting acceleration to the derivative of velocity at $t = 1$ by stating $v_Q'(1) = a_Q(1) = 1.027$ on line 1. Note that if the response presented $v_Q'(1) = 1.027$, without $a_Q(1)$, the first point would still have been earned. The second point was earned for stating that particle Q is slowing down because its velocity is negative while its acceleration is positive on lines 2-3.
- In part (d), the response earned the first point for $v_P(t) = \sin(t^{1.5}) = 0$, $t = 2.145$, and $\int_0^{2.145} \sin(t^{1.5}) dt - \int_{2.145}^{\pi} \sin(t^{1.5}) dt$ on lines 1-3. The second point was earned for the correct total distance $\int_0^{2.145} \sin(t^{1.5}) dt - \int_{2.145}^{\pi} \sin(t^{1.5}) dt = 1.931$ traveled by particle P on line 3. Note that the number line presented on line 2 was not used to determine points earned.

Question 2 (continued)**Sample Identifier: C****Score: 8**

- The response earned 8 points: 3 points in part (a), 2 points in part (b), 2 points in part (c), and 1 point in part (d).
- In part (a), the response earned the first point for $\int_0^1 v_P(t) dt$ on line 1. The response earned the second point for $5 + \int_0^1 v_P(t) dt = 5.371$ on line 1. The response earned the third point for $10 + \int_0^1 v_Q(t) dt = 8.5643$ on line 3. Note that the stated value on line 3 is correct, rounded or truncated, to three decimal places. Note also that the response correctly summarizes the answers on lines 2 and 4. On line 4, the response presents the correct answer, to three decimal places, for the position of Q at $t = 1$.
- In part (b), the response earned the first point for “because $v_P(1) > 0$, particle P is moving right” on lines 6 and 7. The response earned the second point stating that “Particles P and Q are moving towards each other at time $t = 1$,” “particle P is located at $x = 5.371$ and Q is at $x = 8.564$,” “because $v_P(1) > 0$, particle is moving right towards particle Q while $v_Q(1) < 0$, so particle Q is moving left towards particle P .”
- In part (c), the response earned the first point on lines 1 and 2 for $v_Q'(t) = a_Q(t)$ and $a_Q(1) = 1.0268$. Note that if the statement on line 1 was not presented, the first point would not have been earned; $a_Q(1) = 1.0268$ by itself is not sufficient to establish the required connection. Note, also, that the presented decimal answer is accurate to three decimal places on line 2. The response earned the second point for “Because the acceleration and velocity of particle Q at time $t = 1$ have opposite signs, the speed of the particle is decreasing.”
- In part (d), the response earned the first point for $\int_0^\pi |v_P(t)| dt$ on line 1. The response did not earn the second point because the total distance traveled is incorrect.

Question 2 (continued)**Sample Identifier: D****Score: 7**

- The response earned 7 points: 3 points in part (a), no points in part (b), 2 points in part (c), and 2 points in part (d).
- In part (a), the response earned the first point for $\int_0^1 \sin(t^{1.5}) dx$ on line 1. The second point was earned for $5 + \int_0^1 \sin(t^{1.5}) dx = 5 + 0.371$ on line 1. Note the correct position of particle P need not be simplified, however, the response simplified correctly to obtain 5.371. The third point was earned for $10 + \int_0^1 (1.25t(t - 1.8)) dx = 10 + (-1.436) = 8.564$ on line 2. The response was not penalized for the use of dx in place of dt .
- In part (b), no points were earned because the response failed to connect the direction of motion of each particle with the correct signs of the respective velocities at $t = 1$. Also, the response failed to reference the relative positions of P and Q at $t = 1$.
- In part (c), the first point was earned on lines 1 and 2 of the response. Note that the correct expression for $v_Q'(t)$ is given on line 1, however, this was not required. On line 1, the connection between $a_Q(t)$ and $v_Q'(t)$ was made. Note the required connection is also made if the response begins the statement on line 1 with $v_Q'(t)$. On line 2, the correct value of $a_Q(1)$ was given. The second point was earned on lines 3 and 4 by comparing the signs of v_Q and a_Q at $t = 1$, and concluding that the speed of Q is decreasing.
- In part (d), the response earned the first point for $\int_0^\pi (\left| \sin(t^{1.5}) \right|) dx$ on line 1. The response was not penalized for the use of dx in place of dt . The second point was earned for the correct total distance traveled, $\int_0^\pi (\left| \sin(t^{1.5}) \right|) dx = 1.931$, on line 1. The response goes on to summarize the result, which is unnecessary but correct, so the response earns the second point on line 2.

Question 2 (continued)**Sample Identifier: E****Score: 6**

- The response earned 6 points: 2 points in part (a), 1 point in part (b), 1 points in part (c), and 2 point in part (d).
- In part (a), the response earned the first point for $\int_0^1 v_P(t) dt$, and the second point for $5 + \int_0^1 v_P(t) dt = 5.37066028$, both on line 1. The response did not earn the third point because the stated position of Q at $t = 1$ is not correct (the second digit after the decimal is incorrect).
- In part (b), the response earned the first point by connecting the leftward motion of particle Q at $t = 1$ with its negative velocity on lines 2 and 3. The second point was not earned because, on line 1, the response states that P and Q are moving away from each other.
- In part (c), the response earned the first point on line 1 with the statement $v_Q'(1) = 1.026856478$. The expression at the beginning of that equation, “ $a_Q(1) =$ ”, is correct but not necessary. The second point was not earned because of the parenthetical phrase “slowing down.”
- In part (d), the response earned the first point for $\int_0^\pi |v_P(t)| dt$ on line 1. The second point was earned for the correct total distance traveled, $\int_0^\pi |v_P(t)| dt = 1.913480748$, on line 1. Note that the presented total distance traveled is correct to the first three decimal places, rounded or truncated.

Sample Identifier: F**Score: 6**

- The response earned 6 points: 3 points in part (a), no points in part (b), 1 points in part (c), and 2 points in part (d).
- In part (a), the response earned the first point for $\int_0^1 \sin(t^{1.5}) dt$ on line 3. The second point was earned for the statement $5 + \int_0^1 \sin(t^{1.5}) dt = 5.371$ on line 3. The third point was earned for the sum of the initial condition for Q and the definite integral of the velocity of Q from $t = 0$ to $t = 1$ on line 6, and presenting the correct position of Q at $t = 1$ on line 7. Note the inconsistent use of $x_P(t)$ and $x_Q(t)$ did not prevent the response from earning all possible points.
- In part (b), the response earned no points because there was no connection made between the direction of motion of either particle and the sign of its velocity. Also, there was no reference to the relative positions of the particles at $t = 1$.
- In part (c), the response did not earn the first point because the attempt to find the derivative of $v_Q(t)$ is not correct. However, with a declared positive value for the acceleration of Q at $t = 1$, the response was eligible to earn the second point. The second point was earned for the statement “it is decreasing since the signs of acceleration and velocity are different.”
- In part (d), the response earned the first point for $\int_0^\pi |v_P(t)| dt$ on line 1. The second point was earned for the correct total distance traveled, $\int_0^\pi |v_P(t)| dt = 1.931$, on lines 1 and 2.

Question 2 (continued)**Sample Identifier: G****Score: 6**

- The response earned 6 points: 3 points in part (a), no points in part (b), 1 point in part (c), and 2 points in part (d).
- In part (a), the response earned the first point for $\int_0^1 v_P(t) dt$ on line 2. The second point was earned for $5 + \int_0^1 v_P(t) dt = 5.371$ on line 2. The third point was earned for $10 + \int_0^1 v_Q(t) dt = 8.564$ on line 4.
- In part (b), no points were earned because no connection was made between the direction of motion of at least one particle and the sign of its velocity at $t = 1$, and there was no reference made to the relative position of the particles at $t = 1$.
- In part (c), the response earned the first point on line 1 by connecting the acceleration of Q at $t = 1$ to the derivative of its velocity at $t = 1$ and stating the correct acceleration. Note the required connection is also made if the response stated only $v_Q'(1)=1.027$. The second point was not earned because an incorrect conclusion was made.
- In part (d), the response earned the first point for $\int_0^\pi |v_P(t)| dt$ on line 1. The second point was earned for the correct total distance traveled, $\int_0^\pi |v_P(t)| dt = 1.931$, on line 1.

Sample Identifier: H**Score: 5**

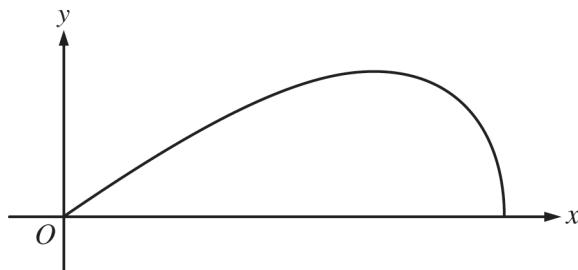
- The response earned 5 points: 3 points in part (a), no points in part (b), no points in part (c), and 2 points in part (d).
- In part (a), the response earned the first point for $\int_0^1 \sin(t^{1.5}) dt$ on line 3 on the left. The second point was earned for $5 + \int_0^1 \sin(t^{1.5}) dt = 5.371$ on lines 3 and 4 on the left. The third point was earned for $10 + \int_0^1 (t - 1.8) \cdot 1.25^t dt = 8.564$ on lines 3 and 4 on the right.
- In part (b), the response earned no points because there was no connection made between the direction of motion of either particle and the sign of its velocity. Also, there was no reference to the relative positions of the particles at $t = 1$.
- In part (c), the response did not earn the first point because there was no connection made between v_Q' and 1.027. The value 1.027 alone is not sufficient to demonstrate that connection. The second point was not earned because the response stated “is increasing at $t = 1$.”
- In part (d), the response earned the first point for $\int_0^\pi |\sin(t^{1.5})| dt$ on line 2. The second point was earned for the correct total distance traveled, $\int_0^\pi |\sin(t^{1.5})| dt = 1.931$, on lines 2 and 3.

Question 2 (continued)**Sample Identifier: I****Score: 5**

- The response earned 5 points: 1 point in part (a), no points in part (b), 2 points in part (c), and 2 points in part (d).
- In part (a), the response earned the first point for $\int_0^1 v_P(t) dt$ on line 3. The second and third points were not earned because both positions are incorrect.
- In part (b), the response earned no points because there was no connection made between the direction of motion of either particle and the sign of its velocity. Also, there was no reference to the relative positions of the particles at $t = 1$, and the response incorrectly concluded that the particles are moving away from each other.
- In part (c), the first point is earned on lines 2 and 3 where the connection between $v'(t)$ and $a(t)$ is made explicit, and the correct value of $a(1) = 1.0269$ is stated. The second point was earned for “The speed of particle Q is decreasing at time $t = 1$ because the velocity is negative and the acceleration is positive, which are different signs.” The response goes on to state the signs of $v(1) < 0$ and $a(1) > 0$ on lines 7 and 8.
- In part (d), the response earned the first point for the definite integral $\int_0^\pi |v_P(t)| dt$ on line 1. The second point was earned for $\int_0^\pi |v_P(t)| dt$ and the correct total distance traveled, 1.9315, stated on line 3. Note that the stated answer of 1.9315, which is given to four decimal places, is correct when truncated to three decimal places.

Sample Identifier: J**Score: 5**

- The response earned 5 points: 3 points in part (a), no points in part (b), 2 points in part (c), and no points in part (d).
- In part (a), the response earned the first point on line 2 for $\int_0^1 v_P(t) dt$. The response earned the second point on line 3 for $5 + \int_0^1 v_P(t) dt = 5.371$. The response earned the third point on line 6 for $10 + \int_0^1 v_Q(t) dt = 8.564$.
- In part (b), the response earned no points because there was no connection made between the direction of motion of either particle and the sign of its velocity. Also, there was no reference to the relative positions of the particles at $t = 1$.
- In part (c), the response earned the first point on line with $v_Q'(1) = 1.027$ on line 1. Note that if the response began with only $a_Q(1)=1.027$, this would not have been sufficient to earn the first point. The second point was earned for the declared value of $v_Q'(1)$ and the statement “speed is decreasing b/c velocity and acceleration of particle Q at $t = 1$ have different signs.”
- In part (d), no points were earned because the response presents the definite integral of velocity instead of the definite integral of speed. Because the answer is incorrect, the response did not earn the second point.



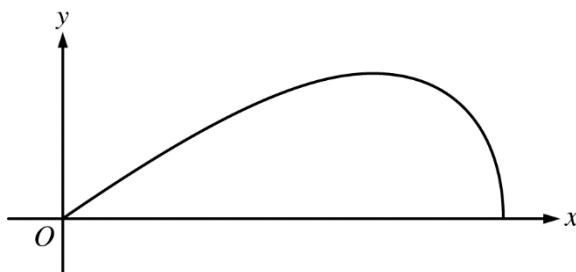
3. A company designs spinning toys using the family of functions $y = cx\sqrt{4 - x^2}$, where c is a positive constant. The figure above shows the region in the first quadrant bounded by the x -axis and the graph of $y = cx\sqrt{4 - x^2}$, for some c . Each spinning toy is in the shape of the solid generated when such a region is revolved about the x -axis. Both x and y are measured in inches.
- (a) Find the area of the region in the first quadrant bounded by the x -axis and the graph of $y = cx\sqrt{4 - x^2}$ for $c = 6$.
- (b) It is known that, for $y = cx\sqrt{4 - x^2}$, $\frac{dy}{dx} = \frac{c(4 - 2x^2)}{\sqrt{4 - x^2}}$. For a particular spinning toy, the radius of the largest cross-sectional circular slice is 1.2 inches. What is the value of c for this spinning toy?
- (c) For another spinning toy, the volume is 2π cubic inches. What is the value of c for this spinning toy?

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

Part B (AB or BC): Graphing calculator not allowed**Question 3****9 points****General Scoring Notes**

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Scoring guidelines and notes contain examples of the most common approaches seen in student responses. These guidelines can be applied to alternate approaches to ensure that these alternate approaches are scored appropriately.



A company designs spinning toys using the family of functions $y = cx\sqrt{4 - x^2}$, where c is a positive constant. The figure above shows the region in the first quadrant bounded by the x -axis and the graph of $y = cx\sqrt{4 - x^2}$, for some c . Each spinning toy is in the shape of the solid generated when such a region is revolved about the x -axis. Both x and y are measured in inches.

Model Solution**Scoring**

- (a) Find the area of the region in the first quadrant bounded by the x -axis and the graph of $y = cx\sqrt{4 - x^2}$ for $c = 6$.

$$6x\sqrt{4 - x^2} = 0 \Rightarrow x = 0, x = 2$$

$$\text{Area} = \int_0^2 6x\sqrt{4 - x^2} dx$$

Let $u = 4 - x^2$.

$$du = -2x dx \Rightarrow -\frac{1}{2} du = x dx$$

$$x = 0 \Rightarrow u = 4 - 0^2 = 4$$

$$x = 2 \Rightarrow u = 4 - 2^2 = 0$$

$$\begin{aligned} \int_0^2 6x\sqrt{4 - x^2} dx &= \int_4^0 6\left(-\frac{1}{2}\right)\sqrt{u} du = -3\int_4^0 u^{1/2} du = 3\int_0^4 u^{1/2} du \\ &= 2u^{3/2} \Big|_{u=0}^{u=4} = 2 \cdot 8 = 16 \end{aligned}$$

The area of the region is 16 square inches.

Integrand

1 point

Antiderivative

1 point

Answer

1 point

Scoring notes:

- Units are not required for any points in this question and are not read if presented (correctly or incorrectly) in any part of the response.
- The first point is earned for presenting $cx\sqrt{4 - x^2}$ or $6x\sqrt{4 - x^2}$ as the integrand in a definite integral. Limits of integration (numeric or alphanumeric) must be presented (as part of the definite integral), but do not need to be correct in order to earn the first point.
- If an indefinite integral is presented with an integrand of the correct form, the first point can be earned if the antiderivative (correct or incorrect) is eventually evaluated using the correct limits of integration.
- The second point can be earned without the first point. The second point is earned for presentation of a correct antiderivative of a function of the form $Ax\sqrt{4 - x^2}$, for any nonzero constant A . If the response has subsequent errors in simplification of the antiderivative or sign errors, the response will earn the second point but will not earn the third point.
- Responses that use u -substitution and have incorrect limits of integration or do not change the limits of integration from x - to u -values are eligible for the second point.
- The response is eligible for the third point only if it has earned the second point.
- The third point is earned only for the answer 16 or equivalent. In the case where a response only presents an indefinite integral, the use of the correct limits of integration to evaluate the antiderivative must be shown to earn the third point.
- The response cannot correct -16 to $+16$ in order to earn the third point; there is no possible reversal here.

Total for part (a) 3 points

(b)

It is known that, for $y = cx\sqrt{4 - x^2}$, $\frac{dy}{dx} = \frac{c(4 - 2x^2)}{\sqrt{4 - x^2}}$. For a particular spinning toy, the radius of the largest cross-sectional circular slice is 1.2 inches. What is the value of c for this spinning toy?

The cross-sectional circular slice with the largest radius occurs where $cx\sqrt{4 - x^2}$ has its maximum on the interval $0 < x < 2$.

Sets $\frac{dy}{dx} = 0$

1 point

$$\frac{dy}{dx} = \frac{c(4 - 2x^2)}{\sqrt{4 - x^2}} = 0 \Rightarrow x = \sqrt{2}$$

$$x = \sqrt{2} \Rightarrow y = c\sqrt{2}\sqrt{4 - (\sqrt{2})^2} = 2c$$

$$2c = 1.2 \Rightarrow c = 0.6$$

Answer

1 point

Scoring notes:

- The first point is earned for setting $\frac{dy}{dx} = 0$, $\frac{c(4 - 2x^2)}{\sqrt{4 - x^2}} = 0$, or $c(4 - 2x^2) = 0$.
- An unsupported $x = \sqrt{2}$ does not earn the first point.
- The second point can be earned without the first point but is earned only for the answer $c = 0.6$ with supporting work.

Total for part (b) 2 points

- (c) For another spinning toy, the volume is 2π cubic inches. What is the value of c for this spinning toy?

$\text{Volume} = \int_0^2 \pi(cx\sqrt{4-x^2})^2 dx = \pi c^2 \int_0^2 x^2(4-x^2) dx$ $= \pi c^2 \int_0^2 (4x^2 - x^4) dx = \pi c^2 \left(\frac{4}{3}x^3 - \frac{1}{5}x^5 \right _0^2$ $= \pi c^2 \left(\frac{32}{3} - \frac{32}{5} \right) = \frac{64\pi c^2}{15}$ $\frac{64\pi c^2}{15} = 2\pi \Rightarrow c^2 = \frac{15}{32} \Rightarrow c = \sqrt{\frac{15}{32}}$	Form of the integrand Limits and constant Antiderivative Answer	1 point 1 point 1 point 1 point
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Scoring notes:

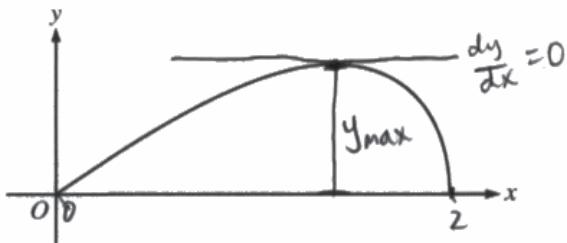
- The first point is earned for presenting an integrand of the form $A(x\sqrt{4-x^2})^2$ in a definite integral with any limits of integration (numeric or alphanumeric) and any nonzero constant A . Mishandling the c will result in the response being ineligible for the fourth point.
- The second point can be earned without the first point. The second point is earned for the limits of integration, $x = 0$ and $x = 2$, and the constant π (but not for 2π) as part of an integral with a correct or incorrect integrand.
- If an indefinite integral is presented with the correct constant π , the second point can be earned if the antiderivative (correct or incorrect) is evaluated using the correct limits of integration.
- A response that presents $2 = \int_0^2 (cx\sqrt{4-x^2})^2 dx$ earns the first and second points.
- The third point is earned for presenting a correct antiderivative of the presented integrand of the form $A(x\sqrt{4-x^2})^2$ for any nonzero A . If there are subsequent errors in simplification of the antiderivative, linkage errors, or sign errors, the response will not earn the fourth point.
- The fourth point cannot be earned without the third point. The fourth point is earned only for the correct answer. The expression does not need to be simplified to earn the fourth point.

Total for part (c) 4 points

Total for question 3 9 points

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Answer QUESTION 3 part (a) on this page.



Response for question 3(a)

$$y = 6 \times \sqrt{4-x^2} = 0$$

$$x=0, x=2$$

$$A = \int_0^2 6 \times \sqrt{4-x^2} dx \quad u = 4-x^2 \\ du = -2x dx$$

$$A = \int_4^0 -3\sqrt{u} du = 3 \int_0^4 u^{\frac{1}{2}} du = 3 \left[u^{\frac{3}{2}} \cdot \frac{2}{3} \right]_0^4$$

$$A = 2(4^{\frac{3}{2}} - 0^{\frac{3}{2}}) = 2(2^{\frac{3}{2}} - 0) = 2(8) = 16$$

$$\boxed{A=16}$$

• 3 3 3 3 NO CALCULATOR ALLOWED 3 3 3 3

Answer QUESTION 3 parts (b) and (c) on this page.

Response for question 3(b)

Largest cross section where y is greatest (maximum of y on graph).

Find MAX:

$$\frac{dy}{dx} = \frac{c(4-2x^2)}{\sqrt{4-x^2}} = 0 \rightarrow c(4-2x^2) = 0 \\ 4 = 2x^2 \\ x = \sqrt{2}$$

At $x = \sqrt{2}$, $y = 1.2$ (largest radius of cross-section equals 1.2, which is max y value)

$$y = cx\sqrt{4-x^2} \\ 1.2 = c\sqrt{2}(\sqrt{4-(\sqrt{2})^2}) = c\sqrt{2}(\sqrt{4-2}) = c\sqrt{2}(\sqrt{2}) = 2c \\ c = \frac{1.2}{2} = 0.6 \rightarrow \boxed{c = 0.6}$$

Response for question 3(c)

$$V = \pi \int_0^2 y^2 dx = \pi \int_0^2 (cx\sqrt{4-x^2})^2 dx = \pi c^2 \int_0^2 x^2(4-x^2) dx = \pi c^2 \int_0^2 (4x^2 - x^4) dx$$

$$V = \pi c^2 \left[\frac{4x^3}{3} - \frac{x^5}{5} \Big|_0^2 \right] = \pi c^2 \left[\left(\frac{4(8)}{3} - \frac{(32)}{5} \right) - (0-0) \right]$$

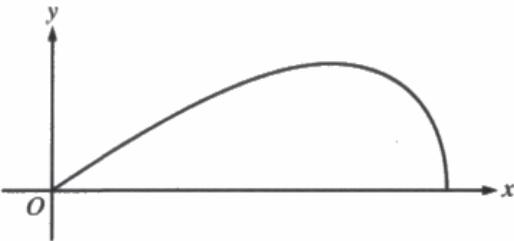
$$V = \pi c^2 \left(\frac{32(5)}{3(5)} - \frac{32(3)}{5(3)} \right) = \pi c^2 \left(\frac{2(32)}{15} \right) = \pi c^2 \left(\frac{64}{15} \right)$$

$$2\pi = \pi c^2 \left(\frac{64}{15} \right)$$

$$c^2 = \frac{30}{64} \rightarrow c = \sqrt{\frac{30}{64}} = \boxed{\frac{\sqrt{30}}{8}}$$

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Answer QUESTION 3 part (a) on this page.



Response for question 3(a)

$$6x\sqrt{9-x^2} = 0$$

$$x=0 \quad x=2$$

$$\text{by } y = (\frac{4}{3}-\frac{x^2}{3})^{1/2}$$

$$\int_0^2 6x\sqrt{9-x^2} dx$$

$$u = 9 - x^2$$

$$du = -2x dx$$

$$-3 \int_{9}^0 \sqrt{u} du = 3 \cdot \frac{2}{3} \cdot (u^{1/2}) \Big|_0^9 = 2 \cdot (8 - 0) \\ = 16$$

The area of the region bounded by the x -axis and the graph of $y = 6x\sqrt{9-x^2}$ is 16 sq. inches

● 3 3 3 3 NO CALCULATOR ALLOWED 3 3 3 3

Answer QUESTION 3 parts (b) and (c) on this page.

Response for question 3(b)

radius = y
radius is largest when $\frac{dy}{dx} = 0$

$$0 = \frac{c(4-2x^2)}{\sqrt{9-x^2}}, \quad c(4-2x^2)=0 \\ x=\sqrt{2}$$

$$1.2 = c \cdot \sqrt{2} \sqrt{9-(\sqrt{2})^2} \quad c=0.6$$

$$= c \cdot \sqrt{2} \cdot \sqrt{2} \\ = 2c$$

The value of c for this spinning toy is 0.6

Response for question 3(c)

$$V = \pi \int_0^6 (cx\sqrt{9-x^2})^2 dx$$

$b=2$ for all c as shown in part a

$$2\pi = \pi \int_0^2 c^2 x^2 (9-x^2) dx$$

The value of c for this spinning toy is $c = \frac{\sqrt{10}}{4}$

$$= c^2 \pi \left(\frac{9}{3} x^3 - \frac{x^5}{5} \right) \Big|_0^2 = c^2 \pi \left(\frac{32}{3} - \frac{32}{5} \right)$$

$$2\pi = c^2 \pi \left(\frac{64}{15} \right)$$

$$\text{Page 9} \quad c^2 = \frac{64}{15} \quad c = \frac{\sqrt{10}}{\sqrt{15}} = \frac{\sqrt{10}}{3\sqrt{5}}$$

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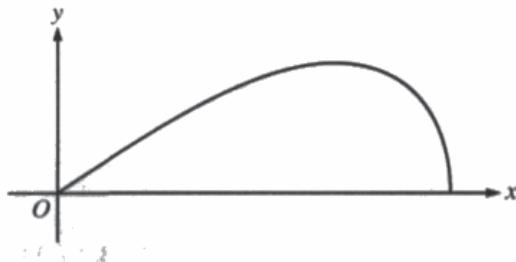
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Answer QUESTION 3 part (a) on this page.



Response for question 3(a)

$$A = \int_0^2 6x(4-x^2)^{1/2} dx = \int_4^0 -3u^{1/2} du = -2u^{3/2} \Big|_4^0$$

$$\text{let } u = 4-x^2 \\ \& du = -2x dx \\ = -2(0)^{3/2} + 2(4)^{3/2} = 16$$

$$u(0) = 4$$

$$A = 16$$

$$u(2) = 0$$

• 3 3 3 3 NO CALCULATOR ALLOWED 3 3 3 3 3

Answer QUESTION 3 parts (b) and (c) on this page.

Response for question 3(b)

$$\frac{C(4-2x^2)}{\sqrt{4-x^2}} = 0 \text{ when } x = \sqrt{2}$$

$$C\sqrt{2}\sqrt{4-(\sqrt{2})^2} = 1.2 \text{ when } C = 0.6$$

Response for question 3(c)

$$\pi \int_0^2 (C \times \sqrt{4-x^2}) dx = 2\pi$$

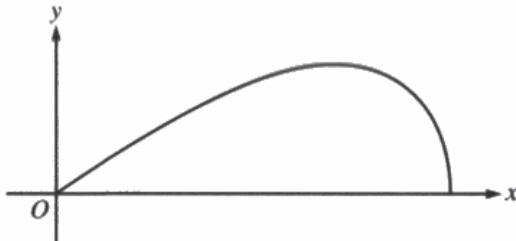
$$\int_0^2 Cx^2(4-x^2) dx = 2 = \int_0^2 C^2(x^2 - x^4) dx$$

$$C^2 \left(\frac{x^3}{3} - \frac{x^5}{5} \right) \Big|_0^2 = C^2 \left(\frac{8}{3} - \frac{32}{5} \right) = 2$$

$$C = \sqrt{\frac{4}{3}} = \frac{16}{5}$$

3 3 3 3 3 NO CALCULATOR ALLOWED 3 3 3 3

Answer QUESTION 3 part (a) on this page.



Response for question 3(a)

$$\begin{aligned}y &= 6x\sqrt{4-x^2} = 0 \\ \Rightarrow 6x &= 0 \text{ or } \sqrt{4-x^2} = 0 \\ x &= 0 \text{ or } x = 2\end{aligned}$$

$$\begin{aligned}\text{Area} &= \int_0^2 (6x\sqrt{4-x^2}) dx = \left[-2(4-x^2)^{3/2} \right]_0^2 \\ &= -2(4-2^2)^{3/2} + 2(4-0^2)^{3/2} \\ &= 0 + 2(\sqrt{4})^3 = 2(2^3) = 16\end{aligned}$$

● 3 3 3 3 3 NO CALCULATOR ALLOWED 3 3 3 3 3

Answer QUESTION 3 parts (b) and (c) on this page.

Response for question 3(b)

$$\text{At } y = 1.2, \frac{dy}{dx} = 0$$

$$\rightarrow c(4 - 2x^2) = 0$$

$$\rightarrow 2x^2 = 4$$

$$x^2 = 2$$

$$x = \sqrt{2}$$

$$1.2 = c(\sqrt{2})\sqrt{4 - (\sqrt{2})^2}$$

$$1.2 = c(\sqrt{2})\sqrt{2} = 2c$$

$$c = \frac{1.2}{2} = \frac{0.3}{0.5} = \frac{3}{5}$$

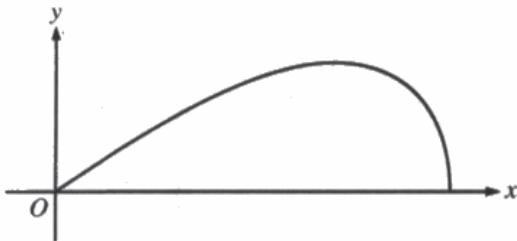
Response for question 3(c)

$$2\pi = \pi \int_0^2 (c \times \sqrt{4-x^2})^2 dx$$

$$2 = \left[\frac{1}{3} (cx\sqrt{4-x^2})^3 \cdot \left(\frac{\sqrt{4-x^2}}{c(4-2x^2)} \right) \right]_0^2$$

3 3 3 3 3 NO CALCULATOR ALLOWED 3 3 3 3

Answer QUESTION 3 part (a) on this page.



Response for question 3(a)

$$y = 6x\sqrt{4-x^2}$$

$$A = \int_0^2 6x\sqrt{4-x^2} dx$$

$$6x\sqrt{4-x^2} = 0$$

$$\begin{aligned} x &= 0 \\ x &= 2 \end{aligned}$$

$$\begin{aligned} u &= 4-x^2 \\ du &= -2x dx \end{aligned}$$

$$-3du = 6x dx$$

$$A = \int_{-3}^0 \sqrt{u} du$$

$$\begin{aligned} u &= 4-2^2 & u &= 4-0^2 \\ u &= 0 & u &= 4 \end{aligned}$$

$$A = -3 \int_4^0 \sqrt{u} du$$

$$\frac{1}{2} + \frac{1}{2} = \frac{3}{2}$$

$$= -3 \left[\frac{2(4-2^2)^{3/2}}{3} - \frac{2(4-0^2)^{3/2}}{3} \right]$$

$$-3 \left[\frac{2(u)^{3/2}}{3} \right] \Big|_4^0$$

$$= -3 \left(0 - \frac{2(4)^{3/2}}{3} \right)$$

$$-3 \left[\frac{2(4-x^2)^{3/2}}{3} \right] \Big|_0^2$$

$$= \frac{6(4)^{3/2}}{3} = 2(4)^{3/2} = 16$$

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Answer QUESTION 3 parts (b) and (c) on this page.

Response for question 3(b)

$$\begin{aligned}
 \frac{dy}{dx} &= 0 \\
 C(4-2x^2) &= 0 \\
 \frac{C(4-2x^2)}{\sqrt{4-x^2}} & \\
 C(4-2x^2) &= 0 \\
 4-2x^2 &= 0 \\
 4 &= 2x^2 \\
 2 &= x^2 \\
 x &= \sqrt{2}
 \end{aligned}
 \quad
 \begin{aligned}
 1.2 &= C\sqrt{2} \times \sqrt{4-(\sqrt{2})^2} \\
 &= C\sqrt{2} \times \sqrt{4-2} \\
 &= C\sqrt{2} \times \sqrt{2} \\
 1.2 &= 2C \\
 C &= \frac{1.2}{2} \\
 C &= 0.6
 \end{aligned}$$

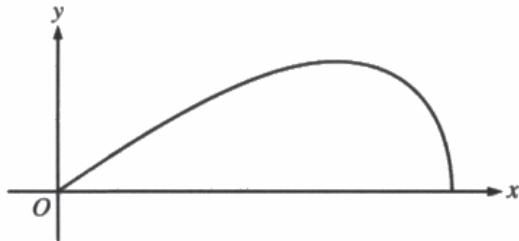
Response for question 3(c)

$$\begin{aligned}
 V &= \pi \int r^2 dx \\
 V &= \pi \int_0^2 Cx \sqrt{4-x^2} dx \\
 C\pi \int_0^2 x \sqrt{4-x^2} dx &= 2\pi \\
 C\pi \int_0^2 \sqrt{u} du &= 2\pi \\
 C\pi \left[\frac{2u^{3/2}}{3} \right]_0^4 &= 2\pi \\
 0 - \frac{2(4)^{3/2}}{3} &= \frac{2}{C} \\
 -\frac{16}{3} &= \frac{2}{C}
 \end{aligned}
 \quad
 \begin{aligned}
 u &= 4-x^2 \\
 \frac{du}{-2} &= -2x dx \\
 du &= x dx
 \end{aligned}
 \quad
 \begin{aligned}
 u &= 4-x^2 = 0 \\
 4-0^2 &= 4 \\
 \frac{2(4)^{3/2}}{3} &= \frac{2}{C} \\
 \frac{-16}{3} &= \frac{2}{C} \\
 C &= \frac{2 \times 3}{-16} \\
 C &= -\frac{3}{8} \\
 C &= -\frac{6}{16}
 \end{aligned}$$

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Answer QUESTION 3 part (a) on this page.



Response for question 3(a)

bounded by 0 and $0 = \sqrt{4-x^2}$

$$x^2 = 4$$

$$x = 2$$

$$\begin{aligned} \text{So area} &= \int_0^2 6x\sqrt{4-x^2} dx && \text{u-substitution:} \\ &= \int_{x=0}^{x=2} -3\sqrt{u} du && u = 4-x^2 \quad u' = -2x \quad dx \\ &= -\frac{1}{2}u^{3/2} \Big|_{x=0}^{x=2} \\ &= -\frac{1}{2}(4-x^2) \Big|_0^2 \\ &= -\frac{1}{2}(4-4) + \frac{1}{2}(4-0) \\ &= 2 \end{aligned}$$

area of region in first quadrant bounded by x-axis
and the graph of $y = 6x\sqrt{4-x^2}$ is 2



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Answer QUESTION 3 parts (b) and (c) on this page.

Response for question 3(b)

largest radius means a maximum for y

$$\text{so } \frac{dy}{dx} = 0 \text{ when } x = 1.2 = \frac{6}{5}$$

$$\frac{dy}{dx} = \frac{c(4-2x^2)}{\sqrt{4-x^2}}$$

$$0 = \frac{c(4-2(\frac{6}{5})^2)}{\sqrt{4-(\frac{6}{5})^2}}$$

$$0 = \frac{c(4-\frac{72}{25})}{\sqrt{4-\frac{36}{25}}}$$

$$0 = c(\frac{28}{25})\sqrt{\frac{25}{64}}$$

$$0 = c(\frac{28}{25})(\frac{5}{8})$$

$$0 = \frac{7}{10}c$$

Response for question 3(c)

$$2\pi r = \pi \int_0^2 (cx\sqrt{4-x^2})^2 dx = 2\pi c^2$$

$$2 = \int_0^2 (cx)^2 (4-x^2) dx$$

$$2 = c^2 \int_0^2 (-x^4 + 4x^2) dx$$

$$2 = c^2 \left(-\frac{1}{5}x^5 + \frac{4}{3}x^3 \right) \Big|_0^2$$

$$2 = c^2 \left(-\frac{32}{5} + \frac{32}{3} \right)$$

$$30 = c^2 (-96 + 160)$$

$$30 = c^2 (64)$$

$$c^2 = \frac{30}{64}$$

$$c = \frac{\sqrt{30}}{8}$$

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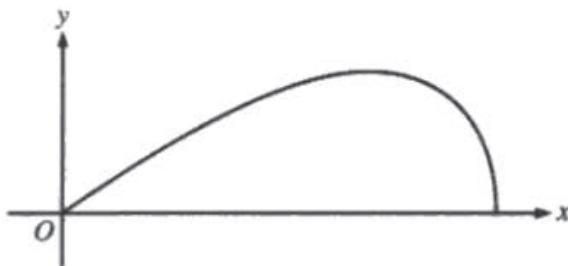
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Answer QUESTION 3 part (a) on this page.



Response for question 3(a)

$$y = 0 \text{ when } x = 0 \text{ and } x = 2$$

$$y = 6x - \sqrt{4-x^2}$$

$$A = \int_0^2 6x - \sqrt{4-x^2} dx$$

• 3 3 3 3 NO CALCULATOR ALLOWED 3 3 3 3

Answer QUESTION 3 parts (b) and (c) on this page.

Response for question 3(b)

Absolute maximum is $y = 1.2$ $\frac{dy}{dx} = 0$ when
 $x = -\sqrt{2}$

$$1.2 = ((-\sqrt{2}) - \sqrt{4 - 2})$$

$$1.2 = -\sqrt{2} + \sqrt{2}$$

$$1.2 = 2(0)$$

$$\frac{1.2}{2} = 0$$

$$\begin{aligned} 2x^2 &= 4 \\ x^2 &= 2 \\ x &= \sqrt{2} \end{aligned}$$

Response for question 3(c)

$$2\pi r = \pi \int_0^2 [x - \sqrt{4-x^2}]^2 dx$$

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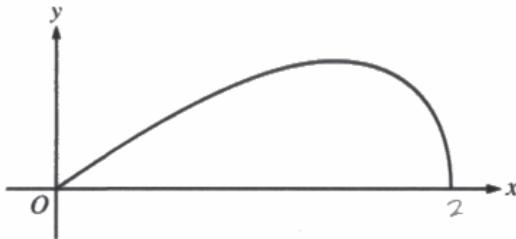
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Answer QUESTION 3 part (a) on this page.



Response for question 3(a)

$$c = 6$$

$$y = 6x \sqrt{4-x^2}$$

$$\begin{aligned} y &= 0 = 6x \\ x &= 0 \end{aligned}$$

$$\begin{aligned} 0 &= \sqrt{4-x^2} \\ 0 &= 4-x^2 \\ x &= 2, \cancel{x=-2} \end{aligned}$$

$$A = \int_0^2 6x \sqrt{4-x^2} dx$$

$$\begin{aligned} u &= 4-x^2 \\ -3du &= -2x dx \end{aligned}$$

$$\rightarrow \int_4^0 -3\sqrt{u} du$$

$$\begin{aligned} 3 \int_0^4 \sqrt{u} du \\ 3 \left[\frac{2}{3} u^{\frac{3}{2}} \right]_0^4 \end{aligned}$$

$$\cancel{8} \left[\frac{16}{3} - 0 \right]$$

$$16 \text{ in}^2$$

● 3 3 3 3 3 NO CALCULATOR ALLOWED 3 3 3 3

Answer QUESTION 3 parts (b) and (c) on this page.

Response for question 3(b)

$$\frac{dy}{dx} = \frac{c(4-2x^2)}{\sqrt{4-x^2}} \quad \left. \frac{dy}{dx} \right|_{x=\sqrt{2}} = \frac{c(4-4)}{\sqrt{2}} = 0$$

$$\text{max at } x = \sqrt{2}$$

$$y(\sqrt{2}) = \frac{1.2}{2} \Rightarrow 0.6$$

$$0.6 = c(\sqrt{2})\sqrt{4-6\sqrt{2}}$$

$$0.6 = c \cancel{\sqrt{2}} \sqrt{2}$$

$$0.6 = 2c$$

$$c = \frac{0.6}{2} = 0.3$$

Response for question 3(c)

$$V = 2\pi \text{ in}^3$$

$$V = \pi \int_0^2 cx \sqrt{4-x^2} dx$$

$$2\pi = \pi \int_0^2 cx \sqrt{4-x^2} dx$$

$$2 = \frac{8c}{3}$$

$$\frac{3 \cdot 2}{8} = c$$

$$\frac{6}{8} = c$$

$$\boxed{c = \frac{3}{4}}$$

$$\begin{aligned} u &= \sqrt{4-x^2} \\ -du &= -2x dx \rightarrow \int_1^0 -\frac{c}{2} \sqrt{u} du \\ -\frac{c}{2} du &= cx dx \\ \frac{c}{2} \int_0^1 \sqrt{u} du & \\ \frac{c}{2} \left[\frac{2}{3} u^{\frac{3}{2}} \right]_0^1 & \\ \frac{c}{2} \left[\frac{16}{3} - 0 \right] & \\ \frac{c}{6} \cancel{- \frac{8c}{3}} & \end{aligned}$$

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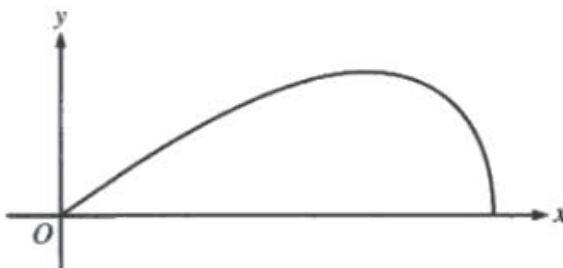
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Answer QUESTION 3 part (a) on this page.



Response for question 3(a)

$$y = 6x \sqrt{4-x^2} \quad y = 3 = 6x \sqrt{4-x^2}$$

$$A = \int_0^2 6x(4-x^2)^{1/2} dx \quad x=0, 2$$

$$A = \frac{1}{2} G \int_0^2 (4-x^2)^{1/2} dx$$

$$A = 3 \left[\frac{2}{3} (4-x^2)^{3/2} \right]_0^2$$

$$A = 3 \left(\frac{2}{3} (0) - \frac{2}{3} (4) \right)$$

$$3 \left(0 - \frac{16}{3} \right)$$

$$|-6| = \boxed{|G| \text{ in } \text{m}^2}$$

• 3 3 3 3 NO CALCULATOR ALLOWED 3 3 3 3

Answer QUESTION 3 parts (b) and (c) on this page.

Response for question 3(b)

$$y = Cx\sqrt{4-x^2} \quad \frac{dy}{dx} = \frac{C(4-2x^2)}{\sqrt{4-x^2}} = 0$$

$$\begin{aligned} 4 &= 2x^2 \\ 2 &= x^2 \\ x &= \sqrt{2} \quad (\text{in } \leftarrow \overbrace{\text{+}+\text{+}+\text{+}+\text{+}}_{\sqrt{2}} \rightarrow) \end{aligned}$$

$$y(\sqrt{2}) = 1.2 = C\sqrt{2}\sqrt{4-2}$$

$$1.2 = C\sqrt{2}\sqrt{2}$$

$$1.2 = 2C$$

$$\boxed{C=0.6}$$

Response for question 3(c)

$$V = 2\pi \int_0^2 x(Cx\sqrt{4-x^2}) dx \quad x \geq 0, 2$$

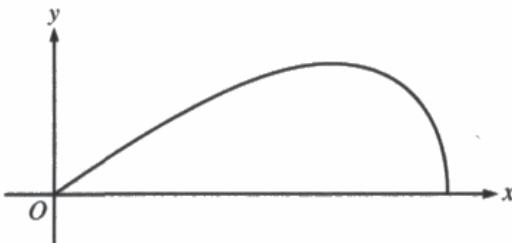
$$V = 2\pi \int_0^2 x(Cx\sqrt{4-x^2}) dx$$

$$\int_0^2 x(Cx\sqrt{4-x^2}) dx = ?$$

$$? = \int_0^2 x(Cx\sqrt{4-x^2}) dx$$

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Answer QUESTION 3 part (a) on this page.



Response for question 3(a)

$$y = 6x\sqrt{u-x^2}$$

$$\theta = 6x\sqrt{u-x^2}$$

$$\theta = x\sqrt{u-x^2}$$

$$\theta = \sqrt{u-x^2}$$

$$\theta = u-x^2$$

$$-u = -x^2$$

$$x = 2$$

$$\int_0^2 6x\sqrt{u-x^2} dx$$

$$\int_0^2 6x\sqrt{v} \frac{dv}{-2x}$$

$$-3 \int_0^u v^{1/2} dv$$

$$-3 \left[\frac{2}{3} v^{3/2} \right]_u^0$$

$$-2 \left[v^{3/2} \right]_u^0$$

$$0 - (-16)$$

$$\boxed{16}$$

$$v = u - x^2$$

$$dv = -2x dx$$

$$u - u = 0$$

$$u - 0 = u$$

● 3 3 3 3 3 NO CALCULATOR ALLOWED 3 3 3 3 3

Answer QUESTION 3 parts (b) and (c) on this page.

Response for question 3(b)

$$\frac{dy}{dx} = \frac{C(u-2x^2)}{\sqrt{u-x^2}}$$

$$0 = \frac{C(u-2(1.2)^2)}{\sqrt{u-1.2^2}} \quad 0 = \frac{C(u-2.88)}{\sqrt{u-1.44}}$$

$$0 = \frac{C(1.22)}{\sqrt{2.64}}$$

$\textcircled{1} = \textcircled{2}$

Response for question 3(c)

$$\int_0^2 Cx\sqrt{u-x^2} dx = 2\pi$$

Question 3**Sample Identifier: A****Score: 9**

- The response earned 9 points: 3 in part (a), 2 in part (b), and 4 in part (c).
- In part (a), the response presented the correct integrand in a definite integral and earned the first point. The antiderivative of $3u^{3/2} \cdot \frac{2}{3}$ with the definition $u = 4 - x^2$ is correct and earned the second point. The response has the correct answer and earned the third point.
- In part (b), the response earned the first point for stating $\frac{dy}{dx} = \frac{c(4 - 2x^2)}{\sqrt{4 - x^2}} = 0$. The answer is correct and the response earned the second point.
- In part (c), the response presented y^2 as the integrand of a definite integral and earned the first point. Note that since $y = cx\sqrt{4 - x^2}$ is given in the statement of the problem, a response can reference the function by using y for the first point. The limits and constant are correct and earned the second point. The antiderivative is correct and earned the third point. The response is eligible for the fourth point. The answer is correct and earned the fourth point. Note that $\frac{\sqrt{30}}{8} = \sqrt{\frac{15}{32}}$.

Sample Identifier: B**Score: 8**

- The response earned 8 points: 3 in part (a), 2 in part (b), and 3 in part (c).
- In part (a), the response presented a correct integrand in a definite integral and earned the first point. The antiderivative $3 \cdot \frac{2}{3} \cdot (u^{3/2})$ with the definition $u = 4 - x^2$ is correct and the response earned the second point. Note that the sign of the antiderivative is consistent with the limits of integration. The response is eligible for the third point. The answer is correct and earned the third point. Note that units were not required for this problem.
- In part (b), the response set $\frac{dy}{dx} = 0$ and earned the first point. The answer is correct and earned the second point.
- In part (c), the response presented an integrand of the correct form in a definite integral in the first line and earned the first point. The limits and constant on the third line are correct and earned the second point. The antiderivative is correct and earned the third point. The response is eligible for the fourth point. There is an error in simplification that results in an incorrect answer and the response did not earn the fourth point.

Question 3 (continued)**Sample Identifier: C****Score: 7**

- The response earned 7 points: 3 points in part (a), 2 points in part (b), and 2 points in part (c).
- In part (a), the response presented the correct integrand in a definite integral and earned the first point. The antiderivative of $-2u^{3/2}$ with the definition of $u = 4 - x^2$ is correct and the response earned the second point. Note that the sign of the antiderivative is consistent with the limits of integration. The response is eligible for the third point. The answer is correct and earned the third point.

- In part (b), the response earned the first point for setting $\frac{c(4 - 2x^2)}{\sqrt{4 - x^2}} = 0$. The answer is correct and earned the second point.
- In part (c), the response presents an integrand of the correct form in a definite integral and earned the first point. The limits and constant are correct and earned the second point. The response incorrectly presents $\left(\frac{x^3}{3} - \frac{x^5}{5}\right)$ as the antiderivative of $(x\sqrt{4 - x^2})^2$ and did not earn the third point. Note that there was an error in using the distributive property that leads to an incorrect antiderivative. Without earning the third point, the response is not eligible for and did not earn the fourth point.

Sample Identifier: D**Score: 7**

- The response earned 7 points: 3 points in part (a), 2 points in part (b), and 2 points in part (c).
- In part (a), the response presented the correct integrand in a definite integral and earned the first point. The antiderivative $-2(4 - x^2)^{3/2}$ is correct and earned the second point. The answer is correct and earned the third point. Note that showing the use of u-substitution is not required to earn the second point.
- In part (b), the response set $\frac{dy}{dx} = 0$ and earned the first point. The answer is correct and earned the second point. Note that simplification of $\frac{1.2}{2}$ was not necessary to earn the second point.
- In part (c), the response presented the correct integrand in a definite integral and earned the first point. The limits and constant are correct and earned the second point. The antiderivative presented is incorrect and did not earn the third point. The response is not eligible for and did not earn the fourth point.

Question 3 (continued)**Sample Identifier: E****Score: 6**

- The response earned 6 points: 3 points in part (a), 2 points in part (b), and 1 point in part (c).
- In part (a), the response presented the correct integrand in a definite integral and earned the first point. The antiderivative $-3\left[\frac{2(u)^{3/2}}{3}\right]$ with the definition $u = 4 - x^2$ is correct and earned the second point. Note that the sign of the antiderivative is consistent with the limits of integration. The response is eligible for the third point. The answer is correct and earned the third point. Note that the substitution of $u = 4 - x^2$ after finding the antiderivative and using the limits of $x = 0$ and $x = 2$ was not necessary to evaluate the antiderivative.
- In part (b), the response earned the first point by stating $\frac{dy}{dx} = 0$. The answer is correct and earned the second point.
- In part (c), the integrand is not of the correct form and the response did not earn the first point. The limits and constant are correct and earned the second point. Because the integrand was not of the correct form, the response is not eligible for and did not earn the third point. Without earning the third point, the response is not eligible for and did not earn the fourth point.

Sample Identifier: F**Score: 6**

- The response earned 6 points: 1 point in part (a), 1 point in part (b), and 4 points in part (c).
- In part (a), the response presented the correct integrand in a definite integral and earned the first point. For $u = 4 - x^2$, the antiderivative $-\frac{1}{2}u^{3/2}$ is incorrect and the response did not earn the second point. The response is not eligible for and did not earn the third point.
- In part (b), the response earned the first point by setting $\frac{dy}{dx} = 0$ in the second line. Note that the first point does not require the response to solve the equation correctly. The response does not have the correct answer and did not earn the second point.
- In part (c), the response presented an integrand of the correct form in a definite integral and earned the first point. The limits and constant are correct and the earned the second point. The antiderivative of the integrand is correct and earned the third point. The response is eligible for the fourth point. The answer is correct and earned the fourth point. Note that the answer of $\frac{\sqrt{30}}{8}$ is equivalent to $\sqrt{\frac{15}{32}}$.

Question 3 (continued)**Sample Identifier: G****Score: 5**

- The response earned 5 points: 1 point in part (a), 2 points in part (b), and 2 points in part (c).
- In part (a), the response presented the correct integrand in a definite integral and earned the first point. The response does not present further work and did not earn the second point or the third point.
- In part (b), the response earned the first point with setting $\frac{dy}{dx} = 0$. The response has the correct answer and earned the second point. Note that the answer did not need to be simplified.
- In part (c), the response has an integrand of the correct form in a definite integral and earned the first point. The limits and constant are correct and the response earned the second point. No further work is presented and the response did not earn the third point and did not earn the fourth point.

Sample Identifier: H**Score: 5**

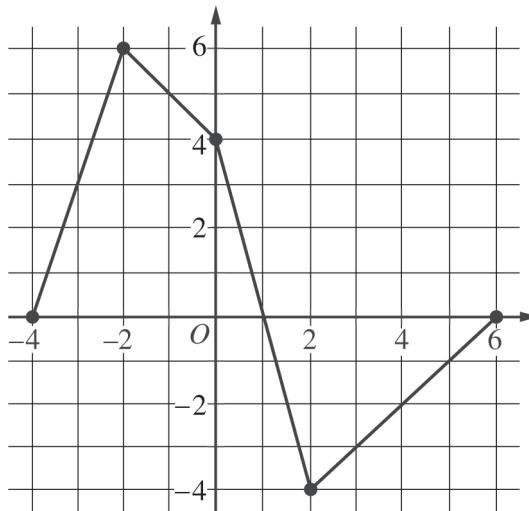
- The response earned 5 points: 3 points in part (a), 1 point in part (b), and 1 point in part (c).
- In part (a), the response presented the correct integrand in a definite integral and earned the first point. The response presented the correct antiderivative as $3\left[\frac{2}{3}(u)^{3/2}\right]$ with the definition $u = 4 - x^2$ and earned the second point. The response was eligible for the third point. The answer was correct and earned the third point. Note that units are not required as part of this problem and did not need to be stated to earn points.
- In part (b), the response earned the first point with $\frac{dy}{dx}\Big|_{x=\sqrt{2}} = \frac{c(4-\cancel{4})}{\sqrt{2}} = 0$. Note that this statement includes both setting $\frac{dy}{dx} = 0$ and solving the equation in one step. The response does not have the correct answer and did not earn the second point.
- In part (c), the response does not have an integrand of the correct form and did not earn the first point. The limits and constant π are correct and the earned the second point. The response is not eligible for and did not earn the third point since the integrand was not of the correct form. The response is not eligible for and did not earn the fourth point.

Question 3 (continued)**Sample Identifier: I****Score: 3**

- The response earned 3 points: 1 point in part (a), 2 points in part (b), and 0 points in part (c).
- In part (a), the response presented the correct integrand in a definite integral and earned the first point. The antiderivative presented is incorrect since the sign of the antiderivative is incorrect and the response did not earn the second point. The response is not eligible for and did not earn the third point.
- In part (b), the response earned the first point by stating $\frac{dy}{dx} = \frac{c(4 - 2x^2)}{\sqrt{4 - x^2}} = 0$. The answer is correct and the response earned the second point.
- In part (c), the integrand presented is not of the correct form and the response did not earn the first point. The constant 2π is incorrect and the response did not earn the second point. Without an integrand of the correct form, the response is not eligible for the third point and is not eligible for the fourth point. The response did not earn the third point and did not earn the fourth point.

Sample Identifier: J**Score: 3**

- The response earned 3 points: 3 points in part (a), 0 points in part (b), and 0 points in part (c).
- In part (a), the response presented the correct integrand in a definite integral and earned the first point. Note that integration symbol is treated as the symbol for an integral in the presence of correct work. The antiderivative is presented correctly as $-3\left[\frac{2}{3}u^{3/2}\right]$ with the definition $u = 4 - x^2$ and earned the second point. Note that the sign of the antiderivative is consistent with the limits of integration. The response is eligible for the third point. The answer is correct and earned the third point.
- In part (b), the response did not earn the first point. Note that the equation in the first line is not set equal to 0 and did not earn the first point. The first point is earned for the setup of an equation that can be solved for the x -value where the maximum occurs. Substitution of 1.2 for x in the second line results in an expression equal to 0 that cannot be solved for x . The answer is incorrect and did not earn the second point.
- In part (c), the integrand $cx\sqrt{4 - x^2}$ is not of the correct form and did not earn the first point. The limits of integration are correct but the constant π is missing. Therefore, the response did not earn the second point. Without an integrand of the correct form, the response is not eligible for and did not earn the third point and did not earn the fourth point.

Graph of f

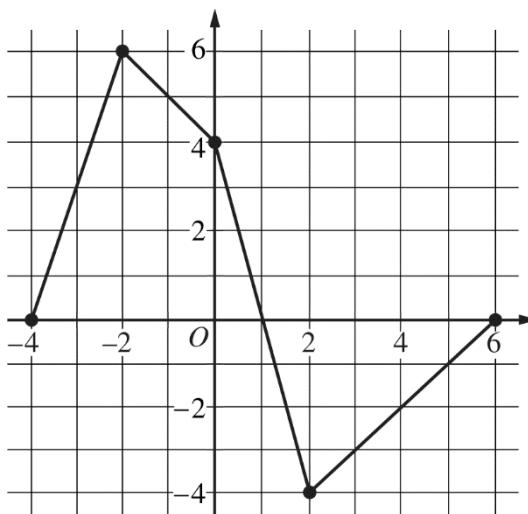
4. Let f be a continuous function defined on the closed interval $-4 \leq x \leq 6$. The graph of f , consisting of four line segments, is shown above. Let G be the function defined by $G(x) = \int_0^x f(t) dt$.
- On what open intervals is the graph of G concave up? Give a reason for your answer.
 - Let P be the function defined by $P(x) = G(x) \cdot f(x)$. Find $P'(3)$.
 - Find $\lim_{x \rightarrow 2} \frac{G(x)}{x^2 - 2x}$.
 - Find the average rate of change of G on the interval $[-4, 2]$. Does the Mean Value Theorem guarantee a value c , $-4 < c < 2$, for which $G'(c)$ is equal to this average rate of change? Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

Part B (AB or BC): Graphing calculator not allowed**Question 4****9 points****General Scoring Notes**

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Scoring guidelines and notes contain examples of the most common approaches seen in student responses. These guidelines can be applied to alternate approaches to ensure that these alternate approaches are scored appropriately.

Graph of f

Let f be a continuous function defined on the closed interval $-4 \leq x \leq 6$. The graph of f , consisting of four line segments, is shown above. Let G be the function defined by $G(x) = \int_0^x f(t) dt$.

Model Solution	Scoring
$G'(x) = f(x)$ in any part of the response	$G'(x) = f(x)$ 1 point

Scoring notes:

- This “global point” can be earned in any one part. Expressions that show this connection and therefore earn this point include: $G' = f$, $G'(x) = f(x)$, $G''(x) = f'(x)$ in part (a), $G'(3) = f(3)$ in part (b), or $G'(2) = f(2)$ in part (c).

Total	1 point
--------------	----------------

- (a) On what open intervals is the graph of G concave up? Give a reason for your answer.

$$G'(x) = f(x)$$

The graph of G is concave up for $-4 < x < -2$ and $2 < x < 6$, because $G' = f$ is increasing on these intervals.

Answer with reason

1 point

Scoring notes:

- Intervals may also include one or both endpoints.

Total for part (a) **1 point**

- (b) Let P be the function defined by $P(x) = G(x) \cdot f(x)$. Find $P'(3)$.

$$P'(x) = G(x) \cdot f'(x) + f(x) \cdot G'(x)$$

$$P'(3) = G(3) \cdot f'(3) + f(3) \cdot G'(3)$$

Product rule

1 point

$$\text{Substituting } G(3) = \int_0^3 f(t) dt = -3.5 \text{ and } G'(3) = f(3) = -3$$

into the above expression for $P'(3)$ gives the following:

$G(3)$ or $G'(3)$

1 point

$$P'(3) = -3.5 \cdot 1 + (-3) \cdot (-3) = 5.5$$

Answer

1 point

Scoring notes:

- The first point is earned for the correct application of the product rule in terms of x or in the evaluation of $P'(3)$. Once earned, this point cannot be lost.
- The second point is earned by correctly evaluating $G(3) = -3.5$, $G'(3) = -3$, or $f(3) = -3$.
- To be eligible to earn the third point, a response must have earned the first two points. Simplification of the numerical value is not required to earn the third point.

Total for part (b) **3 points**

- (c) Find $\lim_{x \rightarrow 2} \frac{G(x)}{x^2 - 2x}$.

$$\lim_{x \rightarrow 2} (x^2 - 2x) = 0$$

Because G is continuous for $-4 \leq x \leq 6$,

$$\lim_{x \rightarrow 2} G(x) = \int_0^2 f(t) dt = 0.$$

Therefore, the limit $\lim_{x \rightarrow 2} \frac{G(x)}{x^2 - 2x}$ is an indeterminate form of

type $\frac{0}{0}$.

Uses L'Hospital's Rule

1 point

Using L'Hospital's Rule,

$$\begin{aligned} \lim_{x \rightarrow 2} \frac{G(x)}{x^2 - 2x} &= \lim_{x \rightarrow 2} \frac{G'(x)}{2x - 2} \\ &= \lim_{x \rightarrow 2} \frac{f(x)}{2x - 2} = \frac{f(2)}{2} = \frac{-4}{2} = -2 \end{aligned}$$

Answer with justification

1 point

Scoring notes:

- To earn the first point, the response must show $\lim_{x \rightarrow 2} (x^2 - 2x) = 0$ and $\lim_{x \rightarrow 2} G(x) = 0$ and must show a ratio of the two derivatives, $G'(x)$ and $2x - 2$. The ratio may be shown as evaluations of the derivatives at $x = 2$, such as $\frac{G'(2)}{2}$.
- To earn the second point, the response must evaluate correctly with appropriate limit notation. In particular the response must include $\lim_{x \rightarrow 2} \frac{G'(x)}{2x - 2}$ or $\lim_{x \rightarrow 2} \frac{f(x)}{2x - 2}$.
- With any linkage errors (such as $\frac{G'(x)}{2x - 2} = \frac{f(2)}{2}$), the response does not earn the second point.

Total for part (c) 2 points

- (d) Find the average rate of change of G on the interval $[-4, 2]$. Does the Mean Value Theorem guarantee a value c , $-4 < c < 2$, for which $G'(c)$ is equal to this average rate of change? Justify your answer.

$G(2) = \int_0^2 f(t) dt = 0$ and $G(-4) = \int_0^{-4} f(t) dt = -16$	Average rate of change $\frac{G(2) - G(-4)}{2 - (-4)} = \frac{0 - (-16)}{6} = \frac{8}{3}$	1 point
Yes, $G'(x) = f(x)$ so G is differentiable on $(-4, 2)$ and continuous on $[-4, 2]$. Therefore, the Mean Value Theorem applies and guarantees a value c , $-4 < c < 2$, such that $G'(c) = \frac{8}{3}$.	Answer with justification	1 point

Scoring notes:

- To earn the first point, a response must present at least a difference and a quotient and a correct evaluation. For example, $\frac{0 + 16}{6}$ or $\frac{G(2) - G(-4)}{6} = \frac{16}{6}$.
- Simplification of the numerical value is not required to earn the first point, but any simplification must be correct.
- The second point can be earned without the first point if the response has the correct setup but an incorrect or no evaluation of the average rate of change. The Mean Value Theorem need not be explicitly stated provided the conditions and conclusion are stated.

Total for part (d)	2 points
Total for question 4	9 points

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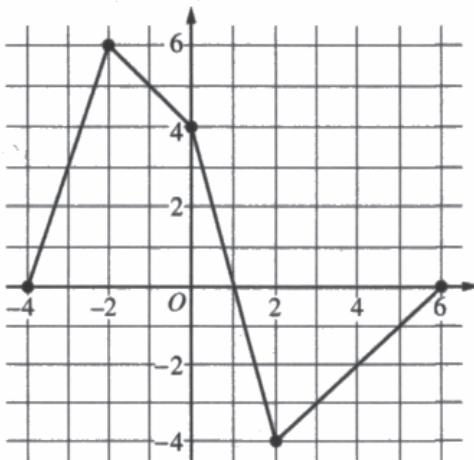
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Answer QUESTION 4 parts (a) and (b) on this page.

Graph of f

Response for question 4(a)

$$G'(x) = f(x)$$

$$G''(x) = f'(x)$$

On $(-4, -2)$ and $(2, 6)$, $G(x)$ is concave up because $f(x)$ (which is equal to $G'(x)$) has a positive slope / is increasing.

Response for question 4(b)

$$P'(x) = G'(x) f(x) + f'(x) G(x)$$

$$P'(3) = G'(3) f(3) + f'(3) G(3)$$

$$\downarrow \\ G'(x) = f(x)$$

$$G'(3) = f(3) = -3$$

$$\downarrow \\ G(3) = \int_0^3 f(t) dt = -\frac{7}{2}$$

$$(P'(3) = (-3)(-3) + (1)(-\frac{7}{2}))$$

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Answer QUESTION 4 parts (c) and (d) on this page.

Response for question 4(c)

$$\lim_{x \rightarrow 2} G(x) = \lim_{x \rightarrow 2} (x^2 - 2x) = 0 \quad \text{Must use l'Hôpital's rule}$$

$$\leftarrow \int_0^2 f(t) dt = 0$$

$$\lim_{x \rightarrow 2} \frac{G'(x)}{2x-2} = \frac{f(2)}{4-2} = \left(\frac{-4}{2} \right)$$

Response for question 4(d)

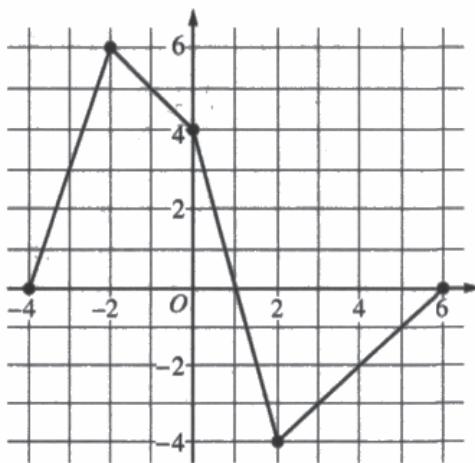
$$\text{AROC of } G = \frac{G(2) - G(-4)}{2 - (-4)} = \frac{0 - (-16)}{2 + 4} = \frac{16}{6} = \frac{8}{3}$$

$$\int_0^2 f(t) dt \quad \int_0^{-4} f(t) dt = -(3+9+3+1) \\ = -16$$

The mean value theorem does guarantee a value c if $-4 < c < 2$, for which $G'(c)$ is equal to average rate of change. This is because $G'(c) = f(t)$ and $x=t$ exists for all values, $-4 \leq x \leq 2$, meaning that $G(x)$ is continuous on the closed interval and differentiable on the open interval.

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Answer QUESTION 4 parts (a) and (b) on this page.



Graph of f

Response for question 4(a)

G is concave up on the intervals $(-4, -2) \cup (2, 6)$

This is because $G''(x) = f''(x)$, $f''(x)$ is positive in the intervals $(-4, -2) \cup (2, 6)$ because the slope of f is positive here. Since $G''(x) = f''(x)$, $G''(x)$ is also positive meaning G is concave up.

Response for question 4(b)

$$P(x) = G'(x) \cdot f(x) + G(x) \cdot f'(x)$$

$$P'(x) = G'(x) \cdot f(x) + G(x) \cdot f'(x)$$

$$P'(3) = G'(3) \cdot f(3) + \int_0^3 f(t) dt \cdot 1$$

$$P'(3) = (-3)(-3) + [1(4)(\frac{1}{2}) - 1(4)(\frac{1}{2}) - (3)(-\frac{1}{2})(\frac{1}{2})]$$

$$P'(3) = 9 + (-3 - \frac{1}{2}) = 9 - \frac{7}{2} = \frac{11}{2} \quad P'(3) = \frac{11}{2}$$

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Answer QUESTION 4 parts (c) and (d) on this page.

Response for question 4(c)

$$\lim_{x \rightarrow 2} \frac{G(x)}{x^2 - 2x} = \lim_{x \rightarrow 2} \frac{\int_0^2 f(t) dt}{x(x-2)} = \frac{0}{0} \quad L'Hopital's \text{ rule}$$

$$\lim_{x \rightarrow 2} \frac{\int_0^x f(t) dt}{x^2 - 2x} = \lim_{x \rightarrow 2} \frac{f(x)}{2x-2} = \frac{f(2)}{2(2)-2} = \frac{-4}{2} = -2$$

$$\lim_{x \rightarrow 2} \frac{G(x)}{x^2 - 2x} = -2$$

Response for question 4(d)

$$G'(c) = \frac{G(2) - G(-4)}{2 - (-4)}$$

$$G'(c) = \frac{\int_0^2 f(t) dt - \int_0^{-4} f(t) dt}{6}$$

$$G'(c) = \frac{0 - \left[(2)(6)\left(\frac{1}{2}\right) + 2(4) + (2)(2)\left(\frac{1}{2}\right) \right](-1)}{6}$$

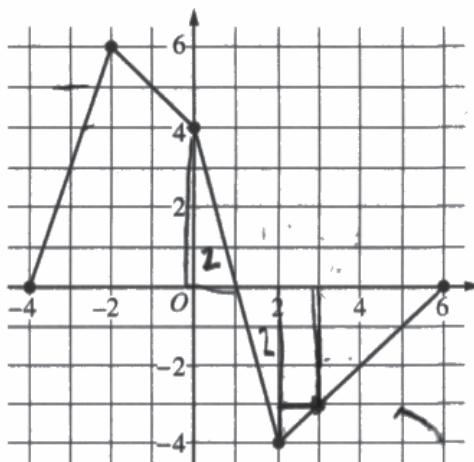
$$G'(c) = \frac{6 + 8 + 2}{6} = \frac{16}{6} = \frac{8}{3}$$

The Mean Value Theorem guarantees a value c for $-4 < c < 2$ for which $G'(c)$ equals the average rate of change because $G(x)$ is completely differentiable on the interval.

$$G'(c) = G'_c(c) = \frac{8}{3}$$

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Answer QUESTION 4 parts (a) and (b) on this page.

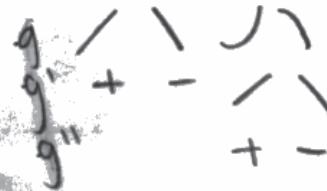
Graph of $f = g'$

Response for question 4(a)

$$f(x) = g'(x)$$

G is concave up on
 $(-4, -2)$ & $(2, 6)$ because

$f(x) = g'(x)$ is increasing
 on these intervals.



Response for question 4(b)

$$P(x) = G(x) \cdot f(x)$$

$$P(x) = G(x) \cdot g'(x)$$

$$P'(x) = g'(x)g'(x) + g(x)g''(x)$$

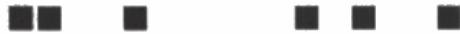
$$\begin{aligned} P'(3) &= (-3)(-3) + (2 - 2 - \frac{1}{2} + 3)(1) \\ &= 9 + (-3 - \frac{1}{2}) \end{aligned}$$

$$\begin{aligned} &= 6 - \frac{1}{2} \\ P'(3) &= \frac{11}{2} \end{aligned}$$

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Answer QUESTION 4 parts (c) and (d) on this page.

Response for question 4(c)

$$\lim_{x \rightarrow 2} \frac{G(x)}{x^2 - 2x} = \frac{0}{0}$$

$$\lim_{x \rightarrow 2} \frac{G'(x)}{2x - 2} = \frac{-4}{2} = \boxed{-2}$$

Response for question 4(d)

$$\text{arvc} = \frac{g'(2) - g'(-4)}{2 - (-4)}$$

$$= \frac{-4 - 0}{6} \\ = \boxed{-\frac{2}{3}}$$

No, the Mean value theorem does not guarantee a value c , $-4 < c < 2$ for which $g'(c)$ is equal to this average rate of change because $G'(c)$ is not differentiable throughout.

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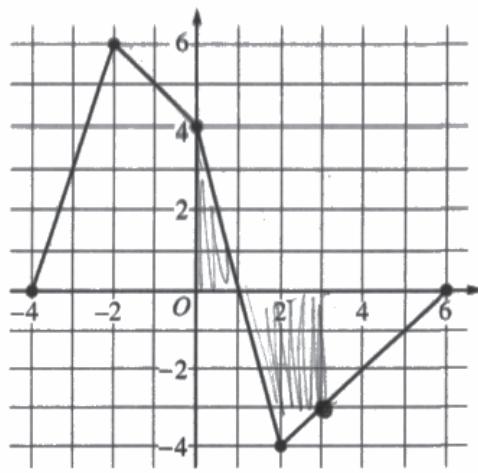
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Answer QUESTION 4 parts (a) and (b) on this page.



$$G(x) = \int_0^x f(t) dt$$

Graph of f

Response for question 4(a)

$$G'(x) = f(x) \leftarrow f(0) = 0$$

$$G''(x) = F'(x)$$

concave up: $[-4, -2]$ and $[2, 6]$

$$G''(x) = F'(x) > 0$$

Response for question 4(b)

$$P(x) = G(x) \cdot F(x)$$

$$P'(x) = G(x) F'(x) + F(x) G'(x)$$

$$P'(3) = G(3) \cdot F'(3) + F(3) \cdot G'(3)$$

$$= \left[\frac{1(4)}{2} - \left(\frac{1(4)}{2} + 3 + \frac{7}{2} \right) \right] 1 + -3(-3)$$

$$= -\frac{7}{2} - 9 = \frac{-7-18}{2} = \boxed{-\frac{25}{2}}$$

Page 10

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Answer QUESTION 4 parts (c) and (d) on this page.

Response for question 4(c)

$$\lim_{x \rightarrow 2} \frac{G(x)}{x^2 - 2x} = \frac{G(2)}{4 - 4} = \frac{0}{0} \quad L'H$$

$$\lim_{x \rightarrow 2} \frac{G'(x)}{2x - 2} = \frac{G'(2)}{2} = \frac{-4}{2} = \boxed{-2}$$

$$G'(2) = f(2) = -4$$

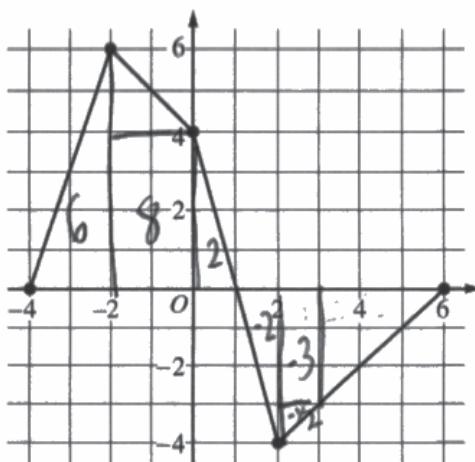
Response for question 4(d)

$$\frac{1}{2+4} \int_{-4}^2 G(x) dx =$$

no because $G(x)$ is not differentiable
at $(-4, 2)$

4 4 4 4 NO CALCULATOR ALLOWED 4 4 4 4

Answer QUESTION 4 parts (a) and (b) on this page.

Graph of f

Response for question 4(a)

The graph of G is concave up on the intervals $(-4, -2) \cup (2, 6)$ because G is concave up when G' is increasing and $G'(x) = f(x)$.

Response for question 4(b)

$$(4)(1)\left(\frac{1}{2}\right) = 2\left(-2 - 3 - \frac{1}{2}\right) = -3 - \frac{1}{2} = -\frac{6}{2} - \frac{1}{2} = -\frac{7}{2}$$

$$P(x) = G(x) \cdot F(x)$$

$$P'(x) = G'(x) \cdot F(x) + F'(x) \cdot G(x) \quad -\frac{5}{18} \cdot -\frac{4}{1}^2 = 10$$

$$P'(3) = G'(3) \cdot F(3) + F'(3) \cdot G(3)$$

$$= (-3) \cdot (-3) + (-4) \cdot \left(-\frac{5}{2}\right)$$

$$= 9 + 10 = 19$$

$$P'(3) \approx 19$$

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0071590



• 4 4 4 4 NO CALCULATOR ALLOWED 4 4 4 4

Answer QUESTION 4 parts (c) and (d) on this page.

Response for question 4(c)

$$\lim_{x \rightarrow 2} \frac{G(x) \rightarrow 0}{x^2 - 2x \rightarrow 0}$$

L'hopital rule

$$\lim_{x \rightarrow 2} \frac{G'(x)}{2x - 2} = \frac{G'(2)}{2(2) - 2} = \frac{-4}{2} = -2$$

Response for question 4(d)

$$\frac{1}{6} \int_{-4}^2 G(x) dx = -\frac{4}{6} = -\frac{2}{3}$$

$$\frac{G(2) - G(-4)}{-4 - 2} = \frac{0 - (-16)}{-6} = \frac{16}{-6} = -\frac{8}{3}$$

The MVT doesn't guarantee the value of c , $-4 < c < 2$
 for which $G'(c)$ is equal to the average rate of
 change of G because the values of the average
 rate of change are different since G' is the derivative G .

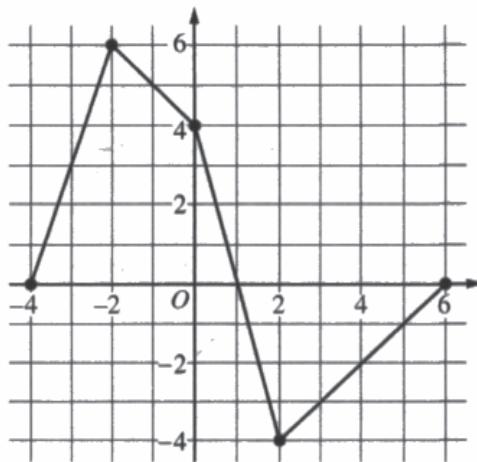
$$\frac{G'(2) - G'(-4)}{-4 - 2} = \frac{-4 - 0}{-6} = \frac{-4}{-6} = \frac{2}{3}$$

$$\frac{1}{2-4} \int_{-4}^2 G(x) dx = \frac{1}{6} \cdot 16 = \frac{16}{6} = \frac{8}{3}$$

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4 4 4 4 NO CALCULATOR ALLOWED 4 4 4 4

Answer QUESTION 4 parts (a) and (b) on this page.

Graph of f

Response for question 4(a)

G is concave up when $G''(x) > 0 \Rightarrow f'(x) > 0$

$$G(x) = \int_1^x f(t) dt$$

$$G'(x) = f(x)$$

$$\underline{G''(x) = f'(x)}$$

$$f'(x) > 0 \text{ on } (-4, -2) \cup (2, 6)$$

\therefore Since $f'(x) > 0$ on $(-4, -2)$ and $(2, 6)$, $G''(x) > 0$ on $(-4, -2)$ and $(2, 6)$. Therefore, G is concave up on the open intervals $(-4, -2) \cup (2, 6)$.

Response for question 4(b)

$$\begin{aligned} P'(x) &= G'(x) f(x) + G(x) f'(x) \\ &= f(x) \cdot f(x) + \int_0^x f(t) dt \cdot f'(x) \\ P'(3) &= f(3) \cdot f(3) + \int_0^3 f(t) dt \cdot f'(3) \\ &= (-3)(-3) + \int_0^3 f(t) dt \cdot 1 \\ &= 9 + \int_0^3 f(t) dt \end{aligned}$$

$$\begin{aligned} &= 9 + [f(3) \cdot 0 - f(0) \cdot 0] \\ &= 9 \end{aligned}$$

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0105057



• 4 4 4 4 NO CALCULATOR ALLOWED 4 4 4 4

Answer QUESTION 4 parts (c) and (d) on this page.

Response for question 4(c)

$$\begin{aligned} \lim_{x \rightarrow 2} \frac{G(x)}{x^2 - 2x} &\xrightarrow{\text{L'Hopital's rule}} \lim_{x \rightarrow 2} \frac{G'(x)}{2x - 2} \\ &= \lim_{x \rightarrow 2} \frac{f(x)}{2x - 2} \\ &= \frac{f(2)}{2(2) - 2} \\ &= \frac{-4}{4 - 2} \\ &= \frac{-4}{2} \\ &= \boxed{-2} \end{aligned}$$

Response for question 4(d)

Average rate of change

$$\begin{aligned} G'(c) &= \frac{G(2) - G(-4)}{2 - (-4)} \\ &= \frac{\int_0^2 f(t) dt - \int_0^{-4} f(t) dt}{6} \\ &= \frac{[f(2) \cdot 0 - f(0) \cdot 0] - [f(-4) \cdot 0 - f(0) \cdot 0]}{6} \\ &= \frac{0}{6} \\ &= 0 \end{aligned}$$

$$G'(c) = f(c)$$

$$0 = f(c)$$

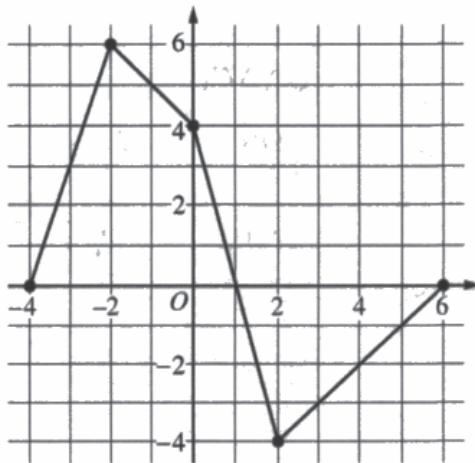
$$\underline{c = 1}$$

The mean value theorem guarantees a value c , $-4 < c < 2$ for which $G'(c)$ is equal to 0, its average rate of change.

$$\underline{G'(c) = 0 \text{ when } c = 1}$$

4 4 4 4 NO CALCULATOR ALLOWED 4 4 4 4

Answer QUESTION 4 parts (a) and (b) on this page.



Graph of f

Response for question 4(a)

f is concave up on the interval $(-4, -2) \cup (2, 6)$ because $f''(x) = g(x)$ and $g(x)$ is positive on these intervals.

Response for question 4(b)

$$\begin{aligned} & \cancel{-3 \int_0^3 t^{\frac{3}{2}} dt} = \cancel{-3(\frac{9}{2})} \\ & F(x) = G(x) + f(x) = G(x) \\ & P(3) = \left(\frac{-27}{2}\right) + -\frac{9}{2}. \end{aligned}$$

Page 10

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0001480



• 4 4 4 4 NO CALCULATOR ALLOWED 4 4 4 4

Answer QUESTION 4 parts (c) and (d) on this page.

Response for question 4(c)

$$\lim_{x \rightarrow 2} \frac{0}{0}$$

chopital's

$$\lim_{x \rightarrow 2} \frac{6x}{2x-2} = \frac{-4}{2}$$

$$\lim_{x \rightarrow 2} \frac{6x}{2x-2} = -2$$

Response for question 4(d)

A) The average rate of change

$$\begin{array}{l} S_4 \\ S_2 \\ S_{-2} \end{array} \text{ Slope is } 3 \quad \begin{array}{l} S_1 \\ S_0 \end{array} \text{ Slope is } -4 \quad \frac{\sqrt{3^2 - 1^2 + 4^2}}{3}$$

B) The mean value theorem does not apply because there

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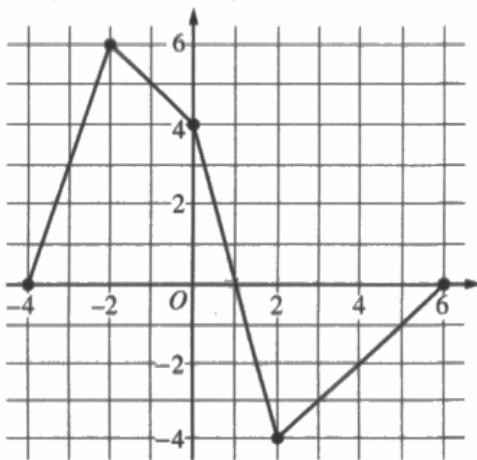
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Answer QUESTION 4 parts (a) and (b) on this page.

Graph of f

$$q'(x) = f(x)$$

Response for question 4(a)

$-4 < x < -2$ and $2 < x < 4$ because the f function
is increasing

Response for question 4(b)

$$p(x) = G(x) \cdot f(x)$$

$$p'(3) = q'(3) \cdot f(3) + f'(3) \cdot g(3)$$

$$(-3)(-3) + (1) ($$

0092377



• 4 4 4 4 NO CALCULATOR ALLOWED 4 4 4 4

Answer QUESTION 4 parts (c) and (d) on this page.

Response for question 4(c)

$$\lim_{x \rightarrow 2} = \frac{G'(x)}{2x - 2}$$

$$\frac{\cancel{G'(2)}}{\cancel{(2)} - 2(2)}$$

~~B~~ $\lim_{x \rightarrow 2}$ does not exist because $\lim_{x \rightarrow 2^-} \neq \lim_{x \rightarrow 2^+}$

Response for question 4(d)

$$\text{B } \frac{G(2) - G(-4)}{2 + 4} = \frac{-4 - 0}{6} = \underline{\underline{-\frac{2}{3}}}$$

No. This function is continuous but not differentiable from $-4 < x < 2$ and therefore MVT cannot be concluded.

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NO CALCULATOR ALLOWED

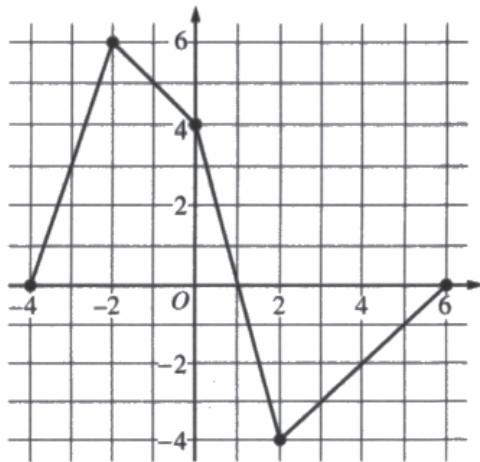
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Answer QUESTION 4 parts (a) and (b) on this page.

Graph of f **Response for question 4(a)**

$$G(x) = \int_0^x f(t) dt$$

$$G(x) = \int f(x) dx$$

$$f(x) = G'(x)$$

Because $f(x)$ is the antiderivative of $G(x)$, that means that $f(x) = G'(x)$. Therefore when the graph $f(x)$ is increasing, then that means $G(x)$ is concave up. G' is concave up on the intervals $(-4, -2) \cup (2, 6)$ b/c f is increasing.

Response for question 4(b)

$$P(x) = G(x) \cdot f(x)$$

$$P'(x) = G'(x) \cdot f(x) + f'(x) \cdot G(x)$$

$$P'(3) = G'(3) \cdot f(3) + f'(3) \cdot G(3)$$

$$P'(3) = -3 \cdot -3 + -3 \cdot -3$$

$$P'(3) = 9 + 9$$

$$P'(3) = 18$$

Page 10

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0087231



• 4 4 4 4 NO CALCULATOR ALLOWED 4 4 4 4

Answer QUESTION 4 parts (c) and (d) on this page.

Response for question 4(c)

$$\lim_{x \rightarrow 2} \frac{G(x)}{x^2 - 2x} \stackrel{\text{L'HOP}}{\rightarrow} \frac{G'(x)}{2x - 2} = \frac{f(x)}{2x - 2}$$

$$= \frac{f(2)}{2(2) - 2}$$

$$= \frac{-4}{2}$$

$$= -2$$

$G'(x) = f(x)$

Response for question 4(d)

$$-4 < c < 2 \quad G(c) = -4 ?$$

$$G(x) = \int_{-4}^2 f(x) dx$$

$$G'(x) = f(x) \Big|_{-4}^2$$

$$= f(2) - f(-4)$$

$$= -4 - 0$$

$$= \boxed{-4}$$

NO b/c c cannot equal
 -4 , it can only be greater
 than it. therefore the Mean
 value theorem does not guarantee
 a value.

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NO CALCULATOR ALLOWED

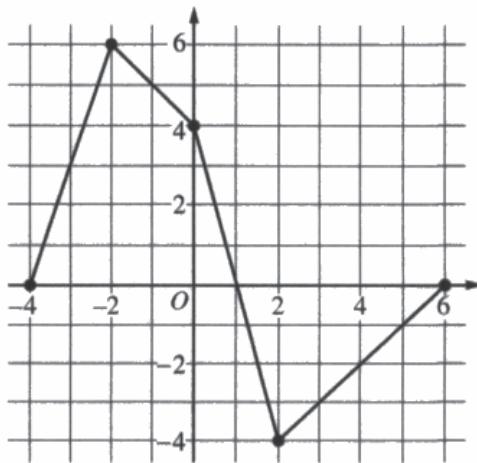
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Answer QUESTION 4 parts (a) and (b) on this page.

Graph of f

Response for question 4(a)

G is concave up $(-4, 1)$ because
 G' switched from negative to positive at
 $x = -4$.

Response for question 4(b)

$$P(x) = G(x) \cdot f(x)$$

$$P'(x) = G'(x) \cdot f(x) + G(x) \cdot f'(x)$$

$$\begin{aligned} P'(3) &= -3 \cdot -3 + \int_0^3 f(t) dt - 1 \\ &\quad 9 + (-3) \end{aligned}$$

$$P'(3) = 6$$

Page 10

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0078646



● 4 4 4 4 NO CALCULATOR ALLOWED 4 4 4 4

Answer QUESTION 4 parts (c) and (d) on this page.

Response for question 4(c)

$$\lim_{x \rightarrow 2} \frac{G(x)}{x^2 - 2x} \quad L'hop rule$$

$$\lim_{x \rightarrow 2} \frac{G'(x)}{2x - 2} = \frac{G'(2)}{2} = -\frac{4}{2} = \boxed{-2}$$

Response for question 4(d)

$$\frac{1}{6} \int_{-4}^2 f(t) dt$$

$$\frac{1}{6} \int_{-4}^2 \frac{1}{2}(2)(t) + \frac{1}{2}(4+6) t^5 - 2 t^{-2} - \frac{7}{2} dt$$

$$\frac{1}{6} \int_{-4}^2 13 + \frac{7}{2} t^2 dt$$

$$\frac{1}{6} \cdot \frac{33}{2} \times \frac{1}{2} (-4)^2$$

$$\frac{26}{2} + \frac{7}{2} = \frac{33}{2}$$

$$\frac{1}{6} (33 - 66)$$

$$\frac{1}{6} (-33)$$

$$\boxed{-\frac{11}{2}}$$

Because of the MVT there is a point c , where $G'(c)$ is equal to this average rate of change.

Page 11

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Question 4**Sample Identifier: A****Score: 9**

- The response earned 9 points: 1 global point, 1 point in part (a), 3 points in part (b), 2 points in part (c), and 2 points in part (d).
- The global point was earned in the first line of part (a) with the statement $G'(x) = f(x)$.
- In part (a) the response earned the point with the presentation of the correct intervals and the reason “ $f(x)$ which is equal to $G'(x)$ has a positive slope/is increasing.”
- In part (b) the response earned the first point with the correct product rule presentation in the first line. The second point was earned for the correct values for both $G'(3)$ and $G(3)$ although only one correct value was necessary for this point. The third point was earned with the expression $(-3)(-3)+(1)\left(-\frac{7}{2}\right)$. Simplification of this expression is not necessary.
- In part (c) the response earned the first point with the extended equation of limits in the first line and the ratio of derivatives in the second line. The second point was earned with the correct ratio of derivatives accompanied by limit notation and the correct (unsimplified) answer.
- In part (d) the response earned the first point for a valid attempt to calculate the average rate of change of G and a correct result. The second point was earned with the answer, “The mean value theorem does guarantee a value c ...,” and the statement that $G(x)$ is both differentiable and continuous.

Sample Identifier: B**Score: 8**

- The response earned 8 points: 1 global point, 1 point in part (a), 3 points in part (b), 2 points in part (c), and 1 point in part (d).
- The global point was earned in the second line of part (a) with the statement $G''(x) = f'(x)$.
- In part (a) the response earned the point with the presentation of the correct intervals and the reason “ $f'(x)$ is positive in these intervals.”
- In part (b) the response earned the first point with the correct product rule in the first line. The second point was earned for the correct values for both $G'(3)$ and $G(3)$ although only one correct value was necessary for this point. The third point was earned with the correct final answer.
- In part (c) the response earned the first point with the $\frac{0}{0}$ in the first line and the ratio of derivatives. (Note: The notation “ $= \frac{0}{0}$ ” is considered acceptable mathematical notation for identifying this indeterminate form. Use of this notation does not impact earning either point in part (c).) The second point was earned with the correct ratio of derivatives accompanied by limit notation and the correct answer.
- In part (d) the response earned the first point for a valid form of the average rate of change of G (using the average value of G') and a correct result. The second point was not earned since the justification does not include a statement of continuity.

Question 4 (continued)**Sample Identifier: C****Score: 7**

- The response earned 7 points: 1 global point, 1 point in part (a), 3 points in part (b), 2 points in part (c), and 0 points in part (d).
- The global point was earned in the first line of part (a) with the statement $f(x) = g'(x)$. This response uses the lower case g to represent the function G in many cases.
- In part (a) the response earned the point with the presentation of the correct intervals and the reason “ $f(x) = g'(x)$ is increasing on these intervals.”
- In part (b) the response earned the first point with the correct product rule in the third line. Note that $f(x)$ was correctly replaced with $g'(x)$ in the second line before the derivative was presented. The second point was earned for the correct values for both $G'(3)$ and $G(3)$ although only one correct value was necessary for this point. The third point was earned with the correct final answer.
- In part (c) the response earned the first point with the $\frac{0}{0}$ in the first line and the ratio of derivatives. (Note: The notation “ $= \frac{0}{0}$ ” is considered acceptable mathematical notation for identifying this indeterminate form. Use of this notation does not impact earning either point in part (c).) The second point was earned with the correct ratio of derivatives accompanied by limit notation and the correct answer.
- In part (d) the response did not earn the first point since an average rate of change of G' is presented (not of G). The response is not eligible for the second point.

Sample Identifier: D**Score: 6**

- The response earned 6 points: 1 global point, 1 point in part (a), 2 points in part (b), 2 points in part (c), and 0 points in part (d).
- The global point was earned in part (a) with the statements $G'(x) = f(x)$ and $G''(x) = f'(x)$ (either statement would have earned the point).
- In part (a) the response earned the point with the presentation of the correct intervals and the reason that $G''(x) = f'(x) > 0$. Note that the inclusion of the endpoints does not affect whether the point is earned.
- In part (b) the response earned the first point with the correct product rule in the second line. The second point was earned for the correct value for $G'(3)$. The value of $G(3)$ is not correct; however, only one correct value was necessary for this point. The third point was not earned since the final answer is not correct.
- In part (c) the response earned the first point with $\frac{0}{0}$ in the first line and the ratio of derivatives in the second line. (Note: The notation “ $= \frac{0}{0}$ ” is considered acceptable mathematical notation for identifying this indeterminate form. Use of this notation does not impact earning either point in part (c).) The second point was earned with the correct ratio of derivatives accompanied by limit notation and the correct answer.
- In part (d) the response did not earn either point since this response gives the average value of G , not the average rate of change of G . The response is not eligible for the second point.

Question 4 (continued)**Sample Identifier: E****Score: 6**

- The response earned 6 points: 1 global point, 1 point in part (a), 2 points in part (b), 2 points in part (c), and 0 points in part (d).
- The global point was earned in part (a) with the statement $G'(x) = f(x)$.
- In part (a) the response earned the point with the presentation of the correct intervals and the reason that G' is increasing.
- In part (b) the response earned the first point with the correct product rule in the second line. The second point was earned with the value of $G'(3)$ as -3 in the fourth line. Note that the incorrect value of $G(3)$ did not affect this point since only one correct value of $G(3)$ or $G'(3)$ is required. The third point was not earned due to the incorrect final answer.
- In part (c) the first point was earned with the arrows pointing from the numerator and denominator to the value 0 and by the ratio of derivatives. The second point was earned with the correct ratio of derivatives accompanied by limit notation and the correct answer.
- In part (d) the first point was not earned since the average rate of change presented is not correct (denominator should be $2 - (-4)$). Because this is not a valid average rate of change form, the response is not eligible for the second point.

Sample Identifier: F**Score: 5**

- The response earned 5 points: 1 global point, 1 point in part (a), 2 points in part (b), 1 point in part (c), and 0 points in part (d).
- The global point was earned in part (a) with the statement $G'(x) = f(x)$.
- In part (a) the response earned the point with the presentation of the correct intervals and the reason that $f'(x) > 0$.
- In part (b) the response earned the first point with the correct product rule in the first line. The second point was earned for the correct value for $G'(3)$ (denoted $f(3)$). The value of $G(3)$ is not correct; however, only one correct value was necessary for this point. The third point was not earned since the final answer is not correct.
- In part (c) the response did not earn the first point since no evidence of $\lim_{x \rightarrow 2} G(x) = 0$ or $\lim_{x \rightarrow 2} (x^2 - 2x) = 0$ is given. The second point was earned with the correct ratio of derivatives accompanied by limit notation and the correct answer.
- In part (d) the response did not earn the first point since the result given is incorrect; however, the response is eligible for the second point since a valid attempt to find the average rate of change of G was presented. The response does not earn the second point since the continuity and differentiability of G is not stated.

Question 4 (continued)**Sample Identifier: G****Score: 5**

- The response earned 5 points: 1 global point, 1 point in part (a), 1 point in part (b), 2 points in part (c), and 0 points in part (d).
- The global point was earned in part (a) with the statement $G''(x) = f'(x)$.
- In part (a) the response earned the point with the presentation of the correct intervals and the reason “ $f'(x)$ is positive on these interval.”
- In part (b) the response earned the first point with the correct product rule presentation. No further points were earned since the values of $G(3)$ and $G'(3)$ are not clearly stated and the final answer is not correct.
- In part (c) the response earned the first point with $\frac{0}{0}$ and the ratio of derivatives. (Note: The notation “ $= \frac{0}{0}$ ” is considered acceptable mathematical notation for identifying this indeterminate form. Use of this notation does not impact earning either point in part (c).) The second point was earned with the correct ratio of derivatives accompanied by limit notation and the correct answer.
- In part (d) the response earned no points since there was not a valid attempt to find the average rate of change of G .

Sample Identifier: H**Score: 4**

- The response earned 4 points: 1 global point, 1 point in part (a), 2 points in part (b), 0 points in part (c), and 0 points in part (d).
- The global point was earned in the statement $g'(x) = f(x)$ above part (a).
- In part (a) the response earned the point with the presentation of the correct intervals and the reason “the f function is increasing.”
- In part (b) the response earned the first point with the correct product rule in the second line. The second point was earned for the correct value for $G'(3)$. Note that only one correct value of $G'(3)$ and $G(3)$ was necessary to earn this point. The third point was not earned as no final answer was presented.
- In part (c) the response did not earn the first point since there is no evidence of $\lim_{x \rightarrow 2} G(x) = 0$ or $\lim_{x \rightarrow 2} (x^2 - 2x) = 0$ given although a ratio of derivatives is presented. The second point was not earned since a final answer was not given.
- In part (d) the response did not earn the first point since the result is incorrect; however, the response is eligible for the second point since a valid attempt to find the average rate of change of G was presented. The second point is not earned since G is stated to be not differentiable.

Question 4 (continued)**Sample Identifier: I****Score: 4**

- The response earned 4 points: 1 global point, 1 point in part (a), 2 points in part (b), 0 points in part (c), and 0 points in part (d).
- The global point was earned in the third line of part (a) with the statement $f(x) = G'(x)$.
- In part (a) the response earned the point with the presentation of the correct intervals and the reason “when $f(x) =$ increasing, then $G(x) =$ concave up.”
- In part (b) the response earned the first point with the correct product rule in the second line. The second point was earned for the correct value for $G(3)$. The value for $G'(3)$ is incorrect, but only one correct value was necessary for this point. The third point was not earned since the final answer is incorrect.
- In part (c) the response did not earn the first point since there is no evidence of $\lim_{x \rightarrow 2} G(x) = 0$ or $\lim_{x \rightarrow 2} (x^2 - 2x) = 0$ given. The second point was not earned since the ratio of derivatives does not have limit notation.
- In part (d) the response did not earn the first point since there is not an attempt to calculate the average rate of change of G . The response is not eligible for the second point.

Sample Identifier: J**Score: 3**

- The response earned 3 points: 0 global points, 0 points in part (a), 2 points in part (b), 1 point in part (c), and 0 points in part (d).
- The global point was not earned since there is no valid connection statement presented in any part of this response.
- In part (a) the response did not earn the point since the interval is incorrect.
- In part (b) the response earned the first point with the correct product rule presentation in the second line. The second point was earned for the correct value for $G'(3)$. Note that only one correct value ($G(3)$ or $G'(3)$) is necessary for this point. The third point was not earned since the final answer is not correct.
- In part (c) the response did not earn the first point since there is no evidence of $\lim_{x \rightarrow 2} G(x) = 0$ or $\lim_{x \rightarrow 2} (x^2 - 2x) = 0$ given. The second point was earned with the correct ratio of derivatives accompanied by limit notation and the correct answer.
- In part (d) the response did not earn the first point since an average value of f was presented. Since the connection of f to G' was never stated, this response is not eligible for points.

5. Consider the function $y = f(x)$ whose curve is given by the equation $2y^2 - 6 = y \sin x$ for $y > 0$.
- (a) Show that $\frac{dy}{dx} = \frac{y \cos x}{4y - \sin x}$.
- (b) Write an equation for the line tangent to the curve at the point $(0, \sqrt{3})$.
- (c) For $0 \leq x \leq \pi$ and $y > 0$, find the coordinates of the point where the line tangent to the curve is horizontal.
- (d) Determine whether f has a relative minimum, a relative maximum, or neither at the point found in part (c). Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

Part B (AB): Graphing calculator not allowed**Question 5****9 points****General Scoring Notes**

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Scoring guidelines and notes contain examples of the most common approaches seen in student responses. These guidelines can be applied to alternate approaches to ensure that these alternate approaches are scored appropriately.

Consider the function $y = f(x)$ whose curve is given by the equation $2y^2 - 6 = y \sin x$ for $y > 0$.

	Model Solution	Scoring
(a) Show that $\frac{dy}{dx} = \frac{y \cos x}{4y - \sin x}$.		
$\frac{d}{dx}(2y^2 - 6) = \frac{d}{dx}(y \sin x) \Rightarrow 4y \frac{dy}{dx} = \frac{dy}{dx} \sin x + y \cos x$	Implicit differentiation	1 point
$\Rightarrow 4y \frac{dy}{dx} - \frac{dy}{dx} \sin x = y \cos x \Rightarrow \frac{dy}{dx}(4y - \sin x) = y \cos x$ $\Rightarrow \frac{dy}{dx} = \frac{y \cos x}{4y - \sin x}$	Verification	1 point

Scoring notes:

- The first point is earned only for correctly implicitly differentiating $2y^2 - 6 = y \sin x$. Responses may use alternative notations for $\frac{dy}{dx}$, such as y' .
- The second point may not be earned without the first point.
- It is sufficient to present $\frac{dy}{dx}(4y - \sin x) = y \cos x$ to earn the second point, provided that there are no subsequent errors.

Total for part (a) 2 points

- (b) Write an equation for the line tangent to the curve at the point $(0, \sqrt{3})$.

At the point $(0, \sqrt{3})$, $\frac{dy}{dx} = \frac{\sqrt{3} \cos 0}{4\sqrt{3} - \sin 0} = \frac{1}{4}$.

An equation for the tangent line is $y = \sqrt{3} + \frac{1}{4}x$.

Answer

1 point

Scoring notes:

- Any correct tangent line equation will earn the point. No supporting work is required. Simplification of the slope value is not required.

Total for part (b) 1 point

- (c) For $0 \leq x \leq \pi$ and $y > 0$, find the coordinates of the point where the line tangent to the curve is horizontal.

$$\frac{dy}{dx} = \frac{y \cos x}{4y - \sin x} = 0 \Rightarrow y \cos x = 0 \text{ and } 4y - \sin x \neq 0$$

Sets $\frac{dy}{dx} = 0$

1 point

$$y \cos x = 0 \text{ and } y > 0 \Rightarrow x = \frac{\pi}{2}$$

$$x = \frac{\pi}{2}$$

1 point

$$\text{When } x = \frac{\pi}{2}, y \sin x = 2y^2 - 6 \Rightarrow y \sin \frac{\pi}{2} = 2y^2 - 6$$

$$y = 2$$

1 point

$$\Rightarrow y = 2y^2 - 6 \Rightarrow 2y^2 - y - 6 = 0$$

$$\Rightarrow (2y + 3)(y - 2) = 0 \Rightarrow y = 2$$

When $x = \frac{\pi}{2}$ and $y = 2$, $4y - \sin x = 8 - 1 \neq 0$. Therefore, the

line tangent to the curve is horizontal at the point $\left(\frac{\pi}{2}, 2\right)$.

1 point

Scoring notes:

- The first point is earned by any of $\frac{dy}{dx} = 0$, $\frac{y \cos x}{4y - \sin x} = 0$, $y \cos x = 0$, or $\cos x = 0$.
- If additional “correct” x -values are considered outside of the given domain, the response must commit to only $x = \frac{\pi}{2}$ to earn the second point. Any presented y -values, correct or incorrect, are not considered for the second point.
- Entering with $x = \frac{\pi}{2}$ does not earn the first point, earns the second point, and is eligible for the third point. The third point is earned for finding $y = 2$. The coordinates do not have to be presented as an ordered pair.
- The third point is not earned with additional points present unless the response commits to the correct point.

Total for part (c) 3 points

- (d) Determine whether f has a relative minimum, a relative maximum, or neither at the point found in part (c). Justify your answer.

$\frac{d^2y}{dx^2} = \frac{(4y - \sin x)\left(\frac{dy}{dx}\cos x - y\sin x\right) - (y\cos x)\left(4\frac{dy}{dx} - \cos x\right)}{(4y - \sin x)^2}$	Considers $\frac{d^2y}{dx^2}$ 1 point
<p>When $x = \frac{\pi}{2}$ and $y = 2$,</p> $\begin{aligned} \frac{d^2y}{dx^2} &= \frac{\left(4 \cdot 2 - \sin \frac{\pi}{2}\right)\left(0 \cdot \cos \frac{\pi}{2} - 2 \cdot \sin \frac{\pi}{2}\right) - \left(2 \cos \frac{\pi}{2}\right)\left(4 \cdot 0 - \cos \frac{\pi}{2}\right)}{\left(4 \cdot 2 - \sin \frac{\pi}{2}\right)^2} \\ &= \frac{(7)(-2) - (0)(0)}{(7)^2} = \frac{-2}{7} < 0. \end{aligned}$	$\frac{d^2y}{dx^2}$ at $\left(\frac{\pi}{2}, 2\right)$ 1 point
<p>f has a relative maximum at the point $\left(\frac{\pi}{2}, 2\right)$ because $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0$.</p>	Answer with justification 1 point

Scoring notes:

- The first point is earned for an attempt to use the quotient rule (or product rule) to find $\frac{d^2y}{dx^2}$.
- The second point is earned for correctly finding $\frac{d^2y}{dx^2}$ and evaluating to find that $\frac{d^2y}{dx^2} < 0$ at $\left(\frac{\pi}{2}, 2\right)$. The explicit value of $-\frac{2}{7}$ or the equivalent does not need to be reported, but any reported values must be correct in order to earn this point.
- The third point can be earned without the second point by reaching a consistent conclusion based on the reported sign of a nonzero value of $\frac{d^2y}{dx^2}$ obtained utilizing $\frac{dy}{dx} = 0$.
- Imports: A response is eligible to earn all 3 points in part (d) with a point of the form $\left(\frac{\pi}{2}, k\right)$ with $k > 0$, imported from part (c).

Alternate Solution for part (d)	Scoring for Alternate Solution
For the function $y = f(x)$ near the point $\left(\frac{\pi}{2}, 2\right)$, $4y - \sin x > 0$ and $y > 0$.	Considers sign of $4y - \sin x$ 1 point
Thus, $\frac{dy}{dx} = \frac{y \cos x}{4y - \sin x}$ changes from positive to negative at $x = \frac{\pi}{2}$.	$\frac{dy}{dx}$ changes from positive to negative at $x = \frac{\pi}{2}$ 1 point
By the First Derivative Test, f has a relative maximum at the point $\left(\frac{\pi}{2}, 2\right)$.	Conclusion 1 point

Scoring notes:

- The first point for considering the sign of $4y - \sin x$ may also be earned by stating that $4y - \sin x$ is not equal to zero.
- The second and third points can be earned without the first point.
- To earn the second point, a response must state that $\frac{dy}{dx}$ (or $\cos x$) changes from positive to negative at $x = \frac{\pi}{2}$.
- The third point cannot be earned without the second point.
- A response that concludes there is a minimum at this point does not earn the third point.

Total for part (d) 3 points**Total for question 5 9 points**

Sample A

5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5

Answer QUESTION 5 parts (a) and (b) on this page.

Response for question 5(a)

$$2y^2 - 6 = y \sin x$$

$$4yy' = y \cos x + y' \sin x$$

$$4yy' - y' \sin x = y \cos x$$

$$y'(4y - \sin x) = y \cos x$$

$$y' = \frac{y \cos x}{4y - \sin x}$$

$$\frac{dy}{dx} = \frac{y \cos x}{4y - \sin x}$$

Response for question 5(b)

b. $\frac{dy}{dx}$ At $(0, \sqrt{3})$ = $\frac{\sqrt{3} \cos(0)}{4\sqrt{3} - \sin(0)} = \frac{\sqrt{3}(1)}{4\sqrt{3}} = \frac{\sqrt{3}}{4\sqrt{3}} = \frac{1}{4}$

$$y - \sqrt{3} = \frac{1}{4}(x - 0)$$



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Answer QUESTION 5 parts (c) and (d) on this page.

Response for question 5(c)

$$\frac{dy}{dx} = \frac{y \cos x}{4y - \sin x} \quad y \neq 0: \quad 2y^2 - 6 = y \sin x$$

$$0 = \frac{y \cos x}{4y - \sin x} \quad y \neq 0$$

$$x = \frac{\pi}{2}, \quad 2y^2 - 6 = y \sin\left(\frac{\pi}{2}\right)$$

$$2y^2 - 6 = y(1)$$

$$0 = y \cos x \quad 2y^2 - y - 6 = 0$$

$$y \neq 0 \quad x \neq 0 \quad (2y+3)(y-2) = 0$$

$$0 = y \cos x \quad \text{since } \cos(0) = 1$$

$$x = \frac{\pi}{2} \quad \text{and } 0 \neq 1$$

$f(x)$ has a horizontal tangent at $(\frac{\pi}{2}, 2)$

$$y = 2 \quad 2y+3=0 \quad (\frac{\pi}{2}, -\frac{3}{2})$$

$$2y = -3$$

$$y = -\frac{3}{2}$$

Response for question 5(d)

$$\frac{dy}{dx} = \frac{y \cos x}{4y - \sin x}$$

Since $\frac{d^2y}{dx^2} < 0$ at $(\frac{\pi}{2}, 2)$, f has a relative maximum at that point.

$$\frac{d^2y}{dx^2} = \frac{(4y - \sin x)[y(-\sin x) + y' \cos x] - (y \cos x)(4y' - \cos x)}{(4y - \sin x)^2}$$

$$\text{At } (\frac{\pi}{2}, 2): \frac{d^2y}{dx^2} = \frac{(4 \cdot 2 - 1)[2(-1) + 0 \cos x] - (2 \cos 0)(4(0) - 0)}{(4(2) - 1)^2}$$

$$\frac{d^2y}{dx^2} = \frac{(8-1)(-2)}{(8-1)^2} = \frac{8(-2)}{7^2} = -\frac{16}{49}$$

concave down

5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5

Answer QUESTION 5 parts (a) and (b) on this page.

Response for question 5(a)

$$\begin{aligned}4y \frac{dy}{dx} - 0 &= y \cos x + \frac{dy}{dx} \sin x \\4y \frac{dy}{dx} - \frac{dy}{dx} \sin x &= y \cos x \\\frac{dy}{dx}(4y - \sin x) &= y \cos x \\\frac{dy}{dx} &= \frac{y \cos x}{4y - \sin x}\end{aligned}$$

Response for question 5(b)

$$\begin{aligned}\frac{dy}{dx} &= \frac{\sqrt{3} \cos 0}{4\sqrt{3} \cdot \sin 0} = \frac{\sqrt{3}}{4\sqrt{3}} = \frac{1}{4} \\y - \sqrt{3} &= \frac{1}{4}(x - 0) \\y &= \frac{1}{4}x + \sqrt{3}\end{aligned}$$



• 5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5

Answer QUESTION 5 parts (c) and (d) on this page.

Response for question 5(c)

$$\frac{dy}{dx} = \frac{y \cos x}{4y - 5 \sin x}$$

$$y \cos x = 0$$

$$x = \frac{\pi}{2}$$

$$2y^2 - 6 = y \sin \frac{\pi}{2}$$

$$2y^2 - 6 = y^{(1)}$$

$$2y^2 - y - 6 = 0$$

$$2y^2 - 4y + 3y - 6 = 0$$

$$2y(y-2) + 3(y-2) = 0$$

$$(2y+3)(y-2) = 0$$

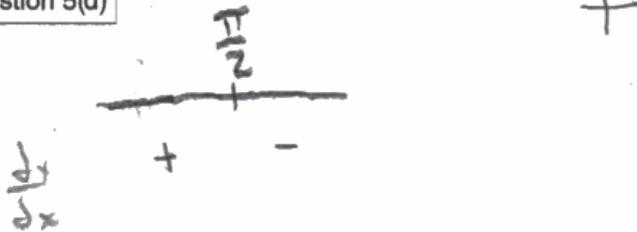
$$(\frac{\pi}{2}, 2)$$

$$y=2$$

$$y=0$$

$$y=2, -\frac{3}{2}$$

Response for question 5(d)



there is a relative max because $\frac{dy}{dx}$
changes from + to -

5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5

Answer QUESTION 5 parts (a) and (b) on this page.

Response for question 5(a)

$$2y^2 - 6 = y \sin x$$

$$4y \frac{dy}{dx} = y \cos x + \frac{dy}{dx} \sin x$$

$$4y \frac{dy}{dx} - \frac{dy}{dx} \sin x = y \cos x$$

$$\frac{dy}{dx} (4y - \sin x) = y \cos x$$

$$\frac{dy}{dx} = \frac{y \cos x}{4y - \sin x}$$

Response for question 5(b)

$$(0, \sqrt{3})$$

$$\frac{dy}{dx} = \frac{\sqrt{3} \cos(0)}{4\sqrt{3} - \sin(0)} = \frac{\sqrt{3}}{4\sqrt{3}} = \boxed{\frac{1}{4}}$$

equation
of tangent:

$$y - \sqrt{3} = \frac{1}{4}(x)$$



• 5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5

Answer QUESTION 5 parts (c) and (d) on this page.

Response for question 5(c)

$$\frac{dy}{dx} = 0 \quad 0 = \frac{y \cos x}{4y - \sin x}$$

$$y \cos x = 0$$

$$\begin{array}{l} \cancel{y \neq 0}, \\ \text{not in range} \end{array} \quad \left| \begin{array}{l} \cos x = 0 \\ x = \frac{\pi}{2} \end{array} \right.$$

$$2y^2 - 6 = y \sin(\frac{\pi}{2})$$

$$2y^2 - 6 = 0$$

$$2y^2 = 6$$

$$\begin{array}{l} y^2 = 3 \\ y = \sqrt{3} \end{array}$$

point $\left(\frac{\pi}{2}, \sqrt{3}\right)$

Response for question 5(d)

$$\frac{d^2y}{dx^2} = \frac{(4y - \sin x)\left(\frac{dy}{dx} \cos x - y \sin x\right) - (y \cos x)\left(4 \frac{dy}{dx} - \cos x\right)}{(4y - \sin x)^2}$$

$$\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} \Big|_{\left(\frac{\pi}{2}, \sqrt{3}\right)} = \frac{(4\sqrt{3} - 1)(0 - \sqrt{3}(1)) - (0)(\dots)}{(4\sqrt{3} - 1)^2}$$

$$= \frac{(4\sqrt{3} - 1)(-\sqrt{3})}{(4\sqrt{3} - 1)^2} \rightarrow \frac{(-)}{(+)}$$

It is a relative max b/c the function is concave down at $x = \frac{\pi}{2}$.

5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5

Answer QUESTION 5 parts (a) and (b) on this page.

Response for question 5(a)

$$2y^2 - 6 = (y \sin x)$$

$$4y \frac{dy}{dx} = (y \cos x) + (\sin x \frac{dy}{dx})$$

$$(4y \frac{dy}{dx} - \sin x \frac{dy}{dx}) = y \cos x$$

$$\frac{dy}{dx} (4y - \sin x) = y \cos x$$

$$\frac{dy}{dx} = \frac{y \cos x}{4y - \sin x}$$

Response for question 5(b)

$$\frac{dy}{dx} = \frac{y \cos x}{4y \sin x} \Big|_{(0, \sqrt{3})} = \frac{3 \cos(0)}{4(\sqrt{3} \sin(0))} = \frac{\sqrt{3}}{4}$$

$$y - \sqrt{3} = \frac{\sqrt{3} \cos(0)}{4(\sqrt{3} \sin(0))} (x - 0)$$

• 5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5

Answer QUESTION 5 parts (c) and (d) on this page.

Response for question 5(c)

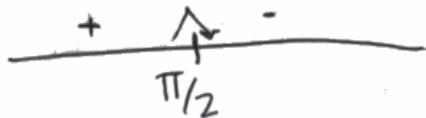
$$\frac{dy}{dx} = \frac{y \cos x}{4y - \sin x} = 0$$

$$y \cos x = 0$$

$$\cos(\pi/2) = 0$$

at the point $(\pi/2, 1)$ the line tangent
to the curve is horizontal

Response for question 5(d)



On the point $(\pi/2, 1)$ f has a relative maximum because the values of $f'(x)$ switch from positive to negative at this x-value.

5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5

Answer QUESTION 5 parts (a) and (b) on this page.

Response for question 5(a)

$$2y^2 - b = y \sin x$$

$$4y \cdot \frac{dy}{dx} - 0 = y(\cos x) + \frac{dy}{dx} \sin x$$

$$(4y - \sin x) \frac{dy}{dx} = y(\cos x)$$

$$\boxed{\frac{dy}{dx} = \frac{y \cos x}{4y - \sin x}}$$

Response for question 5(b)

$$\frac{dy}{dx} = \frac{\sqrt{3} \cos 0}{4\sqrt{3} - \sin 0} = \frac{\sqrt{3}}{4\sqrt{3}} = \frac{1}{4}$$

$$y - y_1 = m(x - x_1)$$

$$\boxed{y - \sqrt{3} = \frac{1}{4}(x - 0)}$$

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0152667



• 5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5

Answer QUESTION 5 parts (c) and (d) on this page.

Response for question 5(c)

$$\frac{dy}{dx} = \frac{y\cos x}{4y - \sin x} > 0$$

$$y\cos x = 0$$

$$y=0.$$

$$2y^2 - b = y \sin x$$

$$0 - b = 0 \cdot \sin x$$

$$X$$

$$\cos x = 0$$

$$x = \frac{\pi}{2}$$

$$2y^2 - b = y \sin \frac{\pi}{2}$$

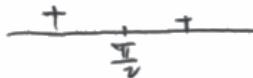
$$2y^2 - b = y$$

$$2y^2 - y - b = 0.$$

$$1 \pm \sqrt{1 - 4(2 \cdot -b)} = 1 \pm \sqrt{1 + 4b} = \frac{1 \pm \sqrt{1 + 4b}}{4} = 2, -\frac{b}{2}$$

$$\boxed{(\frac{\pi}{2}, 2)}$$

Response for question 5(d)



$$x = 0$$

$$2y^2 - b = y \sin \frac{\pi}{2}$$

$$2y^2 - b$$

$$y^2 = \frac{b}{2}$$

$$y = \sqrt{\frac{b}{2}}$$

$$2y^2 - b = y \sin \frac{\pi}{2}$$

$$2y^2 = b$$

$$y^2 = \frac{b}{2}$$

$$y = \sqrt{\frac{b}{2}}$$

$$x = 0$$

$$\frac{dy}{dx} = \frac{y\cos 0}{4y - \sin 0}$$

$$= \frac{y}{4y - 0}$$

$$= \frac{\sqrt{\frac{b}{2}}}{4\sqrt{\frac{b}{2}}}$$

$$= \frac{1}{4}$$

$$x = \pi$$

$$\frac{dy}{dx} = \frac{y\cos \pi}{4y - \sin \pi}$$

$$= \frac{y}{4y - 0}$$

$$= \frac{\sqrt{\frac{b}{2}}}{4\sqrt{\frac{b}{2}}}$$

$$= \frac{1}{4}$$

f has neither at $(\frac{\pi}{2}, 2)$ because $f'(x)$ does not change sign at $x = \frac{\pi}{2}$

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NO CALCULATOR ALLOWED

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Answer QUESTION 5 parts (a) and (b) on this page.

Response for question 5(a)

$$4y(\partial y) = \cos(x)(ay) + \sin(x)(\partial x)y$$

$$(4y - \sin(x))(\partial y) = \cos(x)(\partial x)(y)$$

$$\frac{\partial y}{\partial x} = \frac{\cos(x)(y)}{4y - \sin(x)}$$

Response for question 5(b)

 $\longrightarrow (1, 0)$

$$\frac{\cos(0)\sqrt{3}}{4\sqrt{3} - \sin(0)} = \frac{\sqrt{3} - 1}{4\sqrt{3} - 4}$$

$$y - \sqrt{3} = \frac{1}{4}x$$

$$y = \frac{1}{4}x + \sqrt{3}$$

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0088010



• 5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5

Answer QUESTION 5 parts (c) and (d) on this page.

Response for question 5(c)

$$(0, 1)$$

$$\frac{0}{4y - \sin(y)}$$

$$\frac{y \cos(x)}{4y - \sin(x)} = 0$$

$$\text{when } x = \frac{3\pi}{2}$$

$$y = (\sqrt{3}, 0)$$

$$y = \frac{1}{4}$$

$$\begin{aligned} y' &= \frac{1}{4}x = 0 \\ 2y^2 &= 6 \\ y^2 &= 3 \\ y &= \sqrt{3} \end{aligned}$$

$$\sin(\pi)$$

Response for question 5(d)

$$\frac{y \cos(3\pi/2)}{y}$$

If $f'(x)$ has neither a relative max nor a relative min at the point as $f'(x)$ is continuously positive.

$$\begin{aligned} \frac{2y^2 - 6}{y} &= \sin(x) \\ &= \sin(3\pi/4) \\ &= \left(-\sqrt{2}/2, \sqrt{2}/2\right) \\ \frac{2y^2 - 6}{y} &= \sqrt{2}/2 \end{aligned}$$

5 5 5

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NO CALCULATOR ALLOWED

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Answer QUESTION 5 parts (a) and (b) on this page.

Response for question 5(a)

$$2y^2 - 6 = y \sin x$$

$$4y \frac{dy}{dx} = \frac{dy}{dx} \cdot \sin x + \cos x \cdot y$$

$$4y \frac{dy}{dx} - \frac{dy}{dx} \sin x = \cos x y$$

$$\frac{dy}{dx} (4y - \sin x) = y \cos x$$

$$\frac{dy}{dx} = \frac{y \cos x}{4y - \sin x}$$

Response for question 5(b)

$$\text{at } (0, -\sqrt{3}) \quad \frac{dy}{dx} = \frac{\sqrt{3} \cdot \cos(0)}{4(\sqrt{3}) - \sin(0)} = \frac{\sqrt{3}}{4\sqrt{3}} = \frac{1}{4}$$

$$y - \sqrt{3} = \frac{1}{4}(x - 0)$$

$$y = \frac{1}{4}x + \sqrt{3}$$

Page 12

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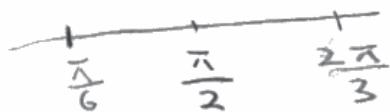
Answer QUESTION 5 parts (c) and (d) on this page.

Response for question 5(c)

$$\frac{y \cos x}{4y - \sin x} = 0$$

$(\frac{\pi}{2}, 1)$ is when $\frac{dy}{dx}$ is horizontal.

Response for question 5(d)



$$2(y^2) - 6 = y \sin(\frac{\pi}{6})$$

$$2y^2 - 6 = y \cdot \frac{1}{2}$$

$$-6 = \frac{1}{2}y - 2y^2$$

$$-12 = y - 2y^2$$

$$-6 = y - y^2$$

$$-y^2 + y + 6 = 0$$

$$(y+3)(y-2)$$

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5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5

Answer QUESTION 5 parts (a) and (b) on this page.

Response for question 5(a)

$$2y^2 - 6 = y \sin x$$

$$4y \frac{dy}{dx} = y \cos(x) + \sin(x) \frac{dy}{dx}$$

$$(4y - \sin(x)) \frac{dy}{dx} = y \cos(x)$$

$$\frac{dy}{dx} = \frac{y \cos(x)}{4y - \sin(x)}$$

Response for question 5(b)

$$\frac{dy}{dx} = \frac{\sqrt{3} \cos(\alpha)}{4\sqrt{3} - \sin(\alpha)} = \frac{\sqrt{3}}{4\sqrt{3}} = \frac{1}{4}$$

$$y - \sqrt{3} = \frac{1}{4}x$$

or $y = \frac{1}{4}x + \sqrt{3}$

Page 12

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0149582



● 5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5 5

Answer QUESTION 5 parts (c) and (d) on this page.

Response for question 5(c)

$$\frac{y \cos(x)}{4y - \sin(x)} = 0 \rightarrow y \cos(x) = 0 \quad y \neq 0 \quad 0 \leq x \leq \pi \\ x = \frac{\pi}{2}$$

$$2y^2 - 6 = y \sin\left(\frac{\pi}{2}\right)$$

$$2y^2 - 6 = 0$$

$$2y^2 = 6$$

$$y^2 = 3$$

$$y = \sqrt{3}$$

$$\left(\frac{\pi}{2}, \sqrt{3}\right)$$

Response for question 5(d)

Maximum: at $x=0$, $\frac{dy}{dx} = \frac{1}{4}$, while at $x=\frac{\pi}{2}$, $\frac{dy}{dx}=0$. This change from a positive derivative to zero proves a relative maximum.

Neither. At $x=0$ and at $x=\frac{\pi}{2}$, y is $\sqrt{3}$. The derivative at $x=0$ is positive, but since y is still $\sqrt{3}$ at $x=\frac{\pi}{2}$, the function was not actually increasing (or decreasing) so there are no extrema. This function does not work like a normal function.

5 5 5 5

NO CALCULATOR ALLOWED

5 5 5 5

Answer QUESTION 5 parts (a) and (b) on this page.

Response for question 5(a)

$$4y \frac{dy}{dx} = \frac{dy \sin x + y \cos x}{dx}$$

$$4y \frac{dy}{dx} - \sin x \frac{dy}{dx} = y \cos x$$

$$\frac{dy}{dx} = \frac{y \cos x}{4y - \sin x}$$

Response for question 5(b)

~~$$2(3) - 6 = \sqrt{3} \sin 0$$~~

~~$$6 - 6 = 0$$~~

$$y - \sqrt{3} = \frac{y \cos x}{4y - \sin x} (x)$$

$$y = \frac{y \cos x}{4y - \sin x} (x) + \sqrt{3}$$

Page 12

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0063086



• 5 5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5

Answer QUESTION 5 parts (c) and (d) on this page.

Response for question 5(c)

$$2y^2 - 6 = y$$

$$x = \pi$$

$$2y^2 - 6 = y \quad (1)$$

$$\frac{2y^2 - 6}{2(y^2 - 3)} = \pm\sqrt{3}$$

$$\frac{dy}{dx} = \frac{y \cos x}{4y - \sin x} = 0$$

$$(0, \sqrt{3})$$

$$x = \pi$$

$$(\pi, 0)$$

$$\frac{dy}{dx} = 0 \quad \frac{y \cos x}{4y - \sin x}$$

Response for question 5(d)

$$\frac{d^2y}{dx^2} = \frac{(4y - \sin x)(\cos x \frac{dy}{dx})(-\sin x) - (4 \frac{dy}{dx} + \cos x)(y \cos x)}{(4y - \sin x)^2}$$

$$\text{if } \frac{d^2y}{dx^2} > 0$$

5

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NO CALCULATOR ALLOWED

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Answer QUESTION 5 parts (a) and (b) on this page.

Response for question 5(a)

$$2y^2 - 6 = y \sin x$$



$$4y = \frac{dy}{dx} \sin x + y \cos x$$

$$\therefore \frac{dy}{dx} = y \cos x - 4y \sin x$$

Response for question 5(b)

The equation is $y - \sqrt{3} = \frac{1}{4}(x - 0)$

by using the $\frac{dy}{dx}$ from the past problem and plugging in the points

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Answer QUESTION 5 parts (c) and (d) on this page.

Response for question 5(c)

The curve is horizontal at $\frac{\pi}{2}$ because
for $\frac{dy}{dx}$ is 0 when $\frac{\pi}{2} = x$ which
means the slope there would also
be 0 meaning its horizontal

Response for question 5(d)

The answer is minimum because
the slope is negative before $\frac{\pi}{2}$ and
positive after meaning it is a
relative minimum.

Question 5**Sample Identifier: A****Score: 9**

- The response earned 9 points: 2 points in part (a), 1 point in part (b), 3 points in part (c), and 3 points in part (d).
- In part (a) the response earned the first point in line 2 with a correct implicit differentiation equation. Having earned the first point, the response is eligible to earn the second point. The response earned the second point with correct algebraic work in lines 3, 4, 5, and 6, verifying the given expression for $\frac{dy}{dx}$. Note that the response would have earned the second point with either line 3 or line 4 leading to either line 5 or line 6.
- In part (b) the response earned the point for a correct equation of the tangent line on line 2.
- In part (c) the response earned the first point at the beginning of line 2 for setting the given expression for $\frac{dy}{dx}$ equal to 0. The response would have earned the second point at the beginning of line 6 for the equation $x = \frac{\pi}{2}$ with no other x -values present. In this case, the response earned the second and third points with the commitment to the single ordered pair $\left(\frac{\pi}{2}, 2\right)$ in the circled statement.
- In part (d) the response earned the first point in line 2 for an attempt to find $\frac{d^2y}{dx^2}$ using the quotient rule. The response earned the second point for a correct expression for $\frac{d^2y}{dx^2}$ found on line 2 followed by a correct evaluation of $\frac{d^2y}{dx^2}$ at the point $\left(\frac{\pi}{2}, 2\right)$ in line 3 with no subsequent errors. The response earned the third point with the circled statement, presenting a correct conclusion with the justification “ $\frac{d^2y}{dx^2} < 0$ at $\left(\frac{\pi}{2}, 2\right)$.”

Question 5 (continued)**Sample Identifier: B****Score: 8**

- The response earned 8 points: 2 points in part (a), 1 point in part (b), 3 points in part (c), and 2 points in part (d).
- In part (a) the response earned the first point in line 1 with a correct implicit differentiation equation. Having earned the first point, the response is eligible to earn the second point. The response earned the second point with correct algebraic work in line 2 and no subsequent errors leading to line 4, verifying the given expression for $\frac{dy}{dx}$.
- In part (b) the response would have earned the point for a correct tangent line equation in line 2. In this case, the response earned the point for an equivalent equation, boxed on line 3.
- In part (c) the response earned the first point on line 2 for the equation $y \cos x = 0$. The response would have earned the second point on line 2 for the statement “ $x = \frac{\pi}{2}$ ” with no other x -values present. In this case, the response earned both the second and third points with the correct boxed ordered pair $\left(\frac{\pi}{2}, 2\right)$.
- In part (d) the response does not present an attempt to find the second derivative as required in the primary solution shown in the scoring guide, so we look to the alternate solution. The response does not reference the sign of $4y - \sin x$, so did not earn the first point. The response earned the second and third points for the statement “there is a relative max because $\frac{dy}{dx}$ changes from + to -.” The response is not required to reference the point $\left(\frac{\pi}{2}, 2\right)$ in the presence of a correct x -value found in part (c), as the prompt specifically states “at the point found in part (c).” If an incorrect x -value is found in part (c), the response must clearly reference $x = \frac{\pi}{2}$ in part (d) to be eligible to earn the second and third points.

Question 5 (continued)**Sample Identifier: C****Score: 7**

- The response earned 7 points: 2 points in part (a), 1 point in part (b), 2 points in part (c), and 2 points in part (d).
- In part (a) the response earned the first point in line 2 with a correct implicit differentiation equation. Having earned the first point, the response is eligible to earn the second point. The response would have earned the second point with either line 3 or line 4 leading to line 5. In this case, the second point is earned for all work correctly presented.
- In part (b) the response earned the point with the boxed equation of the tangent line. Note that any form of this equation would be accepted.
- In part (c) the response earned the first point with the initial statement on line 1 of “ $\frac{dy}{dx} = 0$.” The response would have earned the second point with the boxed result of $x = \frac{\pi}{2}$ on line 4 with no other x -values present. In this case the second point was earned with the boxed ordered pair at the start of line 5, the last line. The y -value in this ordered pair is incorrect, so the response did not earn the third point.
- In part (d) the response earned the first point on line 1 for attempting to find $\frac{d^2y}{dx^2}$. The response earned the second point for a correct expression for $\frac{d^2y}{dx^2}$ on line 1 with a correct evaluation on line 3 consistent with the eligible ordered pair of $\left(\frac{\pi}{2}, \sqrt{3}\right)$ imported from part (c). The response did not earn the third point, as the justification given does not refer to the sign of $\frac{d^2y}{dx^2}$. The response does conclude that the point is a maximum, but the statement “b/c the function is concave down” does not clarify how the sign of the second derivative led to this conclusion.

Question 5 (continued)**Sample Identifier: D****Score: 6**

- The response earned 6 points: 2 points in part (a), no points in part (b), 2 points in part (c), and 2 points in part (d).
- In part (a) the response earned the first point in line 2 with a correct implicit differentiation equation. Having earned the first point, the response is eligible to earn the second point. The response would have earned the second point with the work in either line 3 or line 4 leading to line 5. In this case, the response earned the second point with correct algebraic verification work in lines 3, 4, and 5.
- In part (b) the response did not earn the point because there is an error in the presentation of the slope value, missing the subtraction in the denominator of the expression.
- In part (c) the response earned the first point in line 1 by setting $\frac{dy}{dx}$ equal to 0. The response earned the second point in line 4 with the correct x -value of $\frac{\pi}{2}$ presented in the ordered pair. The response presents an incorrect y -value of 1 in the ordered pair and did not earn the third point.
- In part (d) the response does not present an attempt to find the second derivative as required in the primary solution shown in the scoring guide, so we look to the alternate solution. The response does not reference the sign of $4y - \sin x$ and did not earn the first point. The response is eligible for the second and third points because the response references a point with the correct x -value of $\frac{\pi}{2}$. The response earned the second and third points with the statement “ f has a relative maximum because the values of $f'(x)$ switch from positive to negative at this x -value.” Note that the stem of the question states that $y = f(x)$, thus $f'(x)$ is an acceptable alternative notation for $\frac{dy}{dx}$.

Question 5 (continued)**Sample Identifier: E****Score: 6**

- The response earned 6 points: 2 points in part (a), 1 point in part (b), 3 points in part (c), and no points in part (d).
- In part (a) the response earned the first point in line 2 with a correct implicit differentiation equation. Having earned the first point, the response is eligible to earn the second point. The response earned the second point with correct algebraic work in lines 3 and 4. This response demonstrates a minimum amount of verification work required to earn the second point.
- In part (b) the response earned the point on line 3 with a correct tangent line equation.
- In part (c) the response earned the first point in line 1 with the equation $\frac{dy}{dx} = \frac{y \cos x}{4y - \sin x} = 0$. The response would have earned the second point in line 2 of the second column with the presentation of the single x -value of $x = \frac{\pi}{2}$. In this case, the response earned the second and third points by committing to the correct ordered pair in the final line.
- In part (d) the response does not present an attempt to find the second derivative as required in the primary solution shown in the scoring guide, so we look to the alternate solution. The response does not reference the sign of $4y - \sin x$ and did not earn the first point. The response incorrectly determined that $\frac{dy}{dx}$ is not changing signs at the given point, thus the response is not eligible for the second or third points.

Sample Identifier: F**Score: 5**

- The response earned 5 points: 2 points in part (a), 1 point in part (b), 2 points in part (c), and no points in part (d).
- In part (a) the response earned the first point in line 1 for correctly differentiating the given equation. The response presents this in differential form, which is an accepted alternative notation. Having earned the first point, the response is eligible for the second point. The response earned the second point with correct algebraic work in lines 2 and 3. This response demonstrates a minimum amount of verification work required to earn the second point.
- In part (b) the response would have earned the point for a correct tangent line equation on line 2. In this case, the equivalent equation given in line 3 earned the point.
- In part (c) the response earned the first point at the start of line 1 for setting the given expression for $\frac{dy}{dx}$ equal to 0. The response would have earned the second point in line 2 for the statement “when $x = \frac{\pi}{2}$ ” with no other x -values presented. In this case, the ordered pair given on line 3 earned the second point. The response presents an incorrect value of y and did not earn the third point.
- In part (d) the response does not present an attempt to find the second derivative as required in the primary solution shown in the scoring guide, so we look to the alternate solution. The response does not reference the sign of $4y - \sin x$ and did not earn the first point. The response does not state that $\frac{dy}{dx}$ is changing from positive to negative at the given point, thus is not eligible for the second or third points.

Question 5 (continued)**Sample Identifier: G****Score: 5**

- The response earned 5 points: 2 points in part (a), 1 point in part (b), 2 points in part (c), and no points in part (d).
- In part (a) the response earned the first point in line 2 with a correct implicit differentiation equation. Having earned the first point, the response is eligible to earn the second point. The response earned the second point with correct algebraic work in line 3 and no subsequent errors leading to line 5, verifying the given expression for $\frac{dy}{dx}$.
- In part (b) the response would have earned the point for a correct tangent line equation given in line 2. In this case, the response earned the point for an equivalent equation, boxed on line 3.
- In part (c) the response earned the first point in line 1 by setting the given expression for $\frac{dy}{dx}$ equal to 0. The response earned the second point in line 2 with the declared x -value of $\frac{\pi}{2}$ given as the first coordinate in an ordered pair. The response presents an incorrect y -value and did not earn the third point.
- In part (d) the response presents no relevant work and earned no points.

Sample Identifier: H**Score: 5**

- The response earned 5 points: 2 points in part (a), 1 point in part (b), 2 points in part (c), and no points in part (d).
- In part (a) the response earned the first point in line 2 with a correct implicit differentiation equation. Having earned the first point, the response is eligible to earn the second point. The response earned the second point with correct algebraic work in lines 3 and 4. The response shows the minimum verification work required to earn the second point.
- In part (b) the response would have earned the point for the first circled tangent line equation on line 2. In this case, an equivalent equation is given in the second circled tangent line equation. Both solutions are correct, so the response earned the point.
- In part (c) the response earned the first point for the first equation in line 1, setting the given expression for $\frac{dy}{dx}$ equal to 0. The response earned the second point in line 2 with the statement that $x = \frac{\pi}{2}$. Note that no additional x -values are presented. The response does not earn the third point since an incorrect y -value of $\sqrt{3}$ is given in the final ordered pair.
- In part (d) the response does not present an attempt to find the second derivative as required in the primary solution shown in the scoring guide, so we look to the alternate solution. The response does not reference the sign of $4y - \sin x$ and did not earn the first point. The response does not state that $\frac{dy}{dx}$ is changing from positive to negative at the given point, thus the response is not eligible for the second or third points.

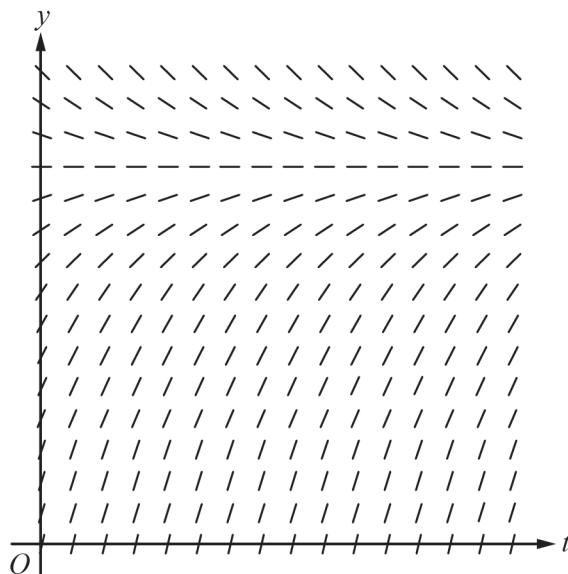
Question 5 (continued)**Sample Identifier: I****Score: 4**

- The response earned 4 points: 2 points in part (a), no points in part (b), 1 point in part (c), and 1 point in part (d).
- In part (a) the response earned the first point in line 1 for a correct implicit differentiation of the given equation. Having earned the first point, the response is eligible to earn the second point. The response earned the second point with correct algebraic work in lines 2 and 3. This response demonstrates a minimum amount of verification work required to earn the second point.
- In part (b) the response did not earn the point. The response does not present a correct numerical expression for the slope in the equation of the tangent line.
- In part (c) the response earned the first point on the last line for the equation $\frac{dy}{dx} = 0$. The response does not present the correct x -value, so did not earn the second point. The response does not present a y -coordinate and so did not earn the third point.
- In part (d) the response earned the first point for an attempt at finding $\frac{d^2y}{dx^2}$ using the quotient rule. The attempt contains errors, so the response is not eligible for the second point. The response presents no further work leading to a consistent conclusion, so the response did not earn the third point.

Sample Identifier: J**Score: 3**

- The response earned 3 points: no points in part (a), 1 point in part (b), 2 points in part (c), and no points in part (d).
- In part (a) the response is missing a factor of $\frac{dy}{dx}$ on the left side of the equation in line 2, thus the response did not earn the first point and is not eligible to earn the second point.
- In part (b) the response earned the point for a correct tangent line equation in line 1.
- In part (c) the response did not earn any points in line 1, since “at $\frac{\pi}{2}$ ” does not specify $x = \frac{\pi}{2}$.
The response earned the first point in line 2, stating that “ f' or $\frac{dy}{dx}$ is 0,” and earned the second point by stating “when $\frac{\pi}{2} = x$.” The response does not present the corresponding y -value ($y = 2$), so did not earn the third point.
- In part (d) the response does not present an attempt to find the second derivative as required in the primary solution shown in the scoring guide, so we look to the alternate solution. The response does not reference the sign of $4y - \sin x$, so it did not earn the first point. In line 1, the response states “the answer is minimum,” so the response is not eligible for the second or third points.

6. A medication is administered to a patient. The amount, in milligrams, of the medication in the patient at time t hours is modeled by a function $y = A(t)$ that satisfies the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$. At time $t = 0$ hours, there are 0 milligrams of the medication in the patient.
- (a) A portion of the slope field for the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$ is given below. Sketch the solution curve through the point $(0, 0)$.



- (b) Using correct units, interpret the statement $\lim_{t \rightarrow \infty} A(t) = 12$ in the context of this problem.
- (c) Use separation of variables to find $y = A(t)$, the particular solution to the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$ with initial condition $A(0) = 0$.

- (d) A different procedure is used to administer the medication to a second patient. The amount, in milligrams, of the medication in the second patient at time t hours is modeled by a function $y = B(t)$ that satisfies the differential equation $\frac{dy}{dt} = 3 - \frac{y}{t+2}$. At time $t = 1$ hour, there are 2.5 milligrams of the medication in the second patient. Is the rate of change of the amount of medication in the second patient increasing or decreasing at time $t = 1$? Give a reason for your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

Part B (AB): Graphing calculator not allowed**Question 6****9 points****General Scoring Notes**

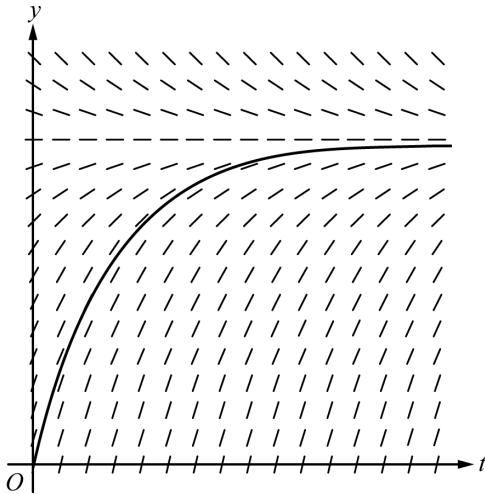
Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Scoring guidelines and notes contain examples of the most common approaches seen in student responses. These guidelines can be applied to alternate approaches to ensure that these alternate approaches are scored appropriately.

A medication is administered to a patient. The amount, in milligrams, of the medication in the patient at time t hours is modeled by a function $y = A(t)$ that satisfies the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$. At time $t = 0$ hours, there are 0 milligrams of the medication in the patient.

Model Solution**Scoring**

- (a) A portion of the slope field for the differential equation $\frac{dy}{dt} = \frac{12 - y}{3}$ is given below. Sketch the solution curve through the point $(0, 0)$.



Solution curve

1 point**Scoring notes:**

- To earn the point the solution curve must pass through the point $(0, 0)$, be generally increasing and concave down, and approach the horizontal asymptote from below as t increases. The point is not earned if two or more solution curves are presented.

Total for part (a) 1 point

- (b) Using correct units, interpret the statement $\lim_{t \rightarrow \infty} A(t) = 12$ in the context of this problem.

Over time the amount of medication in the patient approaches 12 milligrams.	Interpretation	1 point
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Scoring notes:

- To earn the point the interpretation must include “medication in the patient,” “approaches 12,” and units (milligrams), or their equivalents.

Total for part (b) **1 point**

- (c) Use separation of variables to find $y = A(t)$, the particular solution to the differential equation

$$\frac{dy}{dt} = \frac{12 - y}{3} \text{ with initial condition } A(0) = 0.$$

$\frac{dy}{dt} = \frac{12 - y}{3} \Rightarrow \frac{dy}{12 - y} = \frac{dt}{3}$	Separation of variables	1 point
$\int \frac{dy}{12 - y} = \int \frac{dt}{3} \Rightarrow -\ln 12 - y = \frac{t}{3} + C$	Antiderivatives	1 point
$\ln 12 - y = -\frac{t}{3} - C \Rightarrow 12 - y = e^{-t/3 - C}$ $\Rightarrow y = 12 + Ke^{-t/3}$	Constant of integration and uses initial condition	1 point
$0 = 12 + K \Rightarrow K = -12$		
$y = A(t) = 12 - 12e^{-t/3}$	Solves for y	1 point

Scoring notes:

- A response of $\frac{dy}{12 - y} = 3 dt$ is a bad separation and does not earn the first point. However, this response is eligible for the second and third points. It cannot earn the fourth point.
- Absolute value bars are not required in this part.
- A response that correctly separates to $\frac{3 dy}{12 - y} = dt$, but then incorrectly simplifies to $\frac{dy}{4 - y} = dt$ earns the first point (for the initial correct separation), is eligible for the second point (for $-\ln|4 - y| = t$, with or without $+C$), but is not eligible for the third or fourth points.
- $+\ln|12 - y| = \frac{t}{3}$ (with or without $+C$) does not earn the second point and is not eligible for the fourth point; $+\ln|12 - y| = \frac{t}{3} + C$ is eligible for the third point.
- In all other cases, the points are earned consecutively – the second point cannot be earned without the first, the third without the second, etc.

Total for part (c) **4 points**

- (d) A different procedure is used to administer the medication to a second patient. The amount, in milligrams, of the medication in the second patient at time t hours is modeled by a function $y = B(t)$ that satisfies the differential equation $\frac{dy}{dt} = 3 - \frac{y}{t+2}$. At time $t = 1$ hour, there are 2.5 milligrams of the medication in the second patient. Is the rate of change of the amount of medication in the second patient increasing or decreasing at time $t = 1$? Give a reason for your answer.

$\frac{dy}{dt} = 3 - \frac{y}{t+2} \Rightarrow \frac{d^2y}{dt^2} = (-1) \frac{\frac{dy}{dt}(t+2) - y}{(t+2)^2}$	Quotient rule 1 point
$B'(1) = 3 - \frac{B(1)}{3} = 3 - \frac{2.5}{3} = \frac{6.5}{3}$ $B''(1) = -\frac{B'(1) \cdot 3 - B(1)}{3^2} = -\frac{6.5 - 2.5}{9} = -\frac{4}{9} < 0$	$B''(1) < 0$ 1 point
The rate of change of the amount of medication is decreasing at time $t = 1$ because $B''(1) < 0$ and $\frac{d^2y}{dt^2}$ is continuous in an interval containing $t = 1$.	Answer with reason 1 point

Scoring notes:

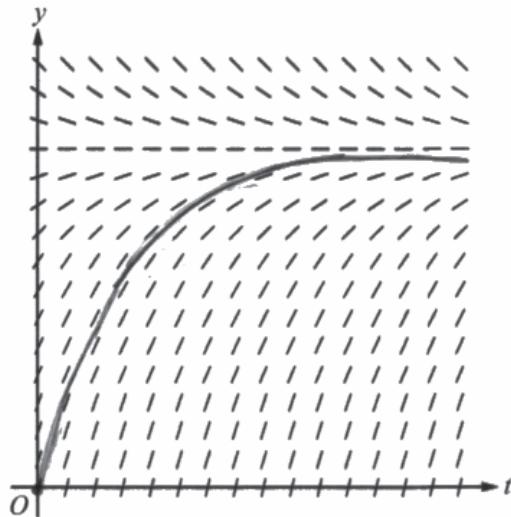
- The first point is for correctly applying the quotient rule to $\frac{y}{t+2}$ or applying the product rule to $y(t+2)^{-1}$. Errors in differentiating the constant, 3, or handling the sign of the second term of $\frac{dy}{dt}$ will result in not earning the second point.
- The second point cannot be earned unless the second derivative $\frac{d^2y}{dt^2}$ is correct.
- For the second point, it is sufficient to state the sign of $B''(1)$ is negative with supporting work. If a value is declared for $B''(1)$, it must be correct in order to earn the second point.
- Eligibility for the third point: An attempt at using the quotient rule (or product rule) to find $B''(1)$. In this case, the third point will be earned for a consistent conclusion based on the declared value (or sign) of $B''(1)$.

Total for part (d) 3 points**Total for question 6 9 points**

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Answer QUESTION 6 parts (a) and (b) on this page.

Response for question 6(a)



Response for question 6(b)

As time goes to infinity, the amount of medication in the patient, in milligrams, is approaching 12.



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Answer QUESTION 6 parts (c) and (d) on this page.

Response for question 6(c)

$$\begin{aligned} \frac{dy}{dt} &= \frac{12-y}{3} \\ \int \frac{dy}{12-y} &= \int \frac{dt}{3} \\ -\ln|12-y| &= \frac{1}{3}t + C \\ -\ln|12-y| &= \frac{1}{3}(0) + C \\ C &= -\ln 12 \\ -\ln|12-y| &= \frac{1}{3}t - \ln 12 \\ \ln|12-y| &= -\frac{1}{3}t + \ln 12 \\ e^{\ln|12-y|} &= e^{-\frac{1}{3}t + \ln 12} \\ 12-y &= e^{-\frac{1}{3}t} \cdot e^{\ln 12} \\ 12-y &= 12e^{-\frac{1}{3}t} \\ -y &= 12e^{-\frac{1}{3}t} - 12 \\ y &= -12e^{-\frac{1}{3}t} + 12 \end{aligned}$$

Response for question 6(d)

$$\begin{aligned} \frac{dy}{dt} &= 3 - \frac{y}{t+2} \\ \text{at } (t,y) = (1, 2.5) &= 3 - \frac{2.5}{1+2} \\ &= 3 - \frac{2.5}{3} \\ &= 3 - \frac{5}{6} < +\# \end{aligned}$$

$$\begin{aligned} \frac{dy}{dt} &= 3 - \frac{y}{t+2} \quad \text{rate of change of the amount} \\ \frac{d^2y}{dt^2} &= -\frac{dy/dx(t+2) - 1(y)}{(t+2)^2} \\ \frac{d^2y}{dt^2} &= -\frac{(3 - \frac{y}{t+2})(t+2) - y}{(t+2)^2} \end{aligned}$$

The rate of change of the amount of medication in the second patient is decreasing at $t=1$ because $\frac{d^2y}{dt^2}|_{(t,y)=(1,2.5)}$ is < 0 .

$$\frac{d^2y}{dt^2}|_{(1,2.5)} = -\frac{(3 - \frac{2.5}{1+2})(1+2) - 2.5}{(1+2)^2} = -\frac{(3 - \frac{5}{3})(3) - 2.5}{3^2} = -\#$$

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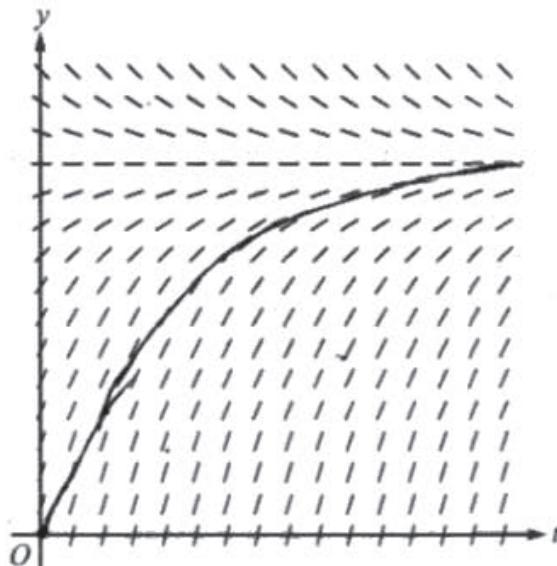
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Answer QUESTION 6 parts (a) and (b) on this page.

Response for question 6(a)



Response for question 6(b) As the amount of time since a medication was administered to a patient approaches 60 hours, the amount, in milligrams, of the medication in the patient's blood approaches 12 milligrams.

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Answer QUESTION 6 parts (c) and (d) on this page.

Response for question 6(c)

$$3 \frac{dy}{dt} = (12-y) dt \Rightarrow \frac{3}{12-y} dy = dt$$

$$\begin{aligned} u &= 12-y \\ \frac{du}{dy} &= -1 \\ -du &= dy \end{aligned}$$

$$\int \frac{3}{12-y} dy = \int dt$$

$$-\int \frac{1}{u} du = \int \frac{1}{3} dt$$

(0,9)

$$-\ln|u| = \frac{1}{3}t + C$$

$$-\ln|12-y| = \frac{1}{3}(t) + C$$

$$-\ln 12 = C$$

$$-\ln|12-y| = \frac{1}{3}(t) - \ln 12$$

$$-|12-y| = e^{\frac{1}{3}(t)-\ln 12}$$

$$-12 - c = y$$

Response for question 6(d)

Because the derivative of the rate of change of the amount of medication in the second patient is negative, the slope of the rate of change of the amount of medication in the second patient is decreasing.

$$\frac{dy}{dt} = 3 - \frac{y}{t+2}$$

$$\frac{dy}{dt} = 3 - \frac{2.5}{3} = \left(3 - \frac{5}{6}\right) = \frac{13}{6}$$

$$\frac{dy}{dt} = \frac{-(t+2)y' + y(1)}{(t+2)^2} \quad t=1, y=2.5$$

$$\frac{dy}{dt} = \frac{-(1+2)\left(\frac{13}{6}\right) + 2.5}{9} = \frac{-\frac{39}{6} + 2.5}{9} =$$

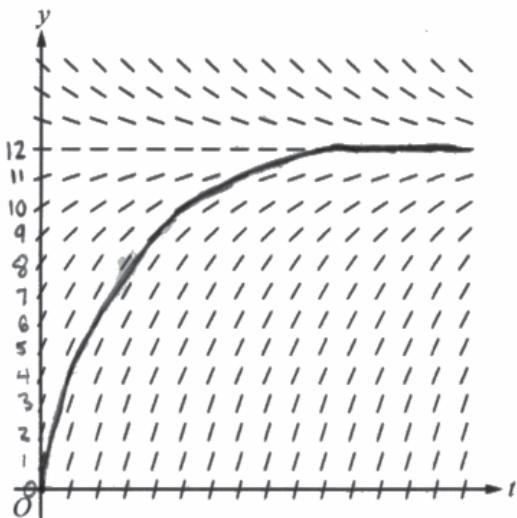
$$\frac{-\frac{39}{6} + \frac{15}{6}}{9} = \text{negative number}$$



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Answer QUESTION 6 parts (a) and (b) on this page.

Response for question 6(a)



Response for question 6(b)

$\lim_{t \rightarrow \infty} A(t) = 12$ means that as time progresses (goes to ∞)

the amount of medication in the patient will approach
12 mg.



• 6 6 6 6 NO CALCULATOR ALLOWED 6 6 6 6

Answer QUESTION 6 parts (c) and (d) on this page.

Response for question 6(c)

$$\int \frac{dy}{12-y} = \int \frac{1}{3} dt$$

$\rightarrow -\ln|12-y| = \frac{1}{3}t + -\ln|12|$
 $e^{-\ln|12-y|} = e^{\frac{1}{3}t + -\ln(12)}$
 $12-y = 12e^{-\frac{1}{3}t}$
 $-y = 12e^{-\frac{1}{3}t} - 12$
 $y = A(t) = 12 - 12e^{-\frac{1}{3}t}$

$v = 12-y$
 $dv = -y dy$
 $-\int \frac{dv}{v} = \frac{1}{3} \int dt$
 $-\ln|12-y| = \frac{1}{3}t + C$
 $-\ln|12| = C$

Response for question 6(d)

$$\frac{d^2y}{dt^2} = 0 - \left[\frac{\frac{dy}{dt}(t+2) - y}{(t+2)^2} \right] = -\frac{\frac{dy}{dt}(t+2) + B(t)}{(t+2)^2}$$

$$\frac{d^2y}{dt^2} = -3 + \frac{y(t+2)}{(t+2)^2} + B(t) = -3 + \frac{B(t) + B(t)}{(t+2)^2}$$

$$\left. \frac{d^2y}{dt^2} \right|_{t=1} = \frac{-3 + 2.5 + 2.5}{9} = \frac{2}{9} > 0$$

The rate of change of the amount of medication in the second patient is increasing at $t=1$ because $\frac{dy}{dt}|_{t=1} > 0$

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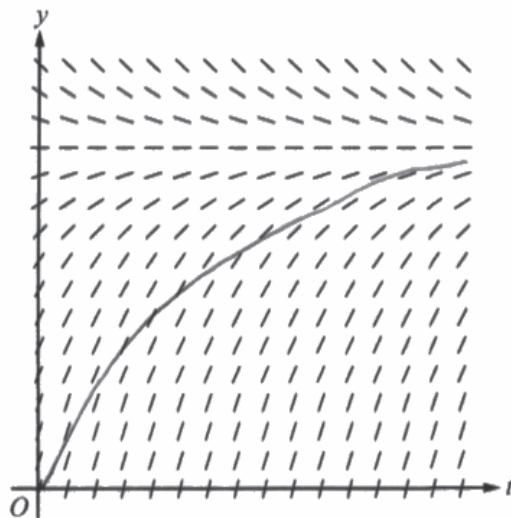
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Answer QUESTION 6 parts (a) and (b) on this page.

Response for question 6(a)



Response for question 6(b)

As time passes since the medication was administered,
the amount of medication in the patient approaches 12mg.

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Answer QUESTION 6 parts (c) and (d) on this page.

Response for question 6(c)

$$\int \frac{1}{12-y} dy = \int \frac{1}{3} dt$$

$$\ln|12-y| = \frac{1}{3}t + C$$

$$\ln|12-y| = \frac{1}{3}(0) + C$$

$$\ln 12 = C$$

$$\ln|12-y| = \frac{1}{3}t + \ln 12$$

$$|12-y| = e^{\frac{1}{3}t + \ln 12}$$

$$12e^{\frac{1}{3}t} = |12-y|$$

$$y = -12e^{\frac{1}{3}t} + 12$$

Response for question 6(d)

$$B(1) = 2.5$$

$$\frac{d^2y}{dt^2} = \frac{(-\frac{dy}{dt})(t+2) - (-y)(1)}{(t+2)^2}$$

$$\frac{d^2y}{dt^2} \Big|_{(1, 2.5)} = \frac{(B - \frac{2.5}{3})(3) + 2.5}{9}$$

The rate of change of medication in the second patient is decreasing at $t=1$ because $\frac{d^2y}{dt^2} < 0$.



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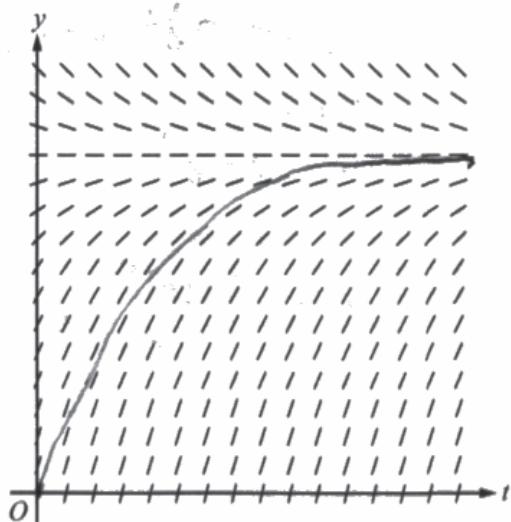
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Answer QUESTION 6 parts (a) and (b) on this page.

Response for question 6(a)



Response for question 6(b)

$\lim_{t \rightarrow \infty} A(t) = 12$ means the amount of medication the patient receives converges to 12 milligrams as hours pass.



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Answer QUESTION 6 parts (c) and (d) on this page.

Response for question 6(c)

$$\frac{dy}{dt} = \frac{12-y}{3}$$

$$\frac{1}{12-y} dy = \frac{1}{3} dt$$

$$-\int \frac{1}{y-12} dy = \int \frac{1}{3} dt$$

$$-\ln|y-12| = \frac{1}{3}t + C$$

$$-\ln|10-12| = \frac{1}{3}(0) + C$$

$$C = -\ln 12$$

$$-\ln|y-12| = \frac{1}{3}t - \ln 12$$

$$\ln|y-12| = -\frac{1}{3}t + \ln 12$$

$$e^{\ln|y-12|} = e^{-\frac{1}{3}t + \ln 12}$$

$$|y-12| = e^{-\frac{1}{3}t} \cdot e^{\ln 12}$$

$$y-12 = \pm 12 e^{-\frac{1}{3}t}$$

$$A(t) = 12 - 12 e^{-\frac{1}{3}t}$$

Response for question 6(d)

$$\frac{d^2y}{dt^2} = \frac{-(t+2) \frac{dy}{dt} + y}{(t+2)^2}$$

$$\left. \frac{d^2y}{dt^2} \right|_{t=1} = \frac{-(1+2)(3 - \frac{2.5}{12}) + 2.5}{(1+2)^2}$$

$$= \frac{-(3)(8.5) + 2.5}{9}$$

$$= \frac{-37.5 + 2.5}{9}$$

The rate of change of the amount of medication in the second patient is decreasing at time $t=1$ since $\left. \frac{d^2y}{dt^2} \right|_{t=1} < 0$.

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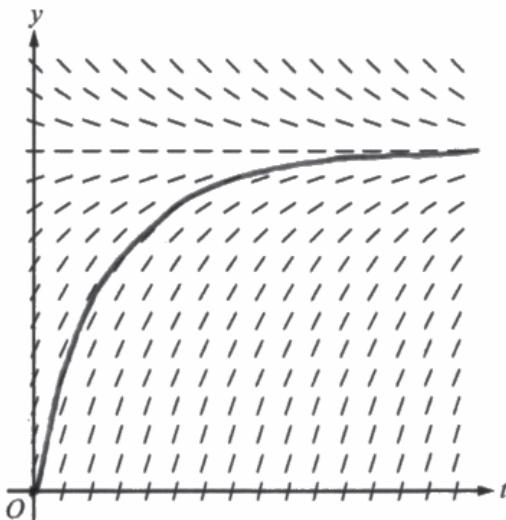
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Answer QUESTION 6 parts (a) and (b) on this page.

Response for question 6(a)



Response for question 6(b)

$\lim_{t \rightarrow \infty} A(t) = 12$ means that 12 milligrams is the maximum amount of medication a patient can receive.

Page 14

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Answer QUESTION 6 parts (c) and (d) on this page.

Response for question 6(c)

Answer QUESTION 6 parts (c) and (d) on this page.

e for question 6(c)

$$3dy = (12-y)dt$$

$$\int \frac{3}{12-y} dy = \int dt$$

$$-3\ln|12-y| = t + C$$

$$e^{-3\ln|12-y|} = e^{t+C}$$

$$12-y = e^{-\frac{1}{3}t - \frac{1}{3}C}$$

$$y = 12 - e^{-\frac{1}{3}t - \frac{1}{3}C}$$

$$0 = 12 - e^{-\frac{1}{3}(0) - \frac{1}{3}C}$$

$$0 = 12 - e^{-\frac{1}{3}C}$$

$$e^{-\frac{1}{3}C} = 12$$

$$-\frac{C}{3} = \ln 12$$

$$C = -3\ln 12$$

$$y = 12 - e^{-\frac{1}{3}t + \ln 12}$$

Response for question 6(d)

$$\frac{dy}{dt^2} = -\frac{(t+2)}{(t+2)^2} - y$$

$$\frac{d^2y}{dt^2} = -\frac{(1+2) - 2.5}{(1+2)^2} = -\frac{0.5}{9} = -\frac{1}{18}$$

The rate of change of the amount of medication is decreasing because the second derivative is less than 0 at $t=1$.

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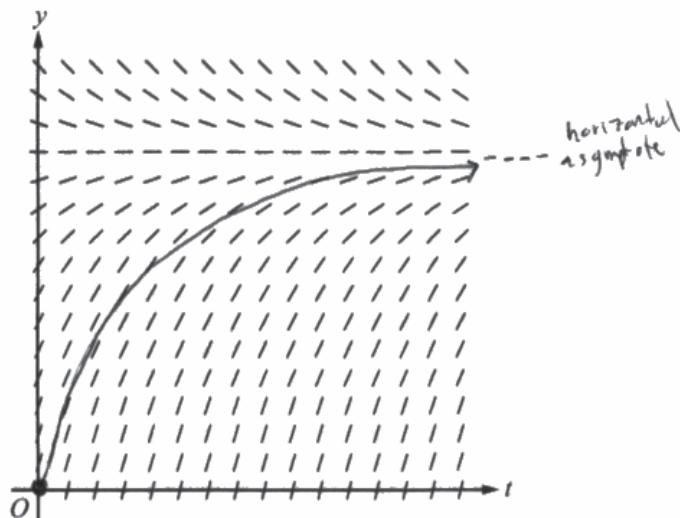
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Answer QUESTION 6 parts (a) and (b) on this page.

Response for question 6(a)



Response for question 6(b)

$$\lim_{t \rightarrow \infty} A(t) = 12$$

So, as time goes on
(approaches infinity)

eventually the amount
of medication in the
patient will level off
at 12 milligrams.

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Answer QUESTION 6 parts (c) and (d) on this page.

Response for question 6(c)

$$\frac{dy}{dt} = \frac{12-y}{3}, (0,0)$$

$$-\ln(12) = C$$

$$\frac{dy}{12-y} = \frac{1}{3} dt$$

$$-\ln(12-y) = \frac{1}{3} t - \ln(12)$$

$$\ln(12-y) = -\frac{1}{3} t + \ln(12)$$

$$-12-y = e^{-\frac{1}{3}t + \ln(12)}$$

$$-y = e^{\frac{1}{3}t} \cdot 12$$

$y = -12e^{\frac{1}{3}t}$

Response for question 6(d)

$$\frac{dy}{dt} = 3 - \frac{y}{t+2}, (1, 2.5)$$

since the solution to the differential equation (derivative) is positive, then the rate of change of medication in the second patient is increasing.

$$\frac{dy}{dt} = 3 - \frac{2.5}{1+2}$$

$$= 3 - \frac{2.5}{3}$$

$$= 3 - \frac{25}{30}$$

$$= 3 - \frac{5}{6}$$

$$= \frac{18}{6} - \frac{5}{6}$$

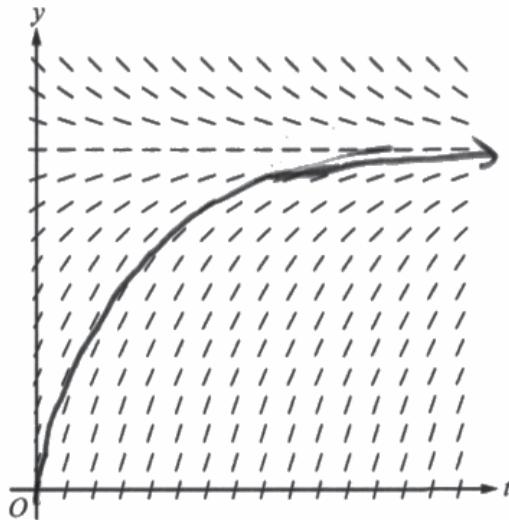
$$= \frac{13}{6}$$



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Answer QUESTION 6 parts (a) and (b) on this page.

Response for question 6(a)



Response for question 6(b)

As time progresses, the amount of medication in the patient will approach but can never reach 12 milligrams.



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Answer QUESTION 6 parts (c) and (d) on this page.

Response for question 6(c)

$$\begin{aligned}\frac{dy}{dt} &= \frac{12-y}{3} \\ \int (12-y)^{-1} dy &< \int dt \\ \ln |12-y| &= 3t + C \\ e^{\ln |12-y|} &= e^{3t+C} \\ |12-y| &= C_1 \cdot e^{3t} \\ 12-y &= C_2 \cdot e^{3t} \\ -y &= C_3 \cdot e^{3t} \\ y &= C_4 \cdot e^{3t}\end{aligned}$$

$$0 = C_4 \cdot e^{3(0)}$$

$$0 = C_4 \cdot e^0$$

$$0 = C_4$$

$$\begin{aligned}e^{C_4} &= C_1 \\ &= C_1 = C_2 \\ C_3 &= C_2 - 12 \\ C_4 &= -C_3\end{aligned}$$

$$(y=0)$$

Response for question 6(d)

$$\begin{aligned}\left. \frac{dy}{dt} \right|_{(1,2.5)} &= - \frac{(t+2) \frac{dy}{dt} - y}{(t+2)^2} \\ &= - \frac{\frac{12}{5}(1+2) - 2.5}{(1+2)^2} \\ &= - \frac{\frac{12}{5} \cdot \frac{3}{1} - \frac{5}{2}}{9} \\ &= - \frac{8}{9} \\ &= - \frac{4}{9}\end{aligned}$$

$$\begin{aligned}\left. \frac{dy}{dt} \right|_{(1,2.5)} &= - \frac{2.5}{1+2} \\ &= - \frac{2.5}{3} \\ &= - \frac{5}{6} \\ &= - \frac{5}{2} \cdot \frac{1}{3} \\ &= - \frac{5}{6} \\ &= \frac{18}{6} - \frac{5}{6} \\ &= \frac{13}{6}\end{aligned}$$

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It is decreasing because $\frac{dy}{dt} < 0$
 $(1,2.5)$ is negative.

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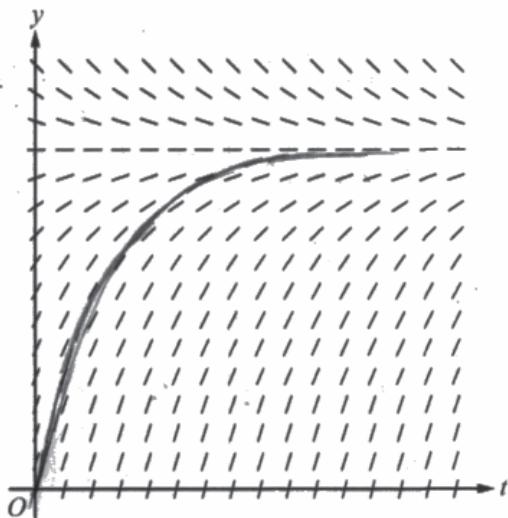
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Answer QUESTION 6 parts (a) and (b) on this page.

Response for question 6(a)



Response for question 6(b)

As time stretches on toward infinity the amount of milligrams of medication in the bloodstream of the patient gets closer and closer to 12 milligrams.

the amount of medication in the bloodstream is always < 12 milligrams
as time goes on .



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Answer QUESTION 6 parts (c) and (d) on this page.

Response for question 6(c)

$$\frac{dy}{dt} = \frac{12-y}{3}$$

$$-\ell^{-\left(\ln|12| + \frac{t}{3}\right)} - 12 = A(t)$$

$$\begin{aligned} \frac{3dy}{12-y} &= \frac{(12-y)dt}{3} \\ \int \frac{3}{12-y} dy &= \int dt \\ -3 \ln|12-y| &= t + C \\ -3 \ln|12-y| + C &= 0 \\ +3 \ln|12| &= C \\ C &= 3 \ln|12| \end{aligned}$$

$$\begin{aligned} -3 \ln|12-y| + 3 \ln|12| &= t \\ -3 \ln|12-y| &= -3 \ln|12| + t \\ \ln|12-y| &= \ln|12| + \frac{t}{-3} \\ e^{\ln|12-y|} &= e^{\ln|12| + \frac{t}{-3}} \\ |12-y| &= |12| e^{\frac{t}{-3}} \\ -12 &= -y \\ -12 &= -y \end{aligned}$$

Response for question 6(d)

$$\frac{dy}{dt} = 3 - \frac{y}{t+2}$$

$$\frac{dy}{dt} = 3 - \frac{2.5}{3}$$

The rate of change of the amount of medication in the patient is increasing at time $t = 1$ hour because the rate is positive at that time

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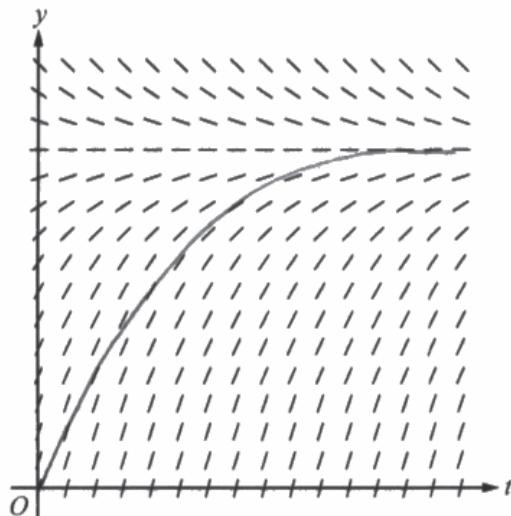
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Answer QUESTION 6 parts (a) and (b) on this page.

Response for question 6(a)



Response for question 6(b)

As t approaches 100 the amount of medication in the patient approaches 12 milligrams of medication.

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Answer QUESTION 6 parts (c) and (d) on this page.

Response for question 6(c)

$$\frac{dy}{dt} = \frac{12-y}{3}$$

$$\frac{dy}{dt} + \frac{y}{3} = 4$$

$$A(t) + \frac{y^2}{6} = 4y$$

$$A(t) = 4y - \frac{y^2}{6}$$

Response for question 6(d)

$$\frac{dy}{dt} = 3 - \frac{y}{t+2}$$

$$\begin{aligned}\frac{d^2y}{dt^2} &= \frac{-(t+2)(\frac{dy}{dt}) - (y)(1)}{(t+2)^2} \\ &= -\left(\frac{(3)(3-\frac{25}{3}) - (2.5)}{9}\right)\end{aligned}$$

$$\frac{dy}{dt} \text{ at } t=1 = 3 - \frac{2.5}{3}$$

The rate of change of the amount of medication in the second patient is decreasing, as " $B''(t)$ " is negative.

Question 6**Sample Identifier: A****Score: 9**

- The response earned 9 points: 1 point in part (a), 1 point in part (b), 4 points in part (c), and 3 points in part (d).
- In part (a) the response earned the point because the solution curve passes through $(0, 0)$, is increasing and concave down, and approaches the horizontal asymptote.
- In part (b) the response was earned because it references both “medication in the patient in milligrams” and “approaching 12.”
- In part (c) the response earned the first point on line 2 on the left side for a correct separation of variables. The second point was earned on line 3 on the left side for correct antiderivatives. The third point was earned for the “ $+ c$ ” on line 3 on the left side with the use of the initial condition on line 4 on the left side. The fourth point was earned for a correct solution presented in the box on the last line on the right side.
- In part (d) the response earned the first point on line 2 for the correct second derivative expression. It is unclear on lines 3 and the first part of line 4 whether or not the leading negative sign is in the numerator or in front of the fraction, however, it is clear on the final presented numerical value. The second point was earned for the correct numeric expression of the second derivative. The expression does not have to be simplified to earn the point. The third point was earned for the answer decreasing with the reasoning based on the sign of the second derivative at $(1, 2.5)$.

Sample Identifier: B**Score: 8**

- The response earned 8 points: 1 point in part (a), 1 point in part (b), 3 points in part (c), and 3 points in part (d).
- In part (a) the response earned the point because the solution curve passes through $(0, 0)$, is increasing and concave down, and approaches the horizontal asymptote. The point can still be earned if the solution curve touches the asymptote.
- In part (b) the response earned the point because of the statement “the amount, in milligrams, of the medication in the patients blood approaches 12 milligrams.”
- In part (c) the response earned the first point on line 1 for a correct form of the separation of variables $\frac{3}{12 - y} dy = dt$. The second point was earned on line 4 for correct antiderivatives with correct u -substitution “ $-\ln|u| = \frac{1}{3}t + c$.” The third point was earned for the “ $+ c$ ” on line 5 with the use of the initial condition on line 6. The response circles an incorrect solution and did not earn the fourth point.
- In part (d) the response earned the first point on line 2 for a correct second derivative expression. The second point was earned on line 3 for the numeric expression of the second derivative at “ $t = 1, y = 2.5$.” The expression does not have to be simplified to earn the point. The third point was earned for the answer decreasing, with the reason “because the derivative of the rate of change of the amount of medication in the second patient @ $t = 1$ is negative.”

Question 6 (continued)**Sample Identifier: C****Score: 8**

- The response earned 8 points: 1 point in part (a), 1 point in part (b), 4 points in part (c), and 2 points in part (d).
- In part (a) the response earned the point because the solution curve passes through $(0, 0)$, is increasing and concave down, and approaches the horizontal asymptote. The solution curve touches the asymptote but does not continue above the asymptote so the point was earned.
- In part (b) the response earned the point because it states “the amount of medication in the patient will approach 12 mg.”
- In part (c) the response earned the first point on line 1 on the left side for the correct separation of variables. The second point was earned on line 5 on the left side for correct antiderivatives. The third point was earned for the “ $+ c$ ” on line 5 with the use of the initial condition on line 6 on the left side. The fourth point was earned for the correct solution on the last line on the right side.
- In part (d) the response earned the first point on line 1 for the correct second derivative expression. The second point was not earned because of the incorrect numeric value $\frac{2}{9}$ resulting from the mistake on the previous line. The third point was earned for the consistent conclusion of increasing based on the declared sign of “ $\left. \frac{d^2y}{dt^2} \right|_{t=1} > 0$.”

Sample Identifier: D**Score: 7**

- The response earned 7 points: 1 point in part (a), 1 point in part (b), 2 points in part (c), and 3 points in part (d).
- In part (a) the response earned the point because the solution curve passes through $(0, 0)$, is increasing and concave down, and approaches the horizontal asymptote.
- In part (b) the response earned the point because it states “the amount of medication in the patient approaches 12 mg.”
- In part (c) the response earned the first point on line 1 on the left side for the correct separation of variables. The second point was not earned because of the incorrect antiderivative, $\ln|12 - y|$ on line 2. It should be $-\ln|12 - y|$. The third point was earned for the “ $+ c$ ” on line 2 on the left side with the use of the initial condition on line 3 on the left side. The response is not eligible for the fourth point because of the incorrect antiderivative.
- In part (d) the response earned the first point on line 2 for a correct second derivative expression. The second point was earned on line 3 for the correct numeric expression of the second derivative at the point $(1, 2.5)$. The expression does not have to be simplified to earn the point. The third point was earned for the answer, decreasing, with the reason “because $\frac{d^2y}{dt^2} < 0$.”

Question 6 (continued)**Sample Identifier: E****Score: 7**

- The response earned 7 points: 1 point in part (a), No points in part (b), 4 points in part (c), and 2 points in part (d).
- In part (a) the response earned the point because the solution curve passes through $(0, 0)$, is increasing and concave down, and approaches the horizontal asymptote.
- In part (b) the response references the amount of medication “converges to 12” which implies the amount approaches 12, however, the point was not earned because the response references “amount of medication the patient receives” instead of medication in the patient.
- In part (c) the response earned the first point on line 2 on the left side for the correct separation of variables. The second point was earned on line 4 on the left side for correct antiderivatives. The third point was earned for the “ $+ c$ ” on line 4 on the left side with the use of the initial condition on line 5 on the left side. The fourth point was earned for the correct solution on the last line on the right side.
- In part (d) the response earned the first point on line 1 for the correct second derivative expression. The second point was not earned because of the incorrect numeric value for the second derivative on line 4 on the left side. The third point was earned for the answer decreasing with the correct reasoning “since $\frac{d^2y}{dt^2} \Big|_{t=1} < 0$.”

Sample Identifier: F**Score: 6**

- The response earned 6 points: 1 point in part (a), no points in part (b), 4 points in part (c), and 1 point in part (d).
- In part (a) the response earned the point because the solution curve passes through $(0, 0)$, is increasing and concave down, and approaches the horizontal asymptote.
- In part (b) the point was not earned because the response states “12 milligrams is the maximum amount” instead of “approaches 12 milligrams” and the response refers to the amount of medication “a patient can receive” instead of the amount of medication in the patient.
- In part (c) the response earned the first point on line 2 on the left side for the correct separation of variables. The second point was earned on line 3 on the left side for the correct antiderivatives. The third point was earned for the “ $+ c$ ” on line 3 on the right side with the use of the initial condition on line 2 on the right side. The fourth point was earned for a correct form of the solution presented in the box on the last line.
- In part (d) the first point was not earned because the expression for the second derivative is incorrect. There is a missing $\frac{dy}{dt}$ in the numerator. The response is not eligible for the second point because the second point cannot be earned without the correct second derivative. The third point was earned for the conclusion, decreasing, based on the reasoning “because the second derivative is less than 0 at $t = 1$.”

Question 6 (continued)**Sample Identifier: G****Score: 5**

- The response earned 5 points: 1 point in part (a), 1 point in part (b), 3 points in part (c), and no points in part (d).
- In part (a) the response earned the point because the solution curve passes through $(0, 0)$, is increasing and concave down, and approaches the horizontal asymptote.
- In part (b) the response earned the point because of the statement “the amount of medication in the patient will level off at 12 milligrams” which implies that it approaches 12 milligrams.
- In part (c) the response earned the first point on line 2 on the left side for the separation of variables. The second point was earned on line 3 on the left side for correct antiderivatives. The response uses “ $-\ln(12 - y)$ ” instead of $-\ln|12 - y|$ which is acceptable since $12 - y$ is greater than 0 for $y = 0$. The third point was earned for the “ $+ c$ ” on line 3 on the left side with the use of the initial condition on line 1 on the right side. The response circles an incorrect solution and did not earn the fourth point.
- In part (d) the response earned no points because there is no reference to the second derivative.

Sample Identifier: H**Score: 4**

- The response earned 4 points: 1 point in part (a), 1 point in part (b), no points in part (c), and 2 points in part (d).
- In part (a) the response earned the point because the solution curve passes through $(0, 0)$, is increasing and concave down, and approaches the horizontal asymptote.
- In part (b) the response earned the point because it states “the amount of medication in the patient will approach but can never reach 12 milligrams.”
- In part (c) the response did not earn the first point because $\int (12 - y)^{-1} dy = \int 3 dt$ is a bad separation. The response is eligible for the second point, which was not earned because of the incorrect antiderivative $\ln|12 - y|$ on line 3 on left side instead of $-\ln|12 - y|$. An algebra mistake on line 7 on the left side caused “ c_3 ” to be handled incorrectly and, as a result, the response is not eligible for the third point. The response is not eligible for the fourth point because of the incorrect antiderivative.
- In part (d) the response did not earn the first point because there is a linkage error on line 1 on the left side. The response equates the numeric second derivative, $\frac{d^2y}{dt^2} \Big|_{(1, 2.5)}$, to the symbolic expression. The response is eligible for the second point because the correct second derivative is given in the middle of line 1. The second point was earned for the correct numeric expression of the second derivative at $(1, 2.5)$ on line 5 on the left. The third point was earned for the circled answer “It is decreasing because $\frac{d^2y}{dt^2}$ at $(1, 2.5)$ is negative.”

Question 6 (continued)**Sample Identifier: I****Score: 4**

- The response earned 4 points: No points in part (a), 1 point in part (b), 3 points in part (c), and no points in part (d).
- In part (a) the response did not earn the point because the solution curve does not extend far enough to the right. It must be within 1 slope field segment of the right side of the window.
- In part (b) the response earned the point because it states “the amount of milligrams of medication in the bloodstream of the patient gets closer and closer to 12 milligrams.”
- In part (c) the response earned the first point on line 3 on the left side for the correct separation of variables $\int \frac{3}{12 - y} dy = \int dt$. The second point was earned on line 4 on the left side for correct antiderivatives. The third point was earned for the “ $+ c$ ” on line 4 on the left side with the use of the initial condition on line 5 on the left side. The fourth point was not earned because the response declares an incorrect expression for $A(t)$, boxed at the top of the right side.
- In part (d) no points were earned because there is no attempt at finding the second derivative.

Sample Identifier: J**Score: 4**

- The response earned 4 points: 1 point in part (a), 1 point in part (b), no points in part (c), and 2 points in part (d).
- In part (a) the response earned the point because the solution curve passes through $(0, 0)$, is increasing and concave down, and approaches the horizontal asymptote.
- In part (b) the response earned the point because it mentions “amount of medication in the patient” and “approaches 12 milligrams.”
- In part (c) the response did not earn the first point because it does not separate the variables. A response that does not separate the variables is not eligible for any other points in part (c).
- In part (d) the response earned the first point on line 2 for the correct second derivative expression. The second point was not earned because equating the symbolic second derivative to the numeric second derivative expression causes a linkage error. In this case, if the linkage error had not occurred, the point would be earned for the correct unsimplified value of the second derivative at $t = 1$. The third point was earned for the answer decreasing with the reason “ $B''(t)$ is negative.”

r (centimeters)	0	1	2	2.5	4
$f(r)$ (milligrams per square centimeter)	1	2	6	10	18

1. The density of a bacteria population in a circular petri dish at a distance r centimeters from the center of the dish is given by an increasing, differentiable function f , where $f(r)$ is measured in milligrams per square centimeter. Values of $f(r)$ for selected values of r are given in the table above.
- (a) Use the data in the table to estimate $f'(2.25)$. Using correct units, interpret the meaning of your answer in the context of this problem.
- (b) The total mass, in milligrams, of bacteria in the petri dish is given by the integral expression

$$2\pi \int_0^4 r f(r) dr.$$
Approximate the value of $2\pi \int_0^4 r f(r) dr$ using a right Riemann sum with the four subintervals indicated by the data in the table.
- (c) Is the approximation found in part (b) an overestimate or underestimate of the total mass of bacteria in the petri dish? Explain your reasoning.
- (d) The density of bacteria in the petri dish, for $1 \leq r \leq 4$, is modeled by the function g defined by

$$g(r) = 2 - 16(\cos(1.57\sqrt{r}))^3.$$
For what value of k , $1 < k < 4$, is $g(k)$ equal to the average value of $g(r)$ on the interval $1 \leq r \leq 4$?

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

2. For time $t \geq 0$, a particle moves in the xy -plane with position $(x(t), y(t))$ and velocity vector $\left\langle (t - 1)e^{t^2}, \sin(t^{1.25}) \right\rangle$. At time $t = 0$, the position of the particle is $(-2, 5)$.
- (a) Find the speed of the particle at time $t = 1.2$. Find the acceleration vector of the particle at time $t = 1.2$.
- (b) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 1.2$.
- (c) Find the coordinates of the point at which the particle is farthest to the left for $t \geq 0$. Explain why there is no point at which the particle is farthest to the right for $t \geq 0$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

Part A (BC): Graphing calculator required**Question 2****9 points****General Scoring Notes**

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Scoring guidelines and notes contain examples of the most common approaches seen in student responses. These guidelines can be applied to alternate approaches to ensure that these alternate approaches are scored appropriately.

For time $t \geq 0$, a particle moves in the xy -plane with position $(x(t), y(t))$ and velocity vector $\langle (t-1)e^{t^2}, \sin(t^{1.25}) \rangle$. At time $t = 0$, the position of the particle is $(-2, 5)$.

Model Solution**Scoring**

(a) Find the speed of the particle at time $t = 1.2$. Find the acceleration vector of the particle at time $t = 1.2$.	
$\sqrt{(x'(1.2))^2 + (y'(1.2))^2} = 1.271488$ <p>At time $t = 1.2$, the speed of the particle is 1.271.</p>	Speed 1 point
$\langle x''(1.2), y''(1.2) \rangle = \langle 6.246630, 0.405125 \rangle$ <p>At time $t = 1.2$, the acceleration vector of the particle is $\langle 6.247 \text{ (or } 6.246\text{), } 0.405 \rangle$.</p>	Acceleration vector 1 point

Scoring notes:

- Unsupported answers do not earn any points in this part.
- The acceleration vector may be presented with other symbols, for example $(,)$ or $[,]$, or the coordinates may be listed separately, as long as they are labeled.
- Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is generally eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode). In degree mode, speed = 0.844 and $y''(1.2) = 0.023$ (or 0.022).

Total for part (a) 2 points

- (b) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 1.2$.

$$\int_0^{1.2} \sqrt{(x'(t))^2 + (y'(t))^2} dt = 1.009817$$

Integrand

1 point

The total distance traveled by the particle over the time interval $0 \leq t \leq 1.2$ is 1.010 (or 1.009).

Answer

1 point**Scoring notes:**

- The first point is earned by presenting the integrand $\sqrt{(x'(t))^2 + (y'(t))^2}$ in a definite integral with any limits. A definite integral with incorrect limits is not eligible for the second point.
- Once earned, the first point cannot be lost. Even in the presence of subsequent copy errors, the correct answer will earn the second point.
- If the first point is not earned because of a copy error, the second point is still earned for a correct answer.
- Unsupported answers will not earn either point.
- Degree mode: distance = 0.677 (or 0.676) (See degree mode statement in part (a).)

Total for part (b) 2 points

- (c) Find the coordinates of the point at which the particle is farthest to the left for $t \geq 0$. Explain why there is no point at which the particle is farthest to the right for $t \geq 0$.

$x'(t) = (t - 1)e^{t^2} = 0 \Rightarrow t = 1$	Sets $x'(t) = 0$	1 point
Because $x'(t) < 0$ for $0 < t < 1$ and $x'(t) > 0$ for $t > 1$, the particle is farthest to the left at time $t = 1$.	Explains leftmost position at $t = 1$	1 point
$x(1) = -2 + \int_0^1 x'(t) dt = -2.603511$	One coordinate of leftmost position	1 point
$y(1) = 5 + \int_0^1 y'(t) dt = 5.410486$		
The particle is farthest to the left at point $(-2.604$ (or -2.603), $5.410)$.	Leftmost position	1 point
Because $x'(t) > 0$ for $t > 1$, the particle moves right for $t > 1$. Also, $x(2) = -2 + \int_0^2 x'(t) dt > -2 = x(0)$, so the particle's motion extends to the right of its initial position after time $t = 1$. Therefore, there is no point at which the particle is farthest to the right.	Explanation	1 point

Scoring notes:

- The second point is earned for presenting a valid reason why the particle is at its leftmost position at time $t = 1$. For example, a response could present the argument shown in the model solution, or it could indicate that the only critical point of $x(t)$ occurs at $t = 1$ and $x'(t)$ changes from negative to positive at this time.
- Unsupported positions $x(1)$ and/or $y(1)$ do not earn the third (or fourth) point(s).
- Writing $x(1) = \int_0^1 x'(t) - 2 = -2.603511$ does not earn the third (or fourth) point, because the missing dt makes this statement unclear or false. However, $x(1) = -2 + \int_0^1 x'(t) = -2.603511$ does earn the third point, because it is not ambiguous. Similarly for $y(1)$.
- For the fourth point, the coordinates of the leftmost point do not have to be written as an ordered pair as long as they are labeled as the x - and y -coordinates.
- To earn the last point, a response must verify that the particle moves to the right of its initial position (as well as moves to the right for all $t > 1$). Note that there are several ways to demonstrate this.
- Degree mode: y -coordinate = 5.008 (or 5.007) (See degree mode statement in part (a).)

Total for part (c)	5 points
Total for question 2	9 points

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Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

$$\text{speed} = \sqrt{(x'(t))^2 + (y'(t))^2}$$

$$\sqrt{((t-1)e^{(t-1)^2})^2 + (\sin(t^{1.25}))^2}$$

$$\text{accel: } \langle (t-1)(2e^{(t-1)^2}) \times (1)e^{(t-1)^2}, \cos(t^{1.25}) \cdot 1.25t^{0.25} \rangle$$

$$\text{speed} = \boxed{1.271}$$

$$+ t - 1 = 1.2$$

$$\text{accel: } \langle 6.247, .405 \rangle$$

Response for question 2(b)

$$\int_0^{1.2} \sqrt{((t-1)e^{(t-1)^2})^2 + (\sin(t^{1.25}))^2} dt$$

$$= \boxed{1.010}$$

• 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 part (c) on this page.

Response for question 2(c)

~~x'(t)~~ $x'(t)$ is negative on $(0, 1)$ and is positive on $(1, \infty)$. ~~thus~~, $x'(1) = 0$. Since $x'(t)$ goes from negative to positive at $t=1$ and $x'(1)=0$, $x(t)$ has a rel. minimum at $t=1$. Since $x(t)$ decreases until $t=1$ and always increases afterwards, $x(1)$ is an abs. min / that is the location of furthest to the left,

$$\star -2 + \int_0^1 (t-1)e^{t^2} dt = x(1)$$

$$x(1) = -2.604$$

$$5 + \int_0^1 (\sin(t^{1.25})) dt = y(1)$$

$$y(1) = 5.410$$

$$(-2.604, 5.410)$$

There is no point furthest to the right because $x'(t)$ increases towards infinity on ~~as~~ $+2\theta$, meaning there is no abs max of $x(t)$ (since ~~as~~ $x(t)$ will thus ~~not~~ be increasing towards ∞) on $t > 1$. This means the particle will ~~not~~ continue to the right for $t > 1$, creating no furthest right point.

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Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

$$\text{Particle speed at } t=1.2 = \sqrt{(x'(1.2))^2 + (y'(1.2))^2} = \text{Magnitude of velocity vector}$$

$$= \boxed{1.271}$$

$$\text{Acceleration vector at } t=1.2 = \langle x''(1.2), y''(1.2) \rangle = \boxed{\langle 6.247, 0.405 \rangle}$$

$$\begin{aligned} x''(t) &= \\ x'''(t) &= \end{aligned}$$

Response for question 2(b)

$$\begin{aligned} x'(t) &= (t-1) e^{t^2} \\ y'(t) &= \sin(t^{1.25}) \end{aligned}$$

$$\begin{aligned} \text{Total distance travelled by particle from } 0 \leq t \leq 1.2 &= \int_0^{1.2} \sqrt{(x'(t))^2 + (y'(t))^2} dt \\ &= \boxed{1.010} \end{aligned}$$

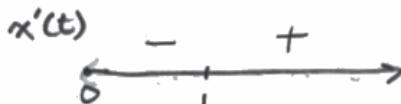
• 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 part (c) on this page.

Response for question 2(c)

Find minimum of $x(t)$

$$x'(t) = (t-1)e^{t^2} = 0 \text{ when } t=1$$



$x(t)$ has a relative minimum at $t=1$ because $x'(t)$ changes from <0 to >0 at $t=1$

t	$x(t)$
0	-2
1	-2.604

$$x(1) = x(0) + \int_0^1 x'(t) dt = -2 + \int_0^1 x'(t) dt \\ = -2.604$$

$x(t)$ has absolute minimum at $t=1$ because $x(1) < x(0)$

$$y(1) = y(0) + \int_0^1 y'(t) dt \\ = 5.410$$

Coordinates: $(-2.604, 5.410)$

The particle continues to move right because $x'(t) = (t-1)e^{t^2}$ always is positive from $(1, \infty)$ \therefore there is no rightmost point for $t \geq 0$.



2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

$$\text{speed} = \sqrt{(x'(t))^2 + (y'(t))^2}$$

$$\text{speed}|_{t=1.2} = 1.27$$

$$x'(t) = (t-1)e^{t^2}$$

$$y'(t) = \sin(t^{1.2})$$

acceleration:

$$\langle a(t) \rangle = \langle x''(t), y''(t) \rangle$$

$$\langle a(1.2) \rangle = \langle 6.247, 0.405 \rangle$$

Response for question 2(b)

$$\text{total distance} = \int_0^{1.2} |v(t)| dt$$

$$\int_0^{1.2} \sqrt{(x'(t))^2 + (y'(t))^2} dt$$

$$= 1.010$$

• 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 part (c) on this page.

Response for question 2(c)

farthest to the left?

$$(t-1)e^{t^2} = 0$$

$$t=1$$

- +
moving left moving right

$$\text{at } t=1$$

$$x(1) = x(0) + \int_0^1 x'(t) dt$$

$$x(1) = -0.604 + (-2)$$

$$x(1) = -2.604$$

$$y(1) = y(0) + \int_0^1 y'(t) dt$$

$$y(1) = 5 + 0.410$$

$$y(1) = 5.410$$

the particle is farthest to the left at $\langle -2.604, 5.410 \rangle$.

b/c $t=1$ is the only critical point and velocity changes from negative to positive at $t=1$, so moving left to right

There is no point where the particle is farthest to the right b/c $x'(t) > 0$ for all $t > 1$, so since it's always increasing and always moving towards the right, there will not be a value where it's farthest right.



2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

$$\begin{aligned}\text{Speed: } & \sqrt{(x'(1.2))^2 + (y'(1.2))^2} \\ &= \sqrt{(1.2-1)e^{1.2^2})^2 + (\sin(1.2^{1.25}))^2} \\ &= 1.271\end{aligned}$$

acceleration vector:

$$\begin{aligned}& \langle x''(1.2), y''(1.2) \rangle \\ &= \langle 6.247, 0.405 \rangle\end{aligned}$$

Response for question 2(b)

$$\begin{aligned}\int_0^{1.2} \sqrt{(x'(t))^2 + (y'(t))^2} dt &= 1.0098 \\ &= 1.010\end{aligned}$$

• 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 part (c) on this page.

Response for question 2(c)

$$\frac{dx}{dt} = 0$$

$$(t-1)e^{t^2} = 0$$

$$t = 1$$

$$x(1) = x(0) + \int_0^1 (x'(t)) dt \quad y(1) = y(0) + \int_0^1 (y'(t)) dt$$

$$= 5 + 0.410$$

$$= -2 + (-0.604)$$

$$= 5.410$$

$$= -2.604$$

$$(-2.604, 5.410)$$

There is no point at which the particle is farthest to the right for $t \geq 0$ because the $\lim_{t \rightarrow 0} x(t)$ does not exist.



2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

Speed = $\sqrt{(x'(t))^2 + (y'(t))^2}$

~~$\sqrt{(6.2966)^2 + (0.4051)^2} = 6.36$~~

~~$\sqrt{0.28164} = \boxed{1.271}$~~

$\sqrt{(-.8441392)^2 + (0.95084)^2} = \boxed{1.271}$

Acceleration Vector at $t=1.2$ $\langle 6.247, 0.405 \rangle$

Response for question 2(b)

Distance $\int_0^{1.2} \sqrt{(x'(t))^2 + (y'(t))^2} dt = \boxed{1.01}$

• 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 part (c) on this page.

Response for question 2(c)

$$t=1$$

$$(-2.604, 5.41)$$

$$x(1) = -2 + \int_0^1 x(t) dt$$

$$y(1) = 5 + \int_0^1 y(t) dt$$

There is no furthest ~~left~~ to the right coordinate because the derivative of $x(t)$ never changes from positive to negative, once the particle starts moving right, it never stops.

2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

$$\text{a. } \sqrt{((+1)e^{+2})^2 + \sin(+1.2\pi)^2}, \quad + = 1.2 \\ \sqrt{(0.344139)^2 + (0.95084775)^2} \\ = 1.271$$

Response for question 2(b)

$$\text{b. } \int_0^{1.2} \sqrt{((+1)e^{+2})^2 + (\sin(+1.2s))^2} ds \\ = 1.010$$

• 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 part (c) on this page.

Response for question 2(c)

$$x'(t) = \text{velocity of } x \quad y'(t) = \text{velocity of } y$$

c. furthest left, minimum x position

$$(t-1) e^{t^2} = 0$$

(critical point: $t=1$ is a minimum because $x''(1) = 2.718 > 0$.

$$\begin{aligned} x(1) &= -2 + \int_0^1 x'(t) dt \\ &= -2.604 \end{aligned}$$

$$y(1) = 5 + \int_0^1 y'(t) dt = 5.4105$$

2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

$$\sqrt{((1.2-1)e^{1.2^2})^2 + (\sin(1.2^{1.25}))^2} = 1.322$$

Speed at $t=1.2$ is 1.322

$$\left\langle \frac{d}{dx}(t-1)e^{t^2} \Big|_{1.2}, \frac{d}{dx}\sin(t^{1.25}) \Big|_{1.2} \right\rangle =$$

acceleration vector at $t=1.2$

$$\langle 6.247, .405 \rangle$$

Response for question 2(b)

total distance

$$\int_0^{1.2} \sqrt{((t-1)e^{t^2})^2 + (\sin(t^{1.25}))^2} dx = 1.010$$



• 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 part (c) on this page.

Response for question 2(c)

There is no farthest point to the right because the velocity stays positive forever after $t = 1$, so the particle keeps going to the right. There's no absolute max. (For the graph of $x'(t)$ because that's used for left/right)

$$-2 + \int_0^1 (t-1)e^{t^2} dx = -2.604$$

$$5 + \int_0^1 \sin(t^{1.25}) dx = 5.410$$

Farthest left at $(-2.604, 5.410)$



2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

$$\sqrt{(x'(1.2))^2 + (y'(1.2))^2} = 1.271$$

$$n(t) = \langle x''(1.2), y''(1.2) \rangle$$

$$\langle 6.247, \cancel{.405} .405 \rangle$$

Response for question 2(b)

$$\int_0^{1.2} \sqrt{(x'(t))^2 + (y'(t))^2} dt = .862$$

• 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 part (c) on this page.

Response for question 2(c)

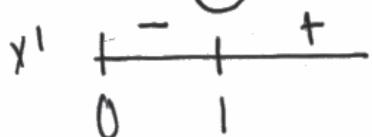
$$x'(t) = (t-1)e^{t^2}$$

$$-2 + \int_0^1 x(t) dt =$$

$$\therefore (t-1)e^{t^2} = 0$$

$$5 + \int_0^1 y(t) dt =$$

$$t = 1$$



$$\int_0^1 \sin(t^{1.25}) - (0) t^{1.25}]_0^1$$

~~REDDITA~~

relative minimum @ $t=1$

~~REDDITA~~

$$(-2.604)$$

There is no point at which the particle is farthest to the right for $t \geq 0$ because there is only one absolute extrema on $t \geq 0$, which is an ~~min~~ absolute minimum at $t=1$, where the particle is farthest to the left. Also, $x(t)$ continues to increase for all t after $t=1$.
INFINITELY

~~WE 1.25~~
~~APR - 1.25~~
~~JUN - 2.44~~
~~JUL - 1.25 E 05~~



2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

$$\text{velocity} \quad \langle (t-1)e^{t^2}, \sin(t^{1.25}) \rangle$$

$$\text{speed} = \sqrt{(t-1)e^{t^2}}^2 + (\sin(t^{1.25}))^2$$

$$\sqrt{((1.2-1)e^{1.2^2})^2 + (\sin(1.2^{1.25}))^2} = 1.271$$

Response for question 2(b)

total distance

$$\int_a^{1.2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$\int_0^{1.2} \sqrt{((t-1)e^{t^2})^2 + (\sin(t^{1.25}))^2} = 0.993$$

• 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 part (c) on this page.

Response for question 2(c)

$$\text{min for } x \\ x'(t) = (-1)e^{t^2}$$

$$(-1)e^{t^2} = 0$$

$$t=1$$



$$x(1) = -2 + \int_0^1 (-1)e^{t^2} dt = -2.604$$

$$y(1) = 5 + \int_0^1 \sin(t^{1.25}) dt = 5.410$$

The particle is farthest to the left at the position $(-2.604, 5.410)$ when $t=1$ b/c $x'(t) = 0$ and changes signs from negative to positive.

There is no point at which the point is farthest to the right for $t \geq 0$, because $x'(t)$ is positive when $t \geq 0$, so $x(t)$ is increasing.



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Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

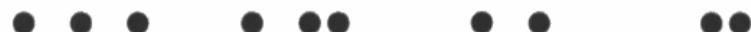
$$S = \sqrt{\left((1.2 - 1)e^{(1.2)^2}\right)^2 + \left(\sin(1.2^{1.25})\right)^2}$$

$$\text{Acc.} = \langle 6.2467, 0.4051 \rangle$$

Response for question 2(b)

$$TDT = \int_0^{1.2} \sqrt{(x'(t))^2 + (y'(t))^2} dt$$

$$= 1.0098$$



• 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 part (c) on this page.

Response for question 2(c)

Right - Left \rightarrow use $x(t)$
 $(t-1)e^{t^2}$ @ $t=0$
 $x = -2$



x component of velocity
is negative from
 $t=0$ to $t=1$ therefore
since x starts at -2
the furthest left point
will be at $t=1$, $(-2.6035, 5.4105)$

There is no point at which
the particle is furthest to
the left because the x
value continues to increase
to infinity, when $t \geq 0$



Question 2**Sample Identifier: A****Score: 9**

- The response earned 9 points: 2 points in part (a), 2 points in part (b), and 5 points in part (c).
- In part (a) the response earned the first point with the radical expression on the second line. Note that the response continues by simplifying and rounding correctly to the boxed answer of 1.271. While simplifying was not necessary, given that it is presented, it must be correct. The response earned the second point on the last line with the boxed answer of $\langle 6.247, .405 \rangle$ and the supporting work found in the vector expression $\left\langle (t - 1)\left(2te^{t^2}\right) + (1)e^{t^2}, \cos(t^{1.25}) \cdot 1.25t^{2.5} \right\rangle$ on the lines above.
- In part (b) the response earned the first point on the first line with the integrand, noting that it is contained within a definite integral with numeric limits. As the limits on the integral are correct and the boxed answer 1.010 is correct, the response earned the second point.
- In part (c) the response earned the first and second points by noting that “ $x'(t)$ is negative on $(0, 1)$ and is positive on $(1, \infty)$.” The second point is reinforced on lines two through six by explaining that due to the fact that decreases until $t = 1$ and always increases afterward, it follows that $x(1)$ is an absolute minimum. The response earned the third point with the equations

$$-2 + \int_0^1 (t - 1)e^{t^2} dt = x(1) \text{ and } x(1) = -2.604. \text{ The response earned the fourth point with the boxed answer } (-2.604, 5.410) \text{ and the supporting equation } 5 + \int_0^1 (\sin(t^{1.25})) dt = y(1) \text{ two lines above.}$$

The response earned the fifth point in the final paragraph by noting that “ $x'(t)$ increases toward infinity on $t > 1$ ” and correctly concluding that “there is no abs max of $x(t)$ (since $x(t)$ will thus be increasing towards ∞ on $t > 1$).”

Question 2 (continued)**Sample Identifier: B****Score: 7**

- The response earned 7 points: 2 points in part (a), 2 points in part (b), and 3 points in part (c).
- In part (a) the response earned the first point with both the radical expression $\sqrt{(x'(1.2))^2 + (y'(1.2))^2}$ and the boxed answer of 1.271. The response earned the second point on the last line with the boxed answer of $\langle 6.247, 0.405 \rangle$ and the supporting work which is the vector expression $\langle x''(1.2), y''(1.2) \rangle$.
- In part (b) the response earned the first point with the integrand $\sqrt{(x'(t))^2 + (y'(t))^2}$ in a definite integral. The response earned the second point with correct limits on the integral and the answer 1.010.
- In part (c) the response earned the first point by setting $x'(t) = 0$. Note that the expression in the middle, $(t - 1)e^{t^2}$, is correct, but it was not necessary to earn the first point. While the response has the correct conclusion that “ $x(t)$ has absolute minimum at $t = 1$,” there is no appeal to a global argument. The response even makes reference on the right to a relative minimum, but does not state explicitly that there is only one critical point. Also note that the sign chart is not sufficient. Thus, the response did not earn the second point. The response earned the third point on the right with the equation $x(1) = x(0) + \int_0^1 x'(t) dt = -2.604$. Note that the middle step in between is correct but not necessary. The response earned the fourth point with both the boxed answer of $(-2.604, 5.410)$, and the additional supporting equation $y(1) = y(0) + \int_0^1 y'(t) dt = 5.410$. In the last boxed paragraph, the response argues that there is not rightmost point due to the fact that “the particle continues to move right because $x'(t) = (t - 1)e^{t^2}$ always positive from $(1, \infty)$.” While this statement about $x'(t)$ is true, it is insufficient without showing that the particle is eventually to the right of the initial point. Hence, the response did not earn the fifth point.

Question 2 (continued)**Sample Identifier: C****Score: 7**

- The response earned 7 points: 2 points in part (a), 2 points in part (b), and 3 points in part (c).
- In part (a) the response earned the first point with the first two lines on the left. The first line is the supporting work for the boxed answer of 1.271. The response earned the second point with the last two lines on the left. The vector $\langle x''(t), y''(t) \rangle$ on the third line is the supporting work to the boxed answer of $\langle 6.247, 0.405 \rangle$.
- In part (b) the response earned the first point on the second line with the integrand $\sqrt{(x'(t))^2 + (y'(t))^2}$. The response earned the second point with correct limits on the integral and the boxed answer 1.010.
- In part (c) the response earned the first point with the equation $(t - 1)e^{t^2} = 0$. The second point is not earned because the explanation provided by the sign chart is not sufficient to support the conclusion of a global minimum for $x(t)$ at $t = 1$. The response earned the third point on the second and third lines on the right with the equation $x(1) = x(0) + \int_0^1 x'(t) dt$ and the equation $x(1) = -0.604 + (-2)$, simplified correctly to -2.604 . The response earned the fourth point with both the boxed answer of $\langle -2.604, 5.410 \rangle$, and the additional supporting equation $y(1) = y(0) + \int_0^1 y'(t) dt$ on the fifth line on the right. Note that the apparent vector notation in this presentation is not penalized. In the last paragraph the response correctly concludes that there is no point farthest right; however, the response does not justify this sufficiently as it does not verify that the particle is eventually to the right of the initial point. Therefore the response did not earn the fifth point.

Question 2 (continued)**Sample Identifier: D****Score: 7**

- The response earned 7 points: 2 points in part (a), 2 points in part (b), and 3 points in part (c).
- In part (a) the response earned the first point with the second line (and subsequent consistent work), as this is the exact answer. As the response continues and provides the answer correct to three decimal places on the third line. The response earned the second point with the last two lines. The vector $\langle x''(1.2), y''(1.2) \rangle$ on the fourth line is the supporting work to the answer $\langle 6.247, 0.405 \rangle$ on the last line.
- In part (b) the response earned the first point on the first line with the integrand $\sqrt{(x'(t))^2 + (y'(t))^2}$, in a definite integral. The response earned the second point with correct limits on the integral and the answer 1.010.
- In part (c) the response earned the first point with the equation $\frac{dx}{dt} = 0$. While the response has the correct conclusion that $t = 1$, there is no argument as to why this corresponds to the leftmost point. Thus, the response did not earn the second point. The response earned the third point with the equation $x(1) = x(0) + \int_0^1 x'(t) dt = -2 + (-0.604)$, simplified correctly to -2.604 . The response earned the fourth point with both the answer of $(-2.604, 5.410)$, and the additional supporting equation $y(1) = y(0) + \int_0^1 y'(t) dt$ three lines above on the right. The response did not earn the fifth point as the limit statement presented on the last line is not sufficient.

Sample Identifier: E**Score: 6**

- The response earned 6 points: 1 point in part (a), 2 points in part (b), and 3 points in part (c).
- In part (a) the response earned the first point with the radical expression $\sqrt{(.8441392)^2 + (0.9508477)^2}$. The response then simplifies and rounds to 1.271, the correct answer rounded to three decimal places. The response did not earn the second point. While the presented vector is correct, there is no indication of the derivatives being evaluated to produce this answer.
- In part (b) the response earned the first point with the integrand presented in a definite integral. As the limits of this integral are correct, the response earned the second point with the answer 1.01.
- In part (c) the response earned the first point in the last sentence by the claim that “the derivative of $x(t)$ never changes from positive to negative.” This statement provides evidence that $x'(t)$ is being compared to 0. The response did not earn the second point as no argument is given supporting an absolute minimum for $x(t)$. The response earned the third point with both the x -coordinate presented within the boxed answer, and with the supporting equation on the second line. The response earned the fourth point with both the correct answer boxed and the supporting equation on the third line. The response did not earn the fifth point as no argument is presented to justify that the particle moves to the right of the initial point.

Question 2 (continued)**Sample Identifier: F****Score: 6**

- The response earned 6 points: 1 point in part (a), 2 points in part (b), and 3 points in part (c).
- In part (a) the response earned the first point with the first two lines. The third line correctly simplifies the expression in the second line, while the first line supports the work leading to the answer. The response did not earn the second point as there is no answer or work presented for the acceleration vector.
- In part (b) the response earned the first point in the first line with the integrand, as it is within a definite integral. The response earned the second point with correct limits on the integral and the answer 1.010.
- In part (c) the response earned the first point with the equation on the third line. The response goes on to make an argument that the x -coordinate is minimized due to the fact that $x''(t) > 0$. This, however, is a local argument. Thus, the response did not earn the second point. The response earned the third point with the equations on the fifth and seventh lines. The response earned the fourth point with the correct value of $x(1)$, and with the equation on the last line. The response did not earn the final point as no answer or argument is given.

Sample Identifier: G**Score: 5**

- The response earned 5 points: 1 point in part (a), 2 points in part (b), and 2 points in part (c).
- In part (a) the response did not earn the first point because the initial radical expression $\sqrt{\left((1.2 - 1)e^{1.2^2}\right)^2 + \left(\sin(1.2^{1.25})\right)^2}$ is missing a right parenthesis in the second term under the radical and the answer 1.322 is incorrect. The response earned the second point with the correct answer $\langle 6.247, .405 \rangle$ and the supporting vector expression in the third line. We note that while the dt in each component in the penultimate line appears to have been misspelled as dx , there is no ambiguity in the presence of the correct answer.
- In part (b) the response earned the first point with the integrand $\sqrt{\left((t - 1)e^{t^2}\right)^2 + \left(\sin(t^{1.25})\right)^2}$ presented within a definite integral with numeric limits. As the limits are correct and the answer of 1.010 is correct, the response earned the second point. We note again that while the dt appears once more to have been misspelled as dx , there is no ambiguity in the presence of the correct answer.
- In part (c) the response did not earn the first point. While the response identifies $t = 1$ and states that “the velocity stays positive...,” there is no connection presented between “velocity” and $x'(t)$. The response did not earn the second point as no argument is given to support an absolute minimum. The response earned the third point with the equation $-2 + \int_0^1 (t - 1)e^{t^2} dt = -2.604$. The response earned the fourth point with the correct coordinates $(-2.604, 5.410)$ and the supporting equation in the line above. We once again note that while the dt appears to have been misspelled as dx in the supporting work for both the third and fourth points, there is no ambiguity in the presence of the correct answers. The response did not earn the fifth point as no consideration is given as to whether the particle ever moves to the right of the initial point.

Question 2 (continued)**Sample Identifier: H****Score: 4**

- The response earned 4 points: 2 points in part (a), 1 point in part (b), and 1 point in part (c).
- In part (a) the response earned the first point with the equation $\sqrt{(x'(1.2))^2 + (y'(1.2))^2} = 1.271$. The response earned the second point with the correct answer of $\langle 6.247, .405 \rangle$ on the last line supported by the expression $\langle x''(1.2), y''(1.2) \rangle$ on the second line.
- In part (b) the response earned the first point with the integrand $\sqrt{(x'(t))^2 + (y'(t))^2}$ presented within a definite integral with numeric limits. The response did not earn the second point because the answer presented is incorrect.
- In part (c) the response earned the first point with the equation $(t - 1)e^{t^2} = 0$ on the second line. The response did not earn the second point as the explanation provided by the sign chart is not sufficient to support the conclusion of a global minimum for $x(t)$ at $t = 1$. The response did not earn the third point. While the x -coordinate -2.604 is correct, the likely expression to support this, found in the upper right corner of the response, has an incorrect integrand. The response did not earn the fourth point as no y -coordinate is presented. The response did not earn the fifth point as no justification is given to support that the particle ever moves to the right of the initial point.

Sample Identifier: I**Score: 4**

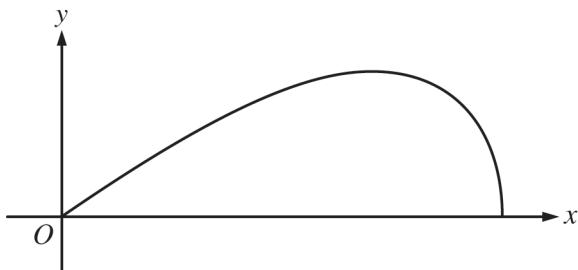
- The response earned 4 points: 1 point in part (a), 0 points in part (b), and 3 points in part (c).
- In part (a) the response earned the first point with the radical expression $\sqrt{(1.2 - 1)e^{1.2^2}}^2 + (\sin(1.2^{1.25}))^2$ given correctly to three decimal places on the third line. The response did not earn the second point because no acceleration vector is presented.
- In part (b) the response did not earn the first point. While the integrand $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$ is presented in a definite integral, the limits are not numeric, and as such, this is a formula. The second integral presented has a parenthesis error; under the radical, there are only four left parentheses, while there are either five or six right parentheses (depending on whether there is a right parenthesis after the first t). In either case, a correct integrand is not presented within a definite integral with numeric limits. The response did not earn the second point because the answer presented 0.993 is incorrect.
- In part (c) the response earned the first point with the equation $(t - 1)e^{t^2} = 0$ on the third line. The response did not earn the second point as the argument given in the first sentence of the final paragraph only references the change of sign of $x'(t)$ at $t = 1$ from negative to positive. While true, this is local argument and does not justify a global minimum when $t = 1$. We note that the sign chart does not provide the global argument. The response earned the third point on the sixth line with the equation $x(1) = -2 + \int_0^1 (t - 1)e^{t^2} dt = -2.604$. The response earned the fourth point with the answer $(-2.604, 5.410)$ supported by the equation $y(1) = 5 + \int_0^1 \sin(t^{1.25}) dt = 5.410$ on the seventh line. In the last paragraph, the response does not support the conclusion that there is not a position farthest to the right because it does not explain that the particle is eventually to the right of the initial point. Therefore the response did not earn the fifth point.

Question 2 (continued)

Sample Identifier: J

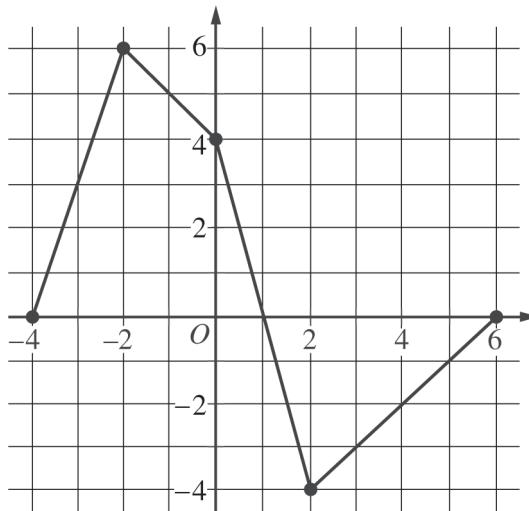
Score: 3

- The response earned 3 points: 1 point in part (a), 2 points in part (b), and 0 points in part (c).
- In part (a) the response earned the first point with the radical expression on the first line. While the vector presented on the second line is correct, there is no supporting work; therefore, the response did not earn the second point.
- In part (b) the response earned the first point on first line with an appropriate integrand within a definite integral with numeric limits. As the limits of this definite integral are correct, the response earned the second point on the second line with the answer 1.0098.
- In part (c) the response did not earn the first point. While the response notes in the middle paragraph that the “ x component of velocity is negative from $t = 0$ to $t = 1\dots$,” no connection has been made between the x -component of “velocity” and $x'(t)$. The response did not earn the second point as the argument given is local and does not support the particle being furthest to the left when $t = 1$. While the coordinates presented at the end of the middle paragraph are correct for the point at $t = 1$, the response earned neither the third nor the fourth points as no supporting work for either of these coordinates is presented. The response did not earn the fifth point because the statement “the x value continues to increase to infinity when $t \geq 0$ ” contradicts that $x(t)$ is decreasing from $t = 0$ to $t = 1$.



3. A company designs spinning toys using the family of functions $y = cx\sqrt{4 - x^2}$, where c is a positive constant. The figure above shows the region in the first quadrant bounded by the x -axis and the graph of $y = cx\sqrt{4 - x^2}$, for some c . Each spinning toy is in the shape of the solid generated when such a region is revolved about the x -axis. Both x and y are measured in inches.
- (a) Find the area of the region in the first quadrant bounded by the x -axis and the graph of $y = cx\sqrt{4 - x^2}$ for $c = 6$.
- (b) It is known that, for $y = cx\sqrt{4 - x^2}$, $\frac{dy}{dx} = \frac{c(4 - 2x^2)}{\sqrt{4 - x^2}}$. For a particular spinning toy, the radius of the largest cross-sectional circular slice is 1.2 inches. What is the value of c for this spinning toy?
- (c) For another spinning toy, the volume is 2π cubic inches. What is the value of c for this spinning toy?

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

Graph of f

4. Let f be a continuous function defined on the closed interval $-4 \leq x \leq 6$. The graph of f , consisting of four line segments, is shown above. Let G be the function defined by $G(x) = \int_0^x f(t) dt$.
- (a) On what open intervals is the graph of G concave up? Give a reason for your answer.
- (b) Let P be the function defined by $P(x) = G(x) \cdot f(x)$. Find $P'(3)$.
- (c) Find $\lim_{x \rightarrow 2} \frac{G(x)}{x^2 - 2x}$.
- (d) Find the average rate of change of G on the interval $[-4, 2]$. Does the Mean Value Theorem guarantee a value c , $-4 < c < 2$, for which $G'(c)$ is equal to this average rate of change? Justify your answer.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

5. Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = y \cdot (x \ln x)$ with initial condition $f(1) = 4$. It can be shown that $f''(1) = 4$.
- (a) Write the second-degree Taylor polynomial for f about $x = 1$. Use the Taylor polynomial to approximate $f(2)$.
- (b) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(2)$. Show the work that leads to your answer.
- (c) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = y \cdot (x \ln x)$ with initial condition $f(1) = 4$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

Part B (BC): Graphing calculator not allowed**Question 5****9 points****General Scoring Notes**

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Scoring guidelines and notes contain examples of the most common approaches seen in student responses. These guidelines can be applied to alternate approaches to ensure that these alternate approaches are scored appropriately.

Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = y \cdot (x \ln x)$ with initial condition $f(1) = 4$. It can be shown that $f''(1) = 4$.

Model Solution**Scoring**

- (a) Write the second-degree Taylor polynomial for f about $x = 1$. Use the Taylor polynomial to approximate $f(2)$.

$$f'(1) = \left. \frac{dy}{dx} \right|_{(x, y)=(1, 4)} = 4 \cdot (1 \ln 1) = 0$$

The second-degree Taylor polynomial for f about $x = 1$ is

$$f(1) + f'(1)(x - 1) + \frac{f''(1)}{2!}(x - 1)^2 = 4 + 0(x - 1) + \frac{4}{2}(x - 1)^2.$$

$$f(2) \approx 4 + 2(2 - 1)^2 = 6$$

Polynomial

1 point

Approximation

1 point**Scoring notes:**

- The first point is earned for $4 + \frac{4 \cdot \ln 1}{1!}(x - 1)^1 + \frac{4}{2!}(x - 1)^2$ or any correctly simplified equivalent expression. A term involving $(x - 1)$ is not necessary. The polynomial must be written about (centered at) $x = 1$.
- If the first point is earned, the second point is earned for just “6” with no additional supporting work.
- If the polynomial is never explicitly written, the first point is not earned. In this case, to earn the second point supporting work of at least “4 + 2(1)” is required.

Total for part (a) 2 points

- (b) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(2)$. Show the work that leads to your answer.

$f(1.5) \approx f(1) + 0.5 \cdot \frac{dy}{dx} \Big _{(x, y)=(1, 4)} = 4 + 0.5 \cdot 0 = 4$	Euler's method with two steps	1 point
$f(2) \approx f(1.5) + 0.5 \cdot \frac{dy}{dx} \Big _{(x, y)=(1.5, 4)}$		
$\approx 4 + 0.5 \cdot 4 \cdot (1.5 \ln 1.5) = 4 + 3 \ln 1.5$	Answer	1 point

Scoring notes:

- The first point is earned for two steps (of size 0.5) of Euler's method, with at most one error. If there is any error, the second point is not earned.
- To earn the first point, a response must contain two Euler steps, $\Delta x = 0.5$, use of the correct expression for $\frac{dy}{dx}$, and use of the initial condition $f(1) = 4$.
 - The two Euler steps may be explicit expressions or may be presented in a table. Here is a minimal example of a (correctly labeled) table.

x	y	$\Delta y = \frac{dy}{dx} \cdot \Delta x$ or $\Delta y = \frac{dy}{dx} \cdot (0.5)$
1	4	0
1.5	4	$3 \ln 1.5$
2	$4 + 3 \ln 1.5$	

- Note: In the presence of the correct answer, such a table does not need to be labeled in order to earn both points. In the presence of an incorrect answer, the table must be correctly labeled in order for the response to earn the first point.
- A single error in computing the approximation of $f(1.5)$ is not considered a second error if that incorrect value is imported correctly into an approximation of $f(2)$.
- Both points are earned for “ $4 + 0.5 \cdot 0 + 0.5 \cdot 4 \cdot (1.5 \ln 1.5)$ ” or “ $4 + 0.5 \cdot 4 \cdot (1.5 \ln 1.5)$ ”.
- Both points are earned for presenting the ordered pair $(2, 4 + 3 \ln 1.5)$ with sufficient supporting work.

Total for part (b) 2 points

- (c) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = y \cdot (x \ln x)$ with initial condition $f(1) = 4$.

$\frac{1}{y} dy = x \ln x dx$	Separation of variables	1 point
Using integration by parts, $u = \ln x \quad du = \frac{1}{x} dx$ $dv = x dx \quad v = \frac{x^2}{2}$ $\int x \ln x dx = \frac{x^2}{2} \cdot \ln x - \int \frac{x^2}{2} \cdot \frac{1}{x} dx = \frac{x^2 \ln x}{2} - \frac{x^2}{4} + C$	Antiderivative for $x \ln x$	1 point
$\ln y = \frac{x^2 \ln x}{2} - \frac{x^2}{4} + C$	Antiderivative for $\frac{1}{y}$	1 point
$\ln 4 = 0 - \frac{1}{4} + C \Rightarrow C = \ln 4 + \frac{1}{4}$	Constant of integration and uses initial condition	1 point
$y = e^{\left(\frac{x^2 \ln x}{2} - \frac{x^2}{4} + \ln 4 + \frac{1}{4}\right)}$	Solves for y	1 point
Note: This solution is valid for $x > 0$.		

Scoring notes:

- A response with no separation of variables earns 0 out of 5 points. If an error in separation results in one side being correct ($\frac{1}{y} dy$ or $x \ln x dx$), the response is only eligible to earn the corresponding antiderivative point.
- The third point (antiderivative of $\frac{1}{y}$) can be earned for either $\ln y$ or $\ln|y|$.
- A response with no constant of integration can earn at most 3 out of 5 points.
- A response is eligible for the fourth point if it has earned the first point and at least 1 of the 2 antiderivative points.
- A response earns the fourth point by correctly including the constant of integration in an equation and then replacing x with 1 and y with 4.
- A response is eligible for the fifth point only if it has earned the first 4 points.

Total for part (c)	5 points
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Total for question 5	9 points
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5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5

Answer QUESTION 5 parts (a) and (b) on this page.

Response for question 5(a)

$$\frac{dy}{dx} = 4 \cdot \ln 1$$

$$P_2(x) = 4 + \ln 1 \cdot (x-1) + \frac{4(x-1)^2}{2!} \quad P_2(x) = 4 + \frac{4(x-1)^2}{2!}$$

$$P_2(2) = 4 + \frac{4(2-1)^2}{2!} = 4 + 2 = \boxed{6}$$

Response for question 5(b)

$$\frac{dy}{dx} = 4 \cdot 1.5 \ln 1.5$$

6 ln 1.5

(x, y)	Δx	$\frac{dy}{dx}$	Δy
(1, 4)	.5	0	0
(1.5, 4)	.5	6 ln 1.5	3 ln 1.5
(2,)			

$$\boxed{f(2) = 4 + 3 \ln 1.5}$$



• 5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5

Answer QUESTION 5 part (c) on this page.

Response for question 5(c)

$$\frac{dy}{dx} = y \cdot x \ln x$$

$$\frac{1}{y} dy = x \ln x dx$$

$$u = \ln x \quad dv = x dx$$

$$\int \frac{1}{y} dy = \int x \ln x dx$$

$$du = \frac{1}{x} dx \quad v = \frac{1}{2} x^2$$

$$\ln y = \frac{1}{2} x^2 \ln x - \int \frac{1}{2} x dx$$

$$\ln y = \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C$$

$$\ln 4 = \frac{1}{2} \ln 1 - \frac{1}{4} + C$$

$$\ln 4 + \frac{1}{4} = C$$

$$\ln y = \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + \ln 4 + \frac{1}{4}$$

$$\boxed{y = 4e^{\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + \frac{1}{4}}}$$

5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5

Answer QUESTION 5 parts (a) and (b) on this page.

Response for question 5(a)

$$\begin{aligned} & \cancel{\text{any two terms}} \quad \cancel{\text{any two terms}} \\ & \cancel{\text{any two terms}} \quad \cancel{\text{any two terms}} \\ & \cancel{\text{any two terms}} \\ & \cancel{\text{any two terms}} \end{aligned}$$

$$f(x) = 4 + 0 \frac{(x-1)^1}{1!} + \frac{4(x-1)^2}{2!}$$

$$f(x) = 4 + 2(x-1)^2$$

Response for question 5(b)

$$(1, 4) \quad \Delta x = 0.5$$

$$\begin{aligned} x_1 &= 1.5 & y_1 &= 4 + 0.5(1.0) \\ &&&= 4 \\ x_2 &= 2 & y_2 &= 4 + 0.5(4 - 1.5)(1.5) \\ &&&= 4 + 2 \cdot 1.5(1.5) \\ &&&= 4 + 3(1.5) \end{aligned}$$

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• 5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5

Answer QUESTION 5 part (c) on this page.

Response for question 5(c)

$$\frac{dy}{dx} = y(\ln x)$$

$$dy \cdot \frac{1}{y} = \ln x \, dx$$

$$\int \frac{1}{y} dy = \int x \ln x \, dx$$

$$\begin{aligned} u &= \ln x & dv &= x \\ du &= \frac{1}{x} dx & v &= \frac{1}{2} x^2 \end{aligned}$$

$$\frac{1}{2} x^2 \cdot \ln x - \int \frac{1}{2} x^2 \frac{1}{x} dx$$

$$\frac{1}{2} x^2 \ln x - \frac{1}{2} \int x \, dx$$

$$\ln |y| = \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C$$

$$\ln 4 = 0 - \frac{1}{4} + C$$

$$\ln 4 + \frac{1}{4} = C$$

$$\ln |y| = \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + (\ln 4 + \frac{1}{4})$$

$$y = e^{\frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + (\ln 4 + \frac{1}{4})}$$

5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5

Answer QUESTION 5 parts (a) and (b) on this page.

Response for question 5(a)

$$(1, 4) \quad \frac{dy}{dx} = y \cdot x \ln x$$

$$T_z f(x) = 4 + \frac{4(x-1)^2}{2}$$

$$f(z) \approx 4 + 2(z-1)^2 = 6$$

Response for question 5(b)

$$(1, 4) \quad \frac{dy}{dx} = 4 \cdot \ln 1 = 0$$

$$y - 4 = 0(x-1) \quad y = 4$$

$$(1.5, 4) \quad \frac{dy}{dx} = 4 \cdot 1.5 \ln(1.5)$$

$$y - 4 = 6 \ln(1.5)(x-1.5)$$

$$y(z) = 6 \ln(1.5)(0.5) + 4$$

$$y(z) = 3 \ln(1.5) + 4$$

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• 5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5

Answer QUESTION 5 part (c) on this page.

Response for question 5(c)

$$\frac{dy}{dx} = y \cdot (x \ln x)$$

$$\int \frac{1}{y} dy = \int x \ln x dx$$

$v = \ln x$ $dv = x$
 $du = \frac{1}{x}$ $u = \frac{1}{2}x^2$

$$\ln |y| = \frac{1}{2}x^2 \ln x - \int \frac{1}{2}x^2 dx$$

$$\ln |y| = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$$

$$|y| = Ae^{\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2}$$

$$y = Ae^{\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2}$$

$$y = Ae^{-\frac{1}{4}} \quad A = 4e^{\frac{1}{4}}$$

$$y = 4e^{\frac{1}{4}} \cdot e^{\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2}$$

5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5

Answer QUESTION 5 parts (a) and (b) on this page.

Response for question 5(a)

$$\frac{dy}{dx} \Big|_{x=1} = 4 - (1 \ln 1) = 0$$

$$4 + \frac{4(x-1)^2}{2!}$$

$$f(2) \approx 4 + \frac{4(1)^2}{2!} = 4 + 2 = 6$$

Response for question 5(b)

$$(x, y) \quad \frac{dy}{dx} \quad \Delta y = \frac{dy}{dx} \Delta x \quad (x + \Delta x, y + \Delta y)$$

$$(1, 4) \quad 4 - 1 \ln 1 = 0 \quad 0(0.5) = 0 \quad (1.5, 4)$$

$$(1.5, 4) \quad 4 - 1.5 \ln 1.5 \quad 2 - 1.5 \ln 1.5 \quad (2, 4 + 2 - 1.5 \ln 1.5)$$

$$f(2) \approx 4 + (2 - 1.5 \ln 1.5)$$



• 5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5

Answer QUESTION 5 part (c) on this page.

Response for question 5(c)

$$\frac{dy}{y} = (x \ln x) dx$$

$$\begin{array}{r} +x \\ -1 \\ +0 \end{array} \quad \begin{array}{r} \ln x \\ \frac{1}{x} \\ -\frac{1}{x^2} \end{array}$$

$$f(1) = 4$$

$$\ln |y| = 1 + \frac{1}{x^2} + C$$

$$\ln 4 = 1 + \frac{1}{1^2} + C$$

$$C = \ln 4 - 2$$

$$\ln |y| = 1 + \frac{1}{x^2} + \ln 4 - 2$$

$$e^{1 + \frac{1}{x^2} + \ln 4 - 2} = y$$

$$4 e^{\frac{1}{x^2} - 1} = y$$

5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5

Answer QUESTION 5 parts (a) and (b) on this page.

Response for question 5(a)

$$f(1) + f'(1)(x-1) + \frac{f''(1)(x-1)^2}{2!}$$

$$T_2 = 4 + 0(x-1) + \frac{4(x-1)^2}{2!}$$

$$f(2) \approx 4 + 0(2-1) + \frac{4(2-1)^2}{2!}$$

$$= 4 + \frac{4}{2} = \boxed{4.5}$$

Response for question 5(b)

x	y	Δx	$\frac{\Delta y}{\Delta x}$	$\Delta y = \Delta x \left(\frac{\Delta y}{\Delta x}\right)$	$(x+\Delta x, y+\Delta y)$
1	4	.5	0	0	(1.5, 4)
1.5	4	.5	$3\ln(1.5)$	$3\ln(1.5)$	$(2, 4+3\ln(1.5))$
2	$4+3\ln(1.5)$				

$$f(2) \approx 4 + 3\ln(1.5)$$



• 5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5

Answer QUESTION 5 part (c) on this page.

Response for question 5(c)

LIPET

$$uv - \int v du \quad u = \ln x \quad v = \frac{1}{2}x^2$$

$$du = \frac{1}{x} dx \quad dv = x dx$$

$$\frac{dy}{dx} = y \cdot (x \ln x)$$

$$\frac{1}{2}x^2 \ln x - \frac{1}{2} \int x^2 \cdot \frac{1}{x} dx$$

$$\frac{dy}{y} = y \cdot (x \ln x) dx$$

$$= \frac{1}{2}x^2 \ln x - \frac{1}{2} \int x dx$$

$$\int \frac{1}{y} dy = \int (x \ln x) dx$$

$$= \frac{1}{2}x^2 \ln x - \frac{1}{2}x^2 + C$$

$$|\ln y| = \frac{1}{2}x^2 \ln x - \frac{1}{2}x^2 + C \quad f(1)=4$$

$$\ln 4 = -\frac{1}{2} + C$$

$$C = \ln 4 + \frac{1}{2}$$

$$|\ln y| = \frac{1}{2}x^2 \ln x - \frac{1}{2}x^2 + \ln 4 + \frac{1}{2}$$

$$y = e^{\frac{1}{2}x^2 \ln x - \frac{1}{2}x^2 + \ln 4 + \frac{1}{2}}$$

$$y = e^{\ln 4 + \frac{1}{2}} \cdot e^{\frac{1}{2}x^2 \ln x - \frac{1}{2}x^2}$$

5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5

Answer QUESTION 5 parts (a) and (b) on this page.

Response for question 5(a)

a) $f(1) = 4$
 $f''(1) = 4$
 $f'(1) = 0$

$$\frac{dy}{dx} = 4 \cdot (1 - \ln x^0)$$

Taylor polynomial = 4 + $\frac{0(x-1)}{1}$ + $\frac{4(x-1)^2}{2!}$

$$4 + \frac{4(2-1)^2}{2} = 4 + \frac{4}{2} = \boxed{6}$$

Response for question 5(b)

(1.4) $4 \cdot (1.5 \ln 1.5)$

b)

Step size = .5

$$f(1) = 4 + .5(6) = 4$$

$$f(1.5) = 4 + .5(4 \cdot 1.5 \ln(1.5)) = 4 + 6 \ln(1.5)$$

$f(2) = 4 + 6 \ln(1.5)$



• 5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5

Answer QUESTION 5 part (c) on this page.

Response for question 5(c)

c)

$$\frac{dy}{dx} = 4x \ln x$$

$$v = \ln x$$

$$dv = \frac{1}{x} dx$$

$$\int \frac{dy}{y} = \int x \ln x dx$$

$$\ln(y) = \ln x + C$$

$$\ln(y) = \ln(x) + C$$

$$C = \ln(4)$$

$$e^{\ln(y)} = Ce^{\ln x}$$

$$y = \ln(4)e^{\ln x}$$

5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5

Answer QUESTION 5 parts (a) and (b) on this page.

Response for question 5(a)

$$T_2(x) = f(1) + f'(1)(x-1) + \frac{f''(1)(x-1)^2}{2!}$$

$$\begin{aligned} f(1) &= 4 \\ \frac{dy}{dx} &= y \cdot (x \ln x) \\ \frac{dy}{dx} &= 4 \cdot (1 \ln 1) \\ &= 0 \end{aligned}$$

$$T_2(2) = 4 + 0(2-1) + \frac{4(2-1)^2}{2!}$$

$$T_2(2) = 4 + 2 = 6$$

Response for question 5(b)

x	y	m	$(\Delta x)(m)$	y_{new}
1	4	0	0	4
1.5	4	$6 \ln(1.5)$	$3 \ln(1.5)$	$2 + 3 \ln(1.5)$
2				

$$f(x) \approx 2 + 3 \ln(1.5)$$



5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5

Answer QUESTION 5 part (c) on this page.

Response for question 5(c)

$$\frac{dy}{dx} = y(x \ln x)$$

$$\int (x \ln x) dx$$

$$u = \ln x \quad dv = x$$

$$du = \frac{1}{x} dx \quad v = \frac{1}{2} x^2$$

$$\int \frac{dy}{y} = \int (x \ln x) dx$$

$$\ln y = (\ln x) \left(\frac{1}{2} x^2 \right) - \int \frac{1}{2} x \, dx + C$$

$$\ln y = (\ln x) \left(\frac{1}{2} x^2 \right) - \frac{1}{4} x^2 + C$$



5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5

Answer QUESTION 5 parts (a) and (b) on this page.

Response for question 5(a)

$$f(1) = 4$$

$$f'(1) = \frac{dy}{dx} \Big|_{(1,4)} = 4(1)\ln 1 = 0$$

$$f''(1) = \frac{d^2y}{dx^2} \Big|_{(1,4)} \Rightarrow \frac{d}{dx} [yx\ln x] \Big|_{(1,4)}$$

$$f \approx P_2(x) = 4 + \frac{f''(1)(x-1)^2}{2} = 4 + \frac{\frac{d}{dx}[yx\ln x] \Big|_{(1,4)} (x-1)^2}{2}$$

$$f(2) \approx 4 + \frac{\frac{d}{dx}[yx\ln x] \Big|_{(1,4)} (2-1)^2}{2} \rightarrow f(2) \approx 4 + \frac{\frac{d}{dx}[yx\ln x] \Big|_{(1,4)}}{2}$$

Response for question 5(b)

$$f(1) = 4$$

$$f(1.5) = 4 + [4 \cdot 1(\ln 1)](0.5) = 6$$

$$f(2) = 6 + [6 \cdot \frac{3}{2} \ln \frac{3}{2}] (0.5)$$



• 5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5

Answer QUESTION 5 part (c) on this page.

Response for question 5(c)

$$\frac{dy}{dx} = y \times \ln x$$

$$\int \frac{1}{y} dy = \int x \ln x dx$$

$$\ln|y| = -\ln|x| + C$$

integration by parts
 $u = x, dv = \ln x dx$
 $du = 1 dx, v = \frac{1}{x}$

$$\int x \ln x dx = x(\frac{1}{x}) - \int \frac{1}{x}(1) dx$$

$$= -\ln|x| + C$$

↳ find C, $f(1) = 4$

$$\ln 4 = -\ln 1 + C$$

$$C = \ln 4 - 1$$

$$\ln|y| = -\ln|x| + \ln 4 - 1$$

$$\ln|y| = \ln 4 - \ln|x|$$

$$e^{\ln 4 - \ln|x|} = y$$

5 5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5 5

Answer QUESTION 5 parts (a) and (b) on this page.

Response for question 5(a)

$$f(x) + f'(x)(x-1) + \frac{f''(x)(x-1)^2}{2!}$$

$$f(x) + f'(x)(x-1) + \frac{f''(x)(x-1)^2}{2!} \quad \text{Taylor polynomial for } f \text{ about } x=1$$

Response for question 5(b)

$$f(1) = 4$$

x	y	$\frac{dy}{dx} \approx \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$
1	4	$(4 + 1.1 \ln(1)) (0.5) + 4$
1.5	4	$(4 + 1.5 \ln(1.5)) (0.5) + 4$
2	$(4 + 1.8 \ln(1.8)) (0.5) + 4$	

$$f(z) = 3 \ln(1.5) + 4$$



• 5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5

Answer QUESTION 5 part (c) on this page.

Response for question 5(c)

$$\frac{dy}{dx} = y \cdot (x \ln x)$$

$$\int \frac{dy}{y} = \int x \ln x \, dx$$

$$\ln y = 1$$

$$e^{\ln y} = e^{1/x}$$

$$y = e^{1/x} + C$$

$$y = e^1 + C$$

$$4 = C$$

$$y = e^{1/x} + 4$$

$$\begin{aligned} \int x \ln x \, dx &= \\ \frac{1}{2}x^2 - \int \frac{1}{x} \, dx &= \\ = 1 - \int x^{-1} \, dx &= 1 - \end{aligned}$$

$$\begin{aligned} u = x &\quad dv = \ln x \\ du = 1 \, dx &\quad v = \frac{1}{x} \end{aligned}$$

$$\begin{aligned} u = \ln(x) &\\ du = \frac{1}{x} \, dx & \end{aligned}$$

$$\int 1 \, du = u + C = \ln x + C$$

$$f(1) = 4 \leftarrow \text{plug in for } C$$

5

5

5

5

NO CALCULATOR ALLOWED

5

5

5

5

Answer QUESTION 5 parts (a) and (b) on this page.

Response for question 5(a)

$$x=1 \quad \boxed{F = 1 + 4x + \frac{2x^2}{2} + \frac{4x^3}{6}}$$

$$f = 1 + 4x + x^2 + \frac{2}{3}x^3$$

$$F(2) \approx 1 + 8 + 4 + \frac{16}{3}$$

$$F(2) = \frac{32}{3} + \frac{16}{3} = \boxed{\frac{56}{3}}$$

$$1 + \int x f(x) dx = \int x f(x) dx$$

Response for question 5(b)

$$f(1) = 4$$

$$4 + (0.5 \times 0)$$

$$f(1.5) = 4$$

$$4 + (0.5 \times 4 + 1.5 \ln(1.5))$$

$$\boxed{F(2) = 4 + 3 \ln(1.5)}$$

• 5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5

Answer QUESTION 5 part (c) on this page.

Response for question 5(c)

$$\frac{dy}{dx} = y \cdot (x \ln x)$$

$$\frac{dy}{y} = x \ln x \cdot dx$$

$$\frac{1}{y} dy = \ln 1 + dx$$

Question 5**Sample Identifier: A****Score: 9**

- The response earned 9 points: 2 points in part (a), 2 points in part (b), and 5 points in part (c).
- In part (a) the response earned the first point with a correct expression for the Taylor polynomial in line 2 on the left. Simplification is not necessary. The response earned the second point with a correct approximation of $f(2)$ in line 3 on the left. Again simplification was not necessary.
- In part (b) the response earned the first point in the table: two Euler steps are visible in the rows, $\Delta x = 0.5$ is given in the second column, the correct values for $\frac{dy}{dx}$ are given in the third column, and the initial condition $f(1) = 4$ was used in the first row. Note that since the final answer is correct in this case, the table does not need to be labelled. The response earned the second point with the correct answer boxed beneath the table.
- In part (c) the response earned the first point with a correct separation of variables in line 2. The response earned the second point with a correct antiderivative of $x \ln x$ in line 5 on the right. The response earned the third point with a correct antiderivative of $\frac{1}{y}$ in line 4 on the left. No absolute value signs are necessary. The response is eligible for the fourth point. The response earned the fourth point with the correct inclusion of the constant of integration in line 5 and the correct substitution of 1 for x and 4 for y in line 6. The response is eligible for the fifth point, which it earned with the correct answer in line 9.

Sample Identifier: B**Score: 8**

- The response earned 8 points: 1 point in part (a), 2 points in part (b), and 5 points in part (c).
- In part (a), the response earned the first point with a correct expression for the Taylor polynomial in line 1. Simplification is not necessary. The response did not earn the second point because no value of $f(2)$ is presented.
- In part (b), the response earned the first point: the two Euler steps are seen in lines 2 and 4, the value $\Delta x = 0.5$ is declared in line 1, lines 2 and 4 demonstrate use of the correct $\frac{dy}{dx}$, and line 2 shows use of the initial condition $f(1) = 4$. The response earned the second point with a correct answer in line 3. Simplification is not necessary.
- In part (c), the response earned the first point with a correct separation of variables in line 2. The response earned the second point with a correct antiderivative of $x \ln x$ in line 8, and earned the third point with a correct antiderivative of $\frac{1}{y}$ in line 4. The response is eligible for the fourth point, and earned the fourth point with a correct inclusion of the constant of integration in line 8 and a correct substitution of 1 for x and 4 for y in line 9. The response is eligible for the fifth point, and earned the fifth point with a correct answer in line 12.

Question 5 (continued)**Sample Identifier: C****Score: 8**

- The response earned 8 points: 2 points in part (a), 2 points in part (b), and 4 points in part (c).
- In part (a) the response earned the first point with a correct expression for the Taylor polynomial in line 3. The response earned the second point with a correct approximation for $f(2)$ in line 3 in the center. Simplification is not necessary.
- In part (b) the response earned the first point: two Euler steps are shown in lines 2 and 4, the step size $\Delta x = 0.5$ is implied in the evaluation of $x - 1.5$ at $x = 2$ in line 5, the correct expression for $\frac{dy}{dx}$ is used in lines 1 and 3, and the initial condition $f(1) = 4$ is used in the point-slope form of the linear equation in line 2. The response earned the second point with a correct answer in line 5. Simplification is not necessary.
- In part (c) the response earned the first point with a correct separation of variables in line 2. The response earned the second point with a correct antiderivative of $x \ln x$ in line 4. The response earned the third point with a correct antiderivative of $\frac{1}{y}$ in line 3. Absolute value signs are included but are not necessary. The response is eligible for the fourth point. The response earned the fourth point with a correct inclusion of the constant of integration in line 4 and a correct substitution of 1 for x and 4 for y in line 7. The response is eligible for the fifth point. The response does not earn the fifth point as the answer is incorrect.

Sample Identifier: D**Score: 7**

- The response earned 7 points: 2 points in part (a), 2 points in part (b), and 3 points in part (c).
- In part (a) the response earned the first point with a correct expression for the Taylor polynomial in line 2. The response earned the second point with a correct approximation for $f(2)$ in line 3 in the center. Simplification is not necessary.
- In part (b) the response earned the first point: two Euler steps are shown in rows 2 and 3 of the table, the step size $\Delta x = 0.5$ is used in column 3 of the table, the correct expression for $\frac{dy}{dx}$ is used in the second column of the table, and the initial condition $f(1) = 4$ is used in the first row of the table. The response earned the second point with a correct answer below the table. Note that the point $(2, 4 + 2 \cdot 1.5 \ln(1.5))$ in the third row and fourth column of the table would also earn the second point.
- In part (c) the response earned the first point with a correct separation of variables in line 1. The response did not earn the second point as the antiderivative of $x \ln x$ is incorrect. The response earned the third point with a correct antiderivative of $\frac{1}{y}$ in line 2. The response is eligible for the fourth point as it earned the first point and one of the two antiderivative points. The response earned the fourth point with a correct inclusion of the constant of integration in line 2 and a correct substitution of 1 for x and 4 for y in line 3. The response is not eligible for the fifth point as it did not earn all of the first four points.

Question 5 (continued)**Sample Identifier: E****Score: 7**

- The response earned 7 points: 1 point in part (a), 2 points in part (b), and 4 points in part (c).
- In part (a) the response earned the first point with a correct expression for the Taylor polynomial in line 2. Simplification is not necessary. The response did not earn the second point. While the correct approximation for $f(2)$ is shown in line 3 and in line 4 on the left, the boxed result was simplified incorrectly.
- In part (b) the response earned the first point: two Euler steps are seen in the second and third rows of the table, the step size $\Delta x = 0.5$ is shown in the third column of the table, the correct expression for $\frac{dy}{dx}$ is used to create the fourth column of the table, and the initial condition $f(1) = 4$ is shown in the first two entries of the second row of the table. Note that since the answer is correct in this case, the table does not need to be labelled. The response earned the second point with a correct answer below the table.
- In part (c) the response earned the first point with a correct separation of variables in line 3 on the left side of the page. The response earned the second point with a correct antiderivative of $x \ln x$ in lines 1-5 on the right side of the page. The response earned the third point with a correct antiderivative of $\frac{1}{y}$ in line 4 on the left side of the page. The response is eligible for the fourth point as it has earned the first point and at least one of the antiderivative points. The response earned the fourth point with a correct inclusion of the constant of integration in line 3 on the left side of the page and the substitution of 1 for x and 4 for y in line 4 on the left side of the page. The response did not earn the fifth point as the answer in line 9 on the left side of the page is incorrect. Note that the answer given in line 8 of the left side of the page is correct, but is simplified incorrectly.

Question 5 (continued)**Sample Identifier: F****Score: 6**

- The response earned 6 points: 2 points in part (a), 1 point in part (b), and 3 points in part (c).
- In part (a) the response earned the first point with a correct expression for the Taylor polynomial in line 4. Simplification is not necessary. The response earned the second point with a correct approximation for $f(2)$ in line 5 on the left. Simplification is not necessary.
- In part (b) the response earned the first point: two Euler steps are shown in lines 4 and 5, the step size $\Delta x = 0.5$ is shown in line 3 and used in lines 4 and 5, the correct expression for $\frac{dy}{dx}$ is used in lines 4 and 5, and the initial condition $f(1) = 4$ is used in line 4. The response did not earn the second point because the final answer in line 5 on the right and in line 6 is incorrect. Note that the answer given in line 5 in the center is correct, but was simplified incorrectly. Note that in part (b), the response mislabels $f(1.5)$ as $f(1)$ and $f(2)$ as $f(1.5)$, but recoups these errors by declaring a consistent $f(2)$.
- In part (c) the response earned the first point with a correct separation of variables in line 2. The response did not earn the second point as the antiderivative for $x \ln x$ in line 3 is incorrect. The response earned the third point with a correct antiderivative for $\frac{1}{y}$ in line 3. The response is eligible for the fourth point since it earned the first point and one of the two antiderivative points. The response earned the fourth point with a correct inclusion of the constant of integration in line 3 and a correct substitution of 1 for x and 4 for y in line 4. The response is not eligible for the fifth point as it did not earn all of the first four points.

Sample Identifier: G**Score: 5**

- The response earned 5 points: 1 point in part (a), 1 point in part (b), and 3 points in part (c).
- In part (a) the response did not earn the first point because there is not a correct expression for the Taylor polynomial. Note that the polynomial in line 1 is generic. The response earned the second point with the supporting work and answer in line 2. Simplification is not necessary. The second point was earned even though the polynomial is not explicitly written.
- In part (b), the response earned the first point: two Euler steps are shown in the second and third rows of the table, the step size $\Delta x = 0.5$ may be inferred from the calculation leading from the $6\ln(1.5)$ to the $3\ln(1.5)$ in the third row of the table, the correct expression for $\frac{dy}{dx}$ may be inferred from the $6\ln(1.5)$ entry in the third row of the table, and the initial condition $f(1) = 4$ is shown in the first row of the table. The response did not earn the second point because the final answer below the table is incorrect.
- In part (c) the response earned the first point with a correct separation of variables in line 2 on the left side of the page. The response earned the second point with a correct antiderivative of $x \ln x$ in line 4 on the left side of the page. The response earned the third point with a correct antiderivative of $\frac{1}{y}$ in line 3 on the left side of the page. The response is eligible for the fourth point, having earned the first three points. The response did not earn the fourth point. While a constant of integration is correctly included in line 3 on the left side of the page, the values of 1 and 4 are not substituted for x and y , respectively. The response is not eligible for the fifth point as it did not earn all of the first four points.

Question 5 (continued)**Sample Identifier: H****Score: 4**

- The response earned 4 points: no points in part (a), 1 point in part (b), and 3 points in part (c).
- In part (a) the response did not earn the first point because no correct Taylor polynomial is presented. The response did not earn the second point because no correct approximation for $f(2)$ is presented.
- In part (b) the response earned the first point: two Euler steps are shown in lines 2 and 3, the step size $\Delta x = 0.5$ is used in lines 2 and 3, the correct expression for the derivative is evaluated in lines 2 and 3, and the initial condition $f(1) = 4$ is stated in line 1. Note that the response contains a single error: the value of the approximation for $f(1.5)$ in line 2 is simplified incorrectly. Importing this incorrect value correctly into line 3 is not an error. Since only one mistake was made, the response is still eligible for the first point, which it earned. The response did not earn the second point because the boxed answer is incorrect.
- In part (c) the response earned the first point with a correct separation of variables in line 2 on the left side of the page. The response did not earn the second point as the antiderivative for $x \ln x$ is incorrect. The response earned the third point with a correct antiderivative for $\frac{1}{y}$ in line 3 on the left side of the page. The response is eligible for the fourth point since it earned the first point and one of the two antiderivative points. The response earned the fourth point with a correct inclusion of a constant of integration in line 3 on the left side of the page and a correct substitution of 1 for x and 4 for y in line 5 on the left side of the page. The response is not eligible for the fifth point as it did not earn all of the first four points.

Sample Identifier: I**Score: 4**

- The response earned 4 points: no points in part (a), 2 points in part (b), and 2 points in part (c).
- In part (a), the response did not earn the first point because it does not present a correct Taylor polynomial for f about $x = 1$. The response did not earn the second point because no approximation for $f(2)$ is presented.
- In part (b), the response earned the first point: two Euler steps are shown in the second and third row of the table, the step size $\Delta x = 0.5$ is stated and used in the third column of the table, the correct expression for $\frac{dy}{dx}$ is used in the second and third columns of the table, and the initial condition $f(1) = 4$ is used in the first row of the table. The response earned the second point with the expression below the table. Note that the expression in the fourth row and second column of the table (which is equal to the correct answer) would earn the second point, as its position in the labelled table indicates that it is an approximation for $f(2)$.
- In part (c), the response earned the first point with a correct separation of variables in line 2 on the left of the page. The response did not earn the second point because the antiderivative for $x \ln x$ is incorrect. The response earned the third point with a correct antiderivative for $\frac{1}{y}$ in line 3 on the left of the page. The response is eligible for the fourth point as it earned the first point and one of the antiderivative points. The response did not earn the fourth point because the constant of integration was included incorrectly. The response is not eligible for the fifth point as it did not earn all of the first four points.

Question 5 (continued)**Sample Identifier: J****Score: 3**

- The response earned 3 points: no points in part (a), 2 points in part (b), and 1 point in part (c).
- In part (a), the response did not earn the first point because a correct Taylor polynomial is not presented. The response did not earn the second point because a correct approximation for $f(2)$ is not presented.
- In part (b) the response earned the first point: two Euler steps are seen in lines 1 and 2 in the center of the page, the step size $\Delta x = 0.5$ is used in the equations in lines 1 and 2 in the center of the page, the correct expression for $\frac{dy}{dx}$ is evaluated in the equations in lines 1 and 2 in the center of the page, and the initial condition $f(1) = 4$ is shown in line 1 on the left side of the page. The response earned the second point with the correct boxed answer. Note that line 2 in the center of the page would also earn the second point. Simplification is not necessary.
- In part (c) the response earned the first point with a correct separation of variables in line 2. The response did not earn the second point because an antiderivative of $x \ln x$ is not presented. The response did not earn the third point because an antiderivative of $\frac{1}{y}$ is not presented. The response is not eligible for the fourth point since it did not earn either of the antiderivative points. The response is not eligible for the fifth point as it did not earn all of the first four points.

6. The function g has derivatives of all orders for all real numbers. The Maclaurin series for g is given by

$$g(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2e^n + 3} \text{ on its interval of convergence.}$$

- (a) State the conditions necessary to use the integral test to determine convergence of the series $\sum_{n=0}^{\infty} \frac{1}{e^n}$.

Use the integral test to show that $\sum_{n=0}^{\infty} \frac{1}{e^n}$ converges.

- (b) Use the limit comparison test with the series $\sum_{n=0}^{\infty} \frac{1}{e^n}$ to show that the series $g(1) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2e^n + 3}$

converges absolutely.

- (c) Determine the radius of convergence of the Maclaurin series for g .

- (d) The first two terms of the series $g(1) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2e^n + 3}$ are used to approximate $g(1)$. Use the alternating

series error bound to determine an upper bound on the error of the approximation.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

Part B (BC): Graphing calculator not allowed**Question 6****9 points****General Scoring Notes**

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Scoring guidelines and notes contain examples of the most common approaches seen in student responses. These guidelines can be applied to alternate approaches to ensure that these alternate approaches are scored appropriately.

The function g has derivatives of all orders for all real numbers. The Maclaurin series for g is given by

$$g(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2e^n + 3} \text{ on its interval of convergence.}$$

Model Solution**Scoring****(a)**

- State the conditions necessary to use the integral test to determine convergence of the series $\sum_{n=0}^{\infty} \frac{1}{e^n}$. Use the integral test to show that $\sum_{n=0}^{\infty} \frac{1}{e^n}$ converges.

e^{-x} is positive, decreasing, and continuous on the interval $[0, \infty)$.	Conditions	1 point
To use the integral test to show that $\sum_{n=0}^{\infty} \frac{1}{e^n}$ converges, show that $\int_0^{\infty} e^{-x} dx$ is finite (converges).	Improper integral	1 point
$\int_0^{\infty} e^{-x} dx = \lim_{b \rightarrow \infty} \int_0^b e^{-x} dx = \lim_{b \rightarrow \infty} \left(-e^{-x} \Big _0^b \right) = \lim_{b \rightarrow \infty} \left(-e^{-b} + e^0 \right) = 1$	Evaluation	1 point
Because the integral $\int_0^{\infty} e^{-x} dx$ converges, the series $\sum_{n=0}^{\infty} \frac{1}{e^n}$ converges.		

Scoring notes:

- To earn the first point, a response must list all three conditions: e^{-x} is positive, decreasing, and continuous.
- The second point is earned for correctly writing the improper integral, or for presenting a correct limit equivalent to the improper integral, for example, $\lim_{b \rightarrow \infty} \int_0^b e^{-x} dx$.
- To earn the third point, a response must correctly use limit notation to evaluate the improper integral, find an evaluation of e^0 (or 1) and conclude that the integral converges or that the series converges.
- If an incorrect lower limit of 1 is used in the improper integral, then the second point is not earned. In this case, if the correct limit ($1/e$) is presented, then the response is eligible for the third point.
- If the response only relies on using a geometric series approach, then no points are earned [0-0-0].
- A response that presents an evaluation with ∞ , such as $e^{-\infty} = 0$, does not earn the third point.

Total for part (a) **3 points**

- (b)** Use the limit comparison test with the series $\sum_{n=0}^{\infty} \frac{1}{e^n}$ to show that the series $g(1) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2e^n + 3}$ converges absolutely.

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{e^n}}{\left| \frac{(-1)^n}{2e^n + 3} \right|} = \lim_{n \rightarrow \infty} \frac{2e^n + 3}{e^n} = 2$$

The limit exists and is positive. Therefore, because the series

$\sum_{n=0}^{\infty} \frac{1}{e^n}$ converges, the series $\sum_{n=0}^{\infty} \left| \frac{(-1)^n}{2e^n + 3} \right|$ converges by the limit comparison test.

Thus, the series $g(1) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2e^n + 3}$ converges absolutely.

Sets up limit comparison

1 point

Explanation

1 point

Scoring notes:

- The first point is earned for setting up the limit comparison, with or without absolute values. Limit notation is required to earn this point.
- The reciprocal of the given ratio is an acceptable alternative; the limit in this case is $1/2$.
- The second point cannot be earned without the use of absolute value symbols, which can occur explicitly or implicitly (e.g., a response might set up the limit comparison initially as

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{e^n}}{\frac{1}{2e^n + 3}}).$$

- Earning the second point requires correctly evaluating the limit and noting that the limit is a positive number. For example, $L = 2 > 0$ or $L = 1/2 > 0$. Therefore, comparing the limit L to 1 does not earn the explanation point.
- A response does not have to repeat that $\sum_{n=0}^{\infty} \frac{1}{e^n}$ converges.
- A response that draws a conclusion based only on the sequence (such as $\frac{1}{e^n}$) without referencing a series does not earn the second point.
- If the response does not explicitly use the limit comparison test, then no points are earned in this part.
- A response cannot earn the second point for just concluding that “the series” converges absolutely because there are multiple series in this part of the problem. The response must specify that the

series $g(1)$ or $\sum_{n=0}^{\infty} \frac{(-1)^n}{2e^n + 3}$ converges absolutely.

Total for part (b) 2 points

- (c) Determine the radius of convergence of the Maclaurin series for g .

$\left \frac{(-1)^{n+1} x^{n+1}}{2e^{n+1} + 3} \cdot \frac{2e^n + 3}{(-1)^n x^n} \right = \left \frac{(2e^n + 3)x^{n+1}}{(2e^{n+1} + 3)x^n} \right = \frac{2e^n + 3}{2e^{n+1} + 3} x $	Sets up ratio	1 point
$\lim_{n \rightarrow \infty} \frac{2e^n + 3}{2e^{n+1} + 3} x = \frac{1}{e} x $	Computes limit of ratio	1 point
$\frac{1}{e} x < 1 \Rightarrow x < e$	Answer	1 point
The radius of convergence is $R = e$.		

Scoring notes:

- The first point is earned for $\frac{(-1)^{n+1} x^{n+1}}{2e^{n+1} + 3} \cdot \frac{2e^n + 3}{(-1)^n x^n}$ or the equivalent. Once earned, this point cannot be lost.
- The second point cannot be earned without the first point.
- To be eligible for the third point, the response must have found a limit for a presented ratio such that the limiting value of the coefficient on $|x|$ is finite and not 0. The third point is earned for setting up an inequality such that the limit is less than 1, solving for $|x|$, and interpreting the result to find the radius of convergence.
- The radius of convergence must be explicitly presented, for example, $R = e$. The third point cannot be earned by presenting an interval, for example $-e < x < e$, with no identification of the radius of convergence.

Total for part (c) **3 points**

(d)

The first two terms of the series $g(1) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2e^n + 3}$ are used to approximate $g(1)$. Use the alternating series error bound to determine an upper bound on the error of the approximation.

The terms of the alternating series $g(1) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2e^n + 3}$ decrease in magnitude to 0.

Answer

1 point

The alternating series error bound for the error of the approximation is the absolute value of the third term of the series.

$$\text{Error} \leq \left| \frac{(-1)^2}{2e^2 + 3} \right| = \frac{1}{2e^2 + 3}$$

Scoring notes:

- A response of $\frac{1}{2e^2 + 3}$ earns this point.

Total for part (d)**1 point****Total for question 6****9 points**

6 6 6 6 NO CALCULATOR ALLOWED 6 6 6 6

Answer QUESTION 6 parts (a) and (b) on this page.

Response for question 6(a)

The function $\frac{1}{e^n}$ is continuous, positive, and decreasing for $n \geq 0$

$$\sum_{n=0}^{\infty} \frac{1}{e^n}$$

converges by integral test

$$\begin{aligned} \int_0^{\infty} e^{-x} dx &= \lim_{b \rightarrow \infty} \int_0^b \frac{1}{e^n} dn = \lim_{b \rightarrow \infty} \int_0^b e^{-n} dn \\ &= \lim_{b \rightarrow \infty} [-e^{-n}]_0^b = \lim_{b \rightarrow \infty} [-e^{-b} - (-e^0)] \\ &= 0 + 1 = 1 \end{aligned}$$

Response for question 6(b)

$$g(1) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2e^n + 3}$$

absolute value of $g(1)$

$$\lim_{n \rightarrow \infty} \frac{1}{\frac{1}{e^n}} = \lim_{n \rightarrow \infty} \frac{e^n}{2e^n + 3} = \frac{1}{2} > 0$$

compare to

$$\sum_{n=0}^{\infty} \frac{1}{e^n}$$

geometric
 $r = \frac{1}{e} < 1$
converges

$$\frac{1}{2e^n + 3} > 0$$

$$\frac{1}{e^n} > 0$$

$g(1)$ converges absolutely
by the Limit Comparison
Test

● 6 6 6 6 NO CALCULATOR ALLOWED 6 6 6 6

Answer QUESTION 6 parts (c) and (d) on this page.

Response for question 6(c)

$$g(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2e^n + 3}$$

$$\lim_{n \rightarrow \infty} \left| \frac{x^{n+1}}{2e^{n+1} + 3} \cdot \frac{2e^n + 3}{x^n} \right| = \left| \frac{x}{e} \right|$$

$$\left| \frac{x}{e} \right| < 1$$

$$|x| < e$$

$$R = e$$

\uparrow
radius of convergence for g

Response for question 6(d)

approximation
using first
two terms

$$\text{Error} \leq \left| \text{third term of the series } g(1) \right|$$

$$\text{Error} \leq \left| \frac{1}{2e^2 + 3} \right|$$

$$\text{Error} \leq \frac{1}{2e^2 + 3}$$

6 6 6 6 NO CALCULATOR ALLOWED 6 6 6 6

Answer QUESTION 6 parts (a) and (b) on this page.

Response for question 6(a)

$\left\{\frac{1}{e^n}\right\}_{n=0}^{\infty}$ has positive, decreasing terms

$$\begin{aligned}& \int_0^{\infty} \frac{1}{e^n} dn \\&= \lim_{b \rightarrow \infty} \int_0^b e^{-n} dn \\&= \lim_{b \rightarrow \infty} [-e^{-n}]_0^b \\&= \lim_{b \rightarrow \infty} (-e^{-b} + e^0) \\&= 0 + 1\end{aligned}$$

= 1 \Rightarrow converge [By the integral test, $\sum_{n=0}^{\infty} \frac{1}{e^n}$ converges]

Response for question 6(b)

$$\sum_{n=0}^{\infty} \left| \frac{(-1)^n}{2e^n + 3} \right| = \sum_{n=0}^{\infty} \frac{1}{2e^n + 3}$$

$\sum_{n=0}^{\infty} \frac{1}{2e^n + 3}$ and $\sum_{n=0}^{\infty} \frac{1}{e^n}$ have positive terms

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{2e^n + 3}}{\frac{1}{e^n}} = \lim_{n \rightarrow \infty} \frac{e^n}{2e^n + 3} = \frac{1}{2} > 0$$

$\sum_{n=0}^{\infty} \frac{1}{e^n}$ converges (part (a))

By the limit comparison test, $\sum_{n=0}^{\infty} \frac{1}{2e^n + 3}$ converges also.

So, $\sum_{n=0}^{\infty} \frac{(-1)^n}{2e^n + 3}$ converges absolutely



● 6 6 6 6 NO CALCULATOR ALLOWED 6 6 6 6

Answer QUESTION 6 parts (c) and (d) on this page.

Response for question 6(c)

$$\begin{aligned} \lim_{n \rightarrow \infty} \left| \frac{\frac{(-1)^{n+1} x^{n+1}}{2e^{n+1} + 3}}{\frac{(-1)^n x^n}{2e^n + 3}} \right| &= \lim_{n \rightarrow \infty} \left| \frac{x^{n+1}}{2e^{n+1} + 3} \cdot \frac{2e^n + 3}{x^n} \right| \\ &= \lim_{n \rightarrow \infty} \left| \frac{x(2e^{n+1} + 3)}{2e^{n+1} + 3} \right| \\ &= |x| \lim_{n \rightarrow \infty} \frac{2e^{n+1} + 3}{2e^{n+1} + 3} \\ &= |x| \cdot \frac{1}{e} \end{aligned}$$

Converges if $\frac{|x|}{e} < 1$

$$|x| < e$$

$$R = e$$

Response for question 6(d)

First 2 terms: $\frac{1}{2e+3} - \frac{1}{2e^2+3}$; Alternating series, so error $\leq |\text{next term}|$

$$\text{Error} \leq \left| \frac{1}{2e^3+3} \right|$$

$$\text{Upper bound} = \frac{1}{2e^3+3}$$

6 6 6 6 NO CALCULATOR ALLOWED 6 6 6 6

Answer QUESTION 6 parts (a) and (b) on this page.

Response for question 6(a)

If $\int_1^\infty \frac{1}{e^n} dn < 1$, $\sum_{n=0}^{\infty} \frac{1}{e^n}$ converges

$$\lim_{b \rightarrow \infty} \int_1^b \frac{1}{e^n} dn = \lim_{b \rightarrow \infty} \left[-e^{-n} \right]_1^b = \lim_{b \rightarrow \infty} \left[e^{-b} + e^{-1} \right] = 0 + \frac{1}{e} < 1$$

$\sum_{n=0}^{\infty} \frac{1}{e^n}$ converges by the Integral Test

Response for question 6(b)

$$\lim_{n \rightarrow \infty} \left| \frac{(-1)^n}{2e^n + 3} \cdot \frac{e^n}{1} \right| = \lim_{n \rightarrow \infty} \left(\frac{e^n}{2e^n + 3} \right) = \lim_{n \rightarrow \infty} \left| \frac{1}{2 + \frac{3}{e^n}} \right| = \frac{1}{2}$$

Since $\sum_{n=0}^{\infty} \frac{1}{e^n}$ converges by the Integral Test, and $\lim_{n \rightarrow \infty} \left| \frac{(-1)^n}{2e^n + 3} \div \frac{1}{e^n} \right| = \frac{1}{2}$,

$\sum_{n=0}^{\infty} \frac{(-1)^n}{2e^n + 3}$ converges absolutely by the Limit Comparison Test



● 6 6 6 6 NO CALCULATOR ALLOWED 6 6 6 6

Answer QUESTION 6 parts (c) and (d) on this page.

Response for question 6(c)

$$\begin{aligned} \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| &= \lim_{n \rightarrow \infty} \left| \frac{(-1)^{n+1} x^{n+1}}{2e^{n+3}} \cdot \frac{2e^n + 3}{(-1)^n x^n} \right| = \lim_{n \rightarrow \infty} \left| \frac{x \cancel{e^n} \cdot (2e^n + 3)}{(2e^{n+1} + 3)} \right| = |x| \lim_{n \rightarrow \infty} \left| \frac{2e^n + 3}{2e^{n+1} + 3} \right| = |x| \lim_{n \rightarrow \infty} \left| \frac{2 + \frac{3}{e^n}}{2e + \frac{3}{e^n}} \right| \\ &= \left| \frac{x}{e} \right| \sum_{n=0}^{\infty} (-1)^n e^n \quad |x| < e \\ &\text{the radius of convergence is } e \end{aligned}$$

Response for question 6(d)

$$\begin{aligned} R &\leq \frac{(-1)^{(z)}}{2e^{(z)+3}} = \frac{1}{2e^{z+3}} \\ |R| &\leq \frac{1}{2e^{z+3}} \end{aligned}$$

6 6 6 6 NO CALCULATOR ALLOWED 6 6 6 6

Answer QUESTION 6 parts (a) and (b) on this page.

$$e^{-x} = -e^{-x}$$

Response for question 6(a)

$$\sum_{n=0}^{\infty} \frac{1}{e^n} = \frac{1}{e^x} \quad * \text{function is equal to series}$$

* series can be approximated by function

$$\int_0^{\infty} \frac{1}{e^x} dx = \lim_{b \rightarrow \infty} \int_0^b \frac{1}{e^x} dx = \lim_{b \rightarrow \infty} -\frac{1}{e^x} \Big|_0^b = \lim_{b \rightarrow \infty} -\frac{1}{e^b} + \frac{1}{e^0} = 1$$

Since $\int_0^{\infty} \frac{1}{e^x} dx = 1$, $\sum_{n=0}^{\infty} \frac{1}{e^n}$ converges by the integral test.

Response for question 6(b)

$$\sum_{n=0}^{\infty} \frac{1}{e^n} > \sum_{n=0}^{\infty} \frac{1}{2e^n+3}$$

$$\lim_{n \rightarrow \infty} \left| \frac{e^n}{2e^n+3} \right| = \lim_{n \rightarrow \infty} \left| \frac{e^n}{2e^n} \right| = \frac{1}{2} < 1$$

So, converges absolutely by limit comparison test.



• 6 6 6 6 NO CALCULATOR ALLOWED 6 6 6 6

Answer QUESTION 6 parts (c) and (d) on this page.

Response for question 6(c)

$$\lim_{n \rightarrow \infty} \left| \frac{(-1)^{n+1} \cdot x^{n+1}}{(-1)^n \cdot x^n} \cdot \frac{2e^n + 3}{2e^{n+1} + 3} \right| = \lim_{n \rightarrow \infty} \left| (-1)(x) \cdot \frac{1}{2e} \right|$$

$$-1 < -\frac{x}{2e} < 1$$

$$2e > x > -2e$$

Radius of convergence = $2e$

Response for question 6(d)

*since first two terms are used for approximation, third term will give the upper bound on error of approximation

$$\text{error} \leq \frac{(-1)^2}{2e^2 + 3} \rightarrow n=2$$

error $\leq \frac{1}{2e^2 + 3}$

6 6 6 6 NO CALCULATOR ALLOWED 6 6 6 6

Answer QUESTION 6 parts (a) and (b) on this page.

Response for question 6(a)

can't equal ∞ , must be a definite number for convergence to be determined
 $\frac{1}{e^x} = e^{-x}$

$$\lim_{b \rightarrow \infty} \int_0^b \frac{1}{e^x} dx$$

$$\lim_{b \rightarrow \infty} \left(-\frac{1}{e^x} \right)_0^b$$

$$\lim_{b \rightarrow \infty} -\left(\frac{1}{e^b} - \frac{1}{e^0} \right)$$

$$-\left(\frac{1}{e^\infty} - \frac{1}{e^0} \right) = 1$$

Response for question 6(b)

$$\lim_{n \rightarrow \infty} \left| \frac{\frac{1}{e^n}}{\frac{1}{2e^n + 3}} \right| = \lim_{n \rightarrow \infty} \left| \frac{1}{e^n} \cdot \frac{2e^n + 3}{1} \right|$$

$$\lim_{n \rightarrow \infty} \left| \frac{2e^n + 3}{e^n} \right|$$

$$\left| \frac{2e^\infty + 3}{e^\infty} \right| = 2 \quad \text{converges absolutely}$$

the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{2e^n + 3}$ converges absolutely as

it converges by the limit comparison test in comparison to $\frac{1}{e^n}$

• 6 6 6 6 NO CALCULATOR ALLOWED 6 6 6 6

Answer QUESTION 6 parts (c) and (d) on this page.

Response for question 6(c)

$$\lim_{n \rightarrow \infty} \left| \frac{(-1)^{n+1} x^{n+1}}{2e^{n+1} + 3} \right| = \lim_{n \rightarrow \infty} \left| \frac{(-1)^{n+1} x^{n+1}}{2e^{n+1} + 3} \cdot \frac{2e^n + 3}{2e^n + 3} \right|$$

$$= \lim_{n \rightarrow \infty} \left| \frac{2e^n + 3}{2e^{n+1} + 3} \cdot -x \right|$$

$$|-x| < 1$$

the radius of convergence is $x = 1$

Response for question 6(d)

first 2 terms

$$\frac{(-1)^0 x^0}{2e^0 + 3} + \frac{(-1)^1 x^1}{2e^1 + 3} = \frac{1}{2e^0 + 3} - \frac{x}{2e^1 + 3}$$

$$g(1) = \frac{1}{2e^0 + 3} - \frac{1}{2e^1 + 3}$$

$$|R_2| < \left| \frac{(-1)^2}{2e^2 + 3} \right|$$

the upper bound of the error of the approximation is $\frac{1}{2e^2 + 3}$

6 6 6 6 6 NO CALCULATOR ALLOWED 6 6 6 6 6

Answer QUESTION 6 parts (a) and (b) on this page.

Response for question 6(a)

To use the integral test, the series has to be continuous, positive and decreasing for all values of x .

$$\begin{aligned} \int_0^{\infty} \frac{1}{e^n} dn &= \lim_{b \rightarrow \infty} \int_0^b \frac{1}{e^n} dn = \lim_{b \rightarrow \infty} \left(\left(\frac{1}{e} \right)^n \right) \Big|_0^b \\ &= \lim_{b \rightarrow \infty} \left[\frac{1}{n+1} \left(\frac{1}{e} \right)^{n+1} \right]_0^b \\ &= \lim_{b \rightarrow \infty} \left[\frac{1}{b+1} \left(\frac{1}{e} \right)^{b+1} - \frac{1}{1} \left(\frac{1}{e} \right)^{0+1} \right] = \frac{1}{e} > 0 \end{aligned}$$

$e^{\frac{1}{n}}$ converges as a geometric series with $r = \frac{1}{e}$, so the series converges.

Response for question 6(b)

$$\lim_{n \rightarrow \infty} \frac{(-1)^n}{2e^n + 3} = \lim_{n \rightarrow \infty} \frac{(-1)^n}{2e^n + 3} \cdot e^{-n} = \frac{1}{2}$$

$\frac{1}{e^n}$ converges absolutely as a geometric series with $r = \frac{1}{e} < 1$, so the series converges absolutely.

• 6 6 6 6 6 NO CALCULATOR ALLOWED 6 6 6 6

Answer QUESTION 6 parts (c) and (d) on this page.

Response for question 6(c)

$$\lim_{n \rightarrow \infty} \left| \frac{(-1)^{n+1} x^{n+1}}{2e^{n+1} + 3} \right| = \frac{2e^n + 3}{(-1)^n x^n}$$

$$= \lim_{n \rightarrow \infty} \left| \frac{-x}{e} \right| = \left| \frac{x}{e} \right| < 1$$

$$|x| < e$$

The radius of convergence for the MacLaurin series for g is e .

Response for question 6(d)

first unused term

$$g(1) \approx \frac{1}{5} - \frac{1}{2e+3}(x-1)$$

$$\text{third term: } \frac{1}{2e^2+3}$$

$$|P_3(1) - g(1)| \leq \left| \frac{1}{(2e^2+3)2!} (x-1)^2 \right|$$

6 6 6 6 NO CALCULATOR ALLOWED 6 6 6 6

Answer QUESTION 6 parts (a) and (b) on this page.

Response for question 6(a)

$\frac{1}{e^n}$ has to be decreasing, continuous, differentiable \Rightarrow condition

$$\begin{aligned} \int_0^\infty \frac{1}{e^x} dx &\Rightarrow \lim_{a \rightarrow \infty} \int_0^a \frac{1}{e^x} dx \\ &= \lim_{a \rightarrow \infty} (-e^{-x}) \Big|_0^a \\ &= \lim_{a \rightarrow \infty} -e^{-a} + e^0 \quad \text{Since } \int_0^\infty \frac{1}{e^x} dx \text{ converges, } \therefore \sum_{n=0}^{\infty} \frac{1}{e^n} \text{ converges} \\ &= -e^{\infty} + 1 = 0 + 1 = 1 \quad \text{Converges} \end{aligned}$$

Response for question 6(b)

$\left(-1\right)^n \frac{1}{2e^n+3} \Rightarrow$ Converges due to the Alternating series theorem

$$a_n = \frac{1}{2e^n+3} \quad b_n = \frac{1}{e^n} \quad \lim_{n \rightarrow \infty} \left| \frac{a_n}{b_n} \right| = \lim_{n \rightarrow \infty} \frac{e^n}{2e^n+3} = \frac{1}{2} > 0$$

Since b_n converges, a_n also converges due to limit comparison test

$\therefore \frac{(-1)^n}{2e^n+3}$ converges absolutely

Page 14

Use a pencil or pen with black or dark blue ink only. Do NOT write your name. Do NOT write outside the box.

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• 6 6 6 6 NO CALCULATOR ALLOWED 6 6 6 6

Answer QUESTION 6 parts (c) and (d) on this page.

Response for question 6(c)

$g(x)$ ROC

$$\lim_{n \rightarrow \infty} \left| \frac{(-1)^{n+1} x^{n+1}}{2e^{n+1} + 3} \cdot \frac{2e^n + 3}{(-1)^n x^n} \right| = \lim_{n \rightarrow \infty} \left| -x \cdot \frac{2e^n + 3}{2e^{n+1} + 3} \right| = \lim_{n \rightarrow \infty} \frac{-x \cdot 2e^n + 3}{2e^n \cdot 2e + 3}$$

$\text{ROC } -\frac{x}{2e} < 1$

Radius of convergence
 $= 2e$

Response for question 6(d)

$$g(x) = \underbrace{\frac{1}{5} - \frac{x}{2e+3} + \frac{x^2}{2e^2+3} - \dots}_{\text{used to calculate error bound}} \quad \therefore \text{Error} = P_2(1) = \frac{1}{2e^2+3}$$

$$g(1) \quad \therefore \sum_{n=2}^{\infty} \frac{(-1)^n}{(2e^n+3)}$$

6 6 6 6 6 NO CALCULATOR ALLOWED 6 6 6 6 6

Answer QUESTION 6 parts (a) and (b) on this page.

Response for question 6(a)

Series must converge by limit test and be continuous

$$\int_0^{\infty} \frac{1}{e^n} dn = \lim_{b \rightarrow \infty} \int_0^b \frac{1}{e^n} dn = -e^{-n} \Big|_0^b$$

$$\lim_{b \rightarrow \infty} -e^{-b} + e^0 = 0 + 1 = 1$$

Convergent

Response for question 6(b)

$$\lim_{n \rightarrow \infty} \sum_{k=0}^n \frac{1}{e^n} = 0$$

Since $\sum_{n=0}^{\infty} \frac{1}{e^n} > \sum_{n=0}^{\infty} \frac{1}{2e^n + 3}$

Converges absolutely



• 6 6 6 6 NO CALCULATOR ALLOWED 6 6 6 6

Answer QUESTION 6 parts (c) and (d) on this page.

Response for question 6(c)

$$\text{ROC} = \lim_{n \rightarrow \infty} \left| \frac{(-x)^n}{2e^n + 3} \cdot \frac{2e^n + 3}{ex^n + 1} \right|$$

$$-1 < -\frac{x}{e} < 1$$

$$e > x > -e$$

Response for question 6(d)

$$g(1) \approx \frac{1}{5} + \frac{1}{2e+3} + \frac{1}{2e^2+3} + \dots + \frac{(-1)^n}{2e^n+3} + \dots$$

$$\left| S_\infty - \left(\frac{1}{5} - \frac{1}{2e+3} \right) \right| < \left| \frac{1}{2e^2+3} \right|$$

$$\text{error} < \frac{1}{2e^2+3}$$

6

6

6

6

NO CALCULATOR ALLOWED

6

6

6

6

Answer QUESTION 6 parts (a) and (b) on this page.

Response for question 6(a)

Conditions are positive, continuous, and decreasing.

$$\int_0^{\infty} e^{-x} dx = -e^{-x} \Big|_0^{\infty} = \frac{1}{e^x} \Big|_0^{\infty} = [0 - \frac{1}{e^0}] = \frac{1}{1} = 1$$

Since the integral went to a finite value in 1,
the series $\sum_{n=0}^{\infty} \frac{1}{e^n}$ converges. (But the integral test does not
say where it converges to though.)

Response for question 6(b)

$$\lim_{n \rightarrow \infty} \frac{1}{e^n} \cancel{e^{2n+3}} \quad \lim_{n \rightarrow \infty} \frac{2e^{n+3}}{(e^n)^{2n+3}} \cancel{e^{2n+3}} \quad \lim_{n \rightarrow \infty} \frac{2e^{n+3}}{e^{2n+3}} \cancel{e^{2n+3}}$$

$$\lim_{n \rightarrow \infty} \frac{e^{12n}}{2e^{n+3}} : \lim_{n \rightarrow \infty} \frac{(e^n)^{e^n}}{2e^{n+3}} \rightarrow \lim_{n \rightarrow \infty} \left(\frac{e^n}{2e^{n+3}} \right) = \frac{1}{2}$$

The limit test converges to less than 1, so it converges there absolutely
and thus there is no need to check conditional convergence.



• 6 6 6 6 NO CALCULATOR ALLOWED 6 6 6 6

Answer QUESTION 6 parts (c) and (d) on this page.

Response for question 6(c)

$$\lim_{n \rightarrow \infty} \left| \frac{(-1)^{n+1} x^{n+1}}{2e^{n+1} + 3} \cdot \frac{2e^{n+3}}{(-1)^n x^n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(-1)x + 3}{2e + 3} \right| \rightarrow \frac{-x + 3}{2e + 3} < 1$$

$$-x + 3 < 2e + 3$$

$$-x < 2e$$

$$x > -2e$$

$3 < x < 2e + 3$

Response for question 6(d)

Need 3rd term

$$\frac{-1}{2e^3 + 3}$$

Upper bound on error of the approximation can only be as big as the next term, thus the maximum error is $-\frac{1}{2e^3 + 3}$

6 6 6 6 NO CALCULATOR ALLOWED 6 6 6 6

Answer QUESTION 6 parts (a) and (b) on this page.

Response for question 6(a)

$$\textcircled{1} \lim_{n \rightarrow \infty} \frac{1}{e^n} = 0 \checkmark$$

$$0 \int_0^{\infty} \frac{1}{e^n} dn = \int_0^{\infty} e^{-n} dn = \left[-\frac{e^{-n}}{n} \right]_0^{\infty} = \frac{1}{e^{\infty}} \times 0 - 0 = 0$$

Because the integral of $\sum_{n=0}^{\infty} \frac{1}{e^n}$ is not ∞ , it converges.

Response for question 6(b)

Since $\sum_{n=0}^{\infty} \frac{1}{e^n}$ converges, $\left| \frac{(-1)^n}{e^n} \right|$ also converges and it converges absolutely.

Since $\left| \frac{(-1)^n}{2e^n + 3} \right| < \left| \frac{(-1)^n}{e^n} \right|$, $\sum_{n=0}^{\infty} \frac{(-1)^n}{2e^n + 3}$ converges absolutely.



• 6 6 6 6 NO CALCULATOR ALLOWED 6 6 6 6

Answer QUESTION 6 parts (c) and (d) on this page.

Response for question 6(c)

$$\left| \frac{(-1)^{n+1} x^{n+1}}{2e^{n+1} + 3} \right| = \left| \frac{x}{\frac{2e^n + 3}{(-1)^n x^n}} \right| < 1$$

$$|x| < 2e$$

$$[-2e < x < 2e]$$

Response for question 6(d)

According to the alternating series error bound, $|g(1)| < \text{next term}$

$$|g(1)| < \frac{1}{2e^3 + 3}$$

Question 6**Sample Identifier: A****Score: 9**

- The response earned 9 points: 3 points in part (a), 2 points in part (b), 3 points in part (c), and 1 point in part (d).
- In part (a) the response earned the first point in the first line by stating that “the function $\frac{1}{e^n}$ is continuous, positive, and decreasing.” The response earned the second point in the fourth line by presenting the limit $\lim_{b \rightarrow \infty} \int_0^b \frac{1}{e^n} dn$. The response earned the third point by presenting a correct antiderivative on the fifth line, correctly evaluating the limit on the sixth line, and stating a correct conclusion on the third line.
- In part (b) the response earned the first point in the second line by presenting the limit (with or without absolute values) $\lim_{n \rightarrow \infty} \frac{\frac{1}{2e^n + 3}}{\frac{1}{e^n}}$. The response earned the second point by correctly evaluating the limit resulting in $\frac{1}{2}$, referencing that $\frac{1}{2} > 0$ and presenting a correct conclusion in the lower right.
- In part (c) the response earned the first point in the second line by presenting the ratio (with or without a limit) $\left| \frac{x^{n+1}}{2e^{n+1} + 3} \cdot \frac{2e^n + 3}{x^n} \right|$. In the absence of the factors $(-1)^{n+1}$ and $(-1)^n$, the response must have absolute values to earn this point. The response earned the second point on the second line by correctly evaluating the limit. The response earned the third point on the fifth line by correctly identifying the radius of convergence.
- In part (d) the response earned the point by providing the correct answer.

Question 6 (continued)**Sample Identifier: B****Score: 7**

- The response earned 7 points: 2 points in part (a), 2 points in part (b), 3 points in part (c), and no points in part (d).
- In part (a) the response did not earn the first point since the response references a sequence and not the function e^{-x} . The response earned the second point on the second line by presenting the improper integral $\int_0^{\infty} \frac{1}{e^n} dn$. The response earned the third point by presenting a correct antiderivative in the fourth line, correctly evaluating the limit, and stating a correct conclusion on the last line.
- In part (b) the response earned the first point in the third line by presenting the limit (with or without absolute values) $\lim_{n \rightarrow \infty} \frac{1}{\frac{2e^n + 3}{e^n}}$. The response earned the second point on the third and fifth lines by correctly evaluating the limit, referencing that $\frac{1}{2} > 0$, and concluding that the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{2e^n + 3}$ converges absolutely.
- In part (c) the response earned the first point in the first line by presenting the ratio (with or without a limit or absolute values) $\frac{\frac{(-1)^{n+1} x^{n+1}}{2e^{n+1} + 3}}{\frac{(-1)^n x^n}{2e^n + 3}}$. The response earned the second point by correctly evaluating the limit in the fourth line. The response earned the third point in the last line by correctly identifying the radius of convergence.
- In part (d) the response did not earn the point since the response presents an incorrect answer.

Question 6 (continued)**Sample Identifier: C****Score: 6**

- The response earned 6 points: 1 point in part (a), 1 point in part (b), 3 points in part (c), and 1 point in part (d).
- In part (a) the response did not earn the first point since the response does not present that the function $\frac{1}{e^x}$ is continuous, decreasing and positive. The response did not earn the second point since the response presents a lower limit in the first line of 1 whereas the correct lower limit is 0. The response earned the third point in the second and third lines by correctly evaluating the limit for their limits of integration and stating a correct conclusion.
- In part (b) the response earned the first point in the first line by presenting the limit (with or without absolute values) $\lim_{n \rightarrow \infty} \left| \frac{(-1)^n}{2e^n + 3} \cdot \frac{e^n}{1} \right|$. The response did not earn the second point since the response does not reference $\frac{1}{2} > 0$.
- In part (c) the response earned the first point in the first line by presenting the ratio (with or without a limit or absolute values) $\frac{(-1)^{n+1} x^{n+1}}{2e^{n+1} + 3} \cdot \frac{2e^n + 3}{(-1)^n x^n}$. The response earned the second point on the second line with a correct evaluation of the limit. The response earned the third point on the third line by correctly identifying the radius of convergence.
- In part (d) the response earned the point for correctly identifying the answer.

Question 6 (continued)**Sample Identifier: D****Score: 6**

- The response earned 6 points: 2 points in part (a), 1 point in part (b), 2 points in part (c), and 1 point in part (d).
- In part (a) the response did not earn the first point since the response does not identify the three needed conditions for the function $\frac{1}{e^x}$. The response earned the second point in the second line by presenting the improper integral (with or without a differential) $\int_0^\infty \frac{1}{e^x}$. The response earned the third point by presenting a correct antiderivative, correctly evaluating the limit and stating a correct conclusion.
- In part (b) the response earned the first point in the second line by presenting the limit (with or without absolute values) $\lim_{n \rightarrow \infty} \left| \frac{e^n}{2e^n + 3} \right|$. The response did not earn the second point since the response compares $\frac{1}{2}$ with 1 and not 0.
- In part (c) the response earned the first point in the first line by presenting the ratio (with or without a limit or absolute values) $\frac{(-1)^{n+1} x^{n+1}}{(-1)^n x^n} \cdot \frac{2e^n + 3}{2e^{n+1} + 3}$. The response did not earn the second point since the response, by bounding $\frac{-x}{2e}$ between both -1 and 1, implies that their limit is $\frac{-x}{2e}$ which is incorrect. The response is eligible for the third point since the response presents a value for the limit and considers an absolute value by presenting $-1 < \frac{-x}{2e} < 1$. The response earned the third point by presenting the radius of convergence $2e$ which is consistent with their limit.
- In part (d) the response earned the point by providing the correct answer.

Question 6 (continued)**Sample Identifier: E****Score: 5**

- The response earned 5 points: 1 point in part (a), 1 point in part (b), 2 points in part (c), and 1 point in part (d).
- In part (a) the response did not earn the first point since the response does not present that the function e^{-x} is continuous, decreasing and positive. The response earned the second point in the fourth line by presenting the limit $\lim_{b \rightarrow \infty} \int_0^b \frac{1}{e^x} dx$. The response did not earn the third point since the response presents $\frac{1}{e^\infty}$ in the last line. In addition, the response does not give a conclusion.
- In part (b) the response earned the first point in the first line by presenting the limit (with or without absolute values) $\lim_{n \rightarrow \infty} \left| \frac{\frac{1}{e^n}}{\frac{1}{2e^n + 3}} \right|$. The response did not earn the second point since the response does not reference that $2 > 0$.
- In part (c) the response earned the first point in the first line by presenting the ratio (with or without a limit or absolute values) $\lim_{n \rightarrow \infty} \left| \frac{\frac{(-1)^{n+1} x^{n+1}}{2e^{n+1} + 3}}{\frac{(-1)^n x^n}{2e^n + 3}} \right|$. The response did not earn the second point since the response incorrectly evaluates the limit. The response earned the third point in the last line for identifying the radius of convergence consistent with their limit.
- In part (d) the response earned the point in the last line by presenting the correct error bound of $\frac{1}{2e^2 + 3}$.

Question 6 (continued)**Sample Identifier: F****Score: 5**

- The response earned 5 points: 1 point in part (a), 1 point in part (b), 3 points in part (c), and no points in part (d).
- In part (a) the response did not earn the first point since the response references “the series” and not the function e^{-x} . The response earned the second point in the third line by presenting the improper integral $\int_0^{\infty} \frac{1}{e^n} dn$. The response did not earn the third point since the response presents an incorrect antiderivative.
- In part (b) the response earned the first point in the first line by presenting the limit (with or without absolute values) $\lim_{n \rightarrow \infty} \frac{(-1)^n}{\frac{1}{e^n}} \frac{2e^n + 3}{e^n}$. The response did not earn the second point since the response does not present any absolute values.
- In part (c) the response earned the first point in the first line by presenting the ratio (with or without a limit or absolute values) $\frac{(-1)^{n+1} x^{n+1}}{2e^{n+1} + 3} \cdot \frac{2e^n + 3}{(-1)^n x^n}$. The response earned the second point in the second line by correctly evaluating the limit. The response earned the third point by correctly identifying the radius of convergence.
- In part (d) the response did not earn the point since the response presents the “third term” in the second line but identifies the error as a polynomial in the third line.

Question 6 (continued)**Sample Identifier: G****Score: 5**

- The response earned 5 points: 1 points in part (a), 1 point in part (b), 2 points in part (c), and 1 point in part (d).
- In part (a) the response did not earn the first point since the response does not state that $\frac{1}{e^n}$ is positive. The response earned the second point in the second line by presenting the improper integral (with or without a differential) $\int_0^\infty \frac{1}{e^n}$. The response did not earn the third point since the response presents $\frac{1}{-e^\infty}$ in the fifth line.
- In part (b) the response earned the first point in the second line by presenting the limit (with or without absolute values) $\lim_{n \rightarrow \infty} \left| \frac{a_n}{b_n} \right|$ in the presence of the definitions of a_n and b_n . The response did not earn the second point since in the third line the response references the n th term of the series and not the series itself.
- In part (c) the response earned the first point in the first line by presenting a correct ratio (with or without a limit or absolute values) $\frac{(-1)^{n+1} x^{n+1}}{2e^{n+1} + 3} \cdot \frac{2e^n + 3}{(-1)^n x^n}$. The response did not earn the second point since the response presents an incorrect limit. The response earned the third point in the last line for identifying the radius of convergence consistent with their limit evaluation.
- In part (d) the response earned the point in the first line by presenting the correct error bound of $\frac{1}{2e^2 + 3}$.

Question 6 (continued)**Sample Identifier: H****Score: 4**

- The response earned 4 points: 1 point in part (a), no points in part (b), 2 points in part (c), and 1 point in part (d).
- In part (a) the response did not earn the first point since the response does not state all three conditions for the function $\frac{1}{e^x}$. The response earned the second point on the second line by presenting the improper integral $\int_0^\infty \frac{1}{e^n} dn$. The response did not earn the third point since the response states “convergent” with no reference to an improper integral or a series.
- In part (b) the response did not earn either point since the response does not apply the Limit Comparison Test.
- In part (c) the response earned the first point in the first line by presenting the ratio (with or without a limit or absolute values) $\frac{(-1)^{n+1} x^{n+1}}{2e^{n+1} + 3} \cdot \frac{2e^n + 3}{(-1)^n x^n}$. The response earned the second point on the second line by presenting $-1 < \frac{-x}{e} < 1$. Since $\frac{-x}{e}$ is bounded between both -1 and 1, the response indicates that their limit is $\left| \frac{-x}{e} \right|$. The response did not earn the third point since the response does not identify a radius of convergence.
- In part (d) the response earned the point by providing the correct answer.

Sample Identifier: I**Score: 3**

- The response earned 3 points: 1 point in part (a), 1 point in part (b), 1 point in part (c), and no points in part (d).
- In part (a) the response did not earn the first point since the response states the correct conditions but does not reference our function $\frac{1}{e^x}$. The response earned the second point in the second line by presenting the improper integral $\int_0^\infty e^{-n} dn$. The response did not earn the third point since on the second line the response applies the Fundamental Theorem of Calculus to an improper integral.
- In part (b) the response earned the first point on the first line by presenting the limit (with or without absolute values) $\lim_{n \rightarrow \infty} \frac{(-1)^n}{\frac{1}{e^n}} \cdot \frac{2e^n + 3}{2e^{n+1} + 3}$. The response did not earn the second point since in the second line the response references a limit “less than 1” whereas the correct application of the Limit Comparison Test would require referencing a limit greater than 0.
- In part (c) the response earned the first point on the first line by presenting the ratio (with or without a limit or absolute values) $\frac{(-1)^{n+1} x^{n+1}}{2e^{n+1} + 3} \cdot \frac{2e^n + 3}{(-1)^n x^n}$. The response did not earn the second point since the evaluation of the limit is incorrect. The response did not earn the third point since the response does not identify a radius of convergence consistent with their limit evaluation.
- In part (d) the response did not earn the point as the response presents the incorrect answer.

Question 6 (continued)**Sample Identifier: J****Score: 2**

- The response earned 2 points: 1 point in part (a), no points in part (b), 1 point in part (c), and no points in part (d).
- In part (a) the response did not earn the first point since the response did not present that the function $\frac{1}{e^x}$ is continuous, decreasing and positive. The response earned the second point in the second line by presenting the improper integral $\int_0^{\infty} \frac{1}{e^n} dn$. The response did not earn the third point since the response applies the Fundamental Theorem of Calculus to an improper integral. In addition, the response will not earn the third point in the presence of the term $\frac{1}{e^{\infty}}$.
- In part (b) the response did not earn either point since the response does not apply the Limit Comparison Test. The response does correctly apply the Comparison Test, but the question explicitly requires the Limit Comparison Test.
- In part (c) the response earned the first point on the first line by presenting the correct ratio (with or without a limit or absolute values) $\left| \frac{(-1)^{n+1} x^{n+1}}{2e^{n+1} + 3} \right| \times \left| \frac{2e^n + 3}{(-1)^n x^n} \right|$. The response did not earn the second point since there is no limit notation. The response is not eligible for the third point since the response does not evaluate a limit.
- In part (d) the response did not earn the point since the response presents the incorrect answer.