2020 AP Calculus BC Exam

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In 2020 before spring had even sprung, schools across the country were transitioning to online instruction in response to the COVID virus. The College Board and ETS decided that administration of the standard multiple choice section and six free response questions for the AP Calculus Exam was not going to be feasible, face-to-face. In addition, the reading could not be held safely face-to-face. Plans were made and many Test Development Committee (TDC) members and others at the College Board crafted and proofed and re-proofed a two question exam and put into place the methods and procedures by which the exam would be both administered and scored online.

This article will discuss procedures and issues related to the scoring of the 2020 AP Calculus Exam. Since no questions and scoring guidelines have been officially released, this article will refer to "mock" questions. These questions and "mock" scoring guidelines were prepared by Tom Dick of Oregon State, former TDC member and long time reader and leader and Stephen Kokoska of Bloomsburg University, former chief reader, continuing reader and leader. Two of these questions were referenced during a TI in Focus webinar broadcast, "Calculus Resources: Results, Technology and Content" September 15, 2020. More information regarding this webinar may be found at https://education.ti.com/en/professional-development/teachers-and-teams/online-learning/on-demand-webinars

Procedures:

Students had explicit instructions regarding when the exam questions were available online and how to upload their responses. All students, worldwide, had to take the exam at the same time, regardless of their time zones. There was a strict timeline during which the problems would be available online. In addition, students were warned that the last five minutes of this time should be used for uploading their responses. After time expired, questions were no longer available and responses could not be uploaded. However, with all students working simultaneously, opportunities for collaboration existed. To help thwart that, there were several forms of questions and variations within forms. For example, a given graph might be slightly varied having different zeros. Related functions were named using different letters for those functions. In a tightly restricted time period this made it more difficult for simple copying and fast collaboration.

Question leaders and "Early Table Leaders" met online for six days before the actual scoring, finalizing the scoring guidelines and preparing responses with scoring commentaries that were used in training readers when they started the virtual Reading. After training, the uploaded student responses were available for readers to view and score as part of a robust software platform. This allowed readers to submit scores and ask questions of table leaders. It also allowed table leaders to backread previously scored exams and make comments to readers. Furthermore, table leaders, question leaders and exam leaders had a platform through which to deal with responses that were either difficult to score or bordering on illegible. On a personal note, I for one was impressed with the solid software online that made fair and consistent scoring possible.

The two questions:

One question was introduced with information including a graph of a continuous function or the derivative of a function. The other question was introduced with information contained in a table of values. Additional given information involved such things as a function defined as an integral of a previously introduced function. A few topics were not considered on this exam in deference to the difficulties and time restrictions teachers and students faced in completing the course appropriately in the online environment imposed quickly upon schools nationwide. These were a few of the later topics outlined in the Course and Exam Description. For example, on the AB test there were no questions regarding volumes, although this did appear on the BC test.

The quick change to a two question exam gave rise to a scoring system different than the traditional 0 to 9 points for a free response question. For example, the one question (and its variations) that I worked on during most of the reading was worth 16 points. This was a question #1 for AB students. Those students also had a question #2 worth 12 points. The BC test was scored 18 points for one question and 11 points for the second question.

The first question referenced below involved functions, their derivatives and questions about a function maximum and/or minimum, points of inflection, the Mean Value Theorem and/or the Intermediate Value Theorem, the fundamental theorem, and other basic AP Calculus concepts concerning function behaviors, with volume perhaps coming into play on the BC exam. The second question relying on values in a table was given in a context such as motion with time in seconds and distance in meters. This question allowed the context of the problem to enter into student solutions and explanations.

The "mock" questions and scoring guidelines appear at the end of this article. They will be referenced in the discussion of the scoring below. Note that these are worth more points than were on the actual exam. This is in an attempt to address a wide variety of topics, perhaps a bit more than students saw, but of the same type. A variety of questions related to the given information and graphs and tables for both the AB and BC exams show a strong attempt at offering a thorough and comprehensive AP Calculus Exam.

Question #1:

THE BC "MOCK" QUESTION #1:

(a) Critical points occur where $g'=0$ or does not exist. Because of the graph and more precisely because g is twice differentiable which implies that g' is continuous, we have that g' does exist. It can be seen that $g'=0$ where $x=9$, and this statement earns the first point. The second point is earned by declaring a relative max for g at this point because g' changes from positive to negative there.

(b) The function g has points of inflection where g' changes from increasing to decreasing or vice versa. Such a change takes place only where $x=4.2$. (Another way to see this is that the slopes of g', which give g'', change sign at this point.)

(c) This area is given by the definite integral of g' from 0 to 9. An antiderivative using the Fundamental Theorem of Calculus is g. The integral $\int_0^9 g'(x)dx$ must be shown to earn the first point. The antiderivative earns the second point and the correct evaluation of $g(9)-g(0)$ earns the third, answer point.

(d) The perimeter of this region includes the arc length of the curve from 0 to 9 and the lengths of the segments from 0 to 1 on the y-axis and 0 to 9 on the x-axis. The first point is awarded for a correct integral that calculates the arc length given by $\int_0^9 \sqrt{1+\left(\frac{d}{dx}(g')\right)^2}\,dx = \int_0^9 \sqrt{1+(g'')^2}\,dx$. The answer point is awarded for adding this to the lengths of the two segments.

(e) This asks for a value $g(c)=0$ between $g(0)$ and $g(9)$. If the function g is continuous, the IVT can be applied. For the first point, a student should state that g is continuous because g is differentiable. The second point is awarded if an interval shows 0 between the values of $g(0)=-7$ and $g(9)=12$.

(f) The first point is for using properties of integrals: the integral of a sum is the sum of the integrals and the contant passes freely to both terms. The second point is for a correct antiderivative of $\sqrt{x}$. The third point is for a correct calculation of the final answer.

(g) Since the numerator and denominator in the expression $\frac{x\cos(x)}{g(x)+2x+7}$ are both continuous, it is a simple matter to state that the limits of both are 0. Recognizing the "form" $\frac{0}{0}$ earns the first point. What can lose this point is explicitly stating that $\lim_{x\to 0}\frac{x\cos(x)}{g(x)+2x+7}=\frac{0}{0}$ with an equal sign. Calculating derivatives of both numerator and denominator earns the second point. Evaluating the limit of the resulting expression earns the answer point.

(h) The work shown in the scoring guidelines earns three points. A perhaps unusual teaching technique which I sometimes use is the following: $h'(x)=\frac{d}{dx}\int_{x^2}^{0} g(x)dx=\frac{d}{dx}\left(\text{Anti}\left(g(0)\right)\right)-\frac{d}{dx}\left(\text{Anti}\left(g\left(x^2\right)\right)\right)=0-g\left(x^2\right)2x$.

This would earn the first two points because the FTC has been applied and the $2x$ indicates application of the chain rule. A correct computation of this numerical value (using the given $g(9)=12$) earns the third point.

(i) The area $A(x)$ is that of a triangular cross section, $\frac{1}{2}(\text{base}\times\text{height})$. The height is given as x and the base of each triangle is the distance from the x-axis to the graph, which is g'. Thus $A(x)=\frac{1}{2}xg'(x)$ for one point.

(j) The volume is found by integrating the cross section areas from 0 to 9. This requires integration by parts. For the first point, the reader would be looking for $xg(x)-\int g(x)dx$. A correct evaluation of the expression from the IBP work earns the second point.

(k) The integrand $\frac{g''(x)}{g'(x)}$ is not defined at one point in the interval given because $g'(9)=0$. Thus one point is awarded for recognizing this as an improper integral and for using the appropriate limit notation. The second point is for the antiderivative $\ln\left|g'(x)\right|$. (Note: students should be trained to always show absolute value in the case of $\int \frac{du}{u} = \ln|u| + C$.) The evaluation needs to show limit notation and the result $-\infty$ or that the limit does not exist.

(l) The first point is earned for the second order Taylor polynomial, showing correct form of the coefficients and corresponding powers of *x*, using 2! appropriately. Using correct values of the derivatives of *g* in this expression, and adding the terms, earns the second point. If "$\cdots$" is added, the second point is not earned. Errors in simplification come off the second point. Only polynomials centered at 0 are eligible for any points.

Question #2:

THE BC "MOCK" QUESTION #2:

(a) Speed is the magnitude of the velocity. At $t = 6$ the velocity vector is $\left\langle [x'(6)]^2,\ [y'(6)]^2 \right\rangle$. This vector does not need to be shown, but its magnitude $\sqrt{[x'(6)]^2 + [y'(6)]^2}$ should be shown in order to earn the first point. Substituting correct values into this expression for both $x'(6)$ and $y'(6)$ earns the second point.

(b) Using the integral $\int_4^b x'(t)dt$ where $b = 12$ earns the first point. The choice of 12 for *b* is needed because the only *x*-coordinate position in the given information is at time $t = 12$: $x(12) = 4$. In order to find $x(4)$, one can consider the fact that $\int_4^{12} x'(t)dt = x(12) - x(4)$ or that the position $x(12) = x(4) + \int_4^{12} x'(t)dt$ (which is the initial position plus the net change in position over the interval $4 \le t \le 12$). Using either equation, it can be seen that $x(4) = x(12) - \int_4^{12} x'(t)dt$. To earn the answer point, the value of the integral must be computed and appropriately combined with $x(12) = 4$.

(c) The first point is earned for presenting the correct first step in Euler's method with at most one error. Tabular work can be shown, but an unlabeled table must be accompanied by the correct answer. If there is at most one error, the student earns the first point but is not eligible for the second point.

(d) One point is for calculation of the slope of this tangent line. Since information about this curve is provided separately for each of the coordinates of a point on the curve, the form $\frac{y'(12)}{x'(12)}$ should be used in student work. A correct form of a tangent line equation is needed in order to earn the second point.

(e) This question asks for the rate of change in distance, $r(t)$, between a particle on the curve and the Origin at the time $t = 12$. The distance between (0, 0) and $\left(x(t), y(t)\right)$ is given by $r(t) = \sqrt{x(t)^2 + y(t)^2}$. This earns the first point. The rate of change in distance is given by $r'(t) = \frac{2x(t)x'(t) + 2y(t)y'(t)}{2\sqrt{x(t)^2 + y(t)^2}}$ and earns the second point. Evaluating this last expression earns the answer point.

(f) This requires an appeal to the sign of the result for $r'(12)$. The correct answer is that the particle is moving further away from the Origin at time $t = 12$. An incorrect answer could not be justified in order to earn this point.

(g) A correct term in this polynomial is $\frac{y^{(p)}(12)(t-12)^p}{p!}$. Presenting two of these terms earns the first point. Adding all four correct terms including the values of $y^{(p)}(12)$ earns the second point. If terms are not centered at 12, no points are earned.

(h) Total distance traveled for $12 \le t \le 15$ is given by $\int_{12}^{15}(speed)\,dt = \int_{12}^{15}\sqrt{x'(t)+y'(t)}\,dt$. Given in the problem is an expression for $x'(t)$. It is stated in part h that $y(t)$ over this interval is given by the Taylor polynomial which was computed in part g. Thus $y'(t)$ is the derivative of the result from part g. A correct computation of $y'(t)$ earns the first point. Including this in the definite integral for speed over $12 \le t \le 15$ earns the second point.

More detailed interpretations of these scoring guidelines as well as short content presentations associated with each problem part are given in the TI in Focus videos mentioned above, which can be found at education.ti.com.