2020 AP Calculus AB Exam

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In 2020 before spring had even sprung, schools across the country were transitioning to online instruction in response to the COVID virus. The College Board and ETS decided that administration of the standard multiple choice section and six free response questions for the AP Calculus Exam was not going to be feasible, face-to-face. In addition, the reading could not be held safely face-to-face. Plans were made and many Test Development Committee (TDC) members and others at the College Board crafted and proofed and re-proofed a two question exam and put into place the methods and procedures by which the exam would be both administered and scored online.

This article will discuss procedures and issues related to the scoring of the 2020 AP Calculus Exam. Since no questions and scoring guidelines have been officially released, this article will refer to "mock" questions. These questions and "mock" scoring guidelines were prepared by Tom Dick of Oregon State, former TDC member and long time reader and leader and Stephen Kokoska of Bloomsburg University, former chief reader, continuing reader and leader. Two of these questions were referenced during a TI in Focus webinar broadcast, "Calculus Resources: Results, Technology and Content" September 15, 2020. More information regarding this webinar may be found at https://education.ti.com/en/professional-development/teachers-and-teams/online-learning/on-demand-webinars

Procedures:

Students had explicit instructions regarding when the exam questions were available online and how to upload their responses. All students, worldwide, had to take the exam at the same time, regardless of their time zones. There was a strict timeline during which the problems would be available online. In addition, students were warned that the last five minutes of this time should be used for uploading their responses. After time expired, questions were no longer available and responses could not be uploaded. However, with all students working simultaneously, opportunities for collaboration existed. To help thwart that, there were several forms of questions and variations within forms. For example, a given graph might be slightly varied having different zeros. Related functions were named using different letters for those functions. In a tightly restricted time period this made it more difficult for simple copying and fast collaboration.

Question leaders and "Early Table Leaders" met online for six days before the actual scoring, finalizing the scoring guidelines and preparing responses with scoring commentaries that were used in training readers when they started the virtual Reading. After training, the uploaded student responses were available for readers to view and score as part of a robust software platform. This allowed readers to submit scores and ask questions of table leaders. It also allowed table leaders to backread previously scored exams and make comments to readers. Furthermore, table leaders, question leaders and exam leaders had a platform through which to deal with responses that were either difficult to score or bordering on illegible. On a personal note, I for one was impressed with the solid software online that made fair and consistent scoring possible.

The two questions:
One question was introduced with information including a graph of a continuous function or the derivative of a function. The other question was introduced with information contained in a table of values. Additional given information involved such things as a function defined as an integral of a previously introduced function. A few topics were not considered on this exam in deference to the difficulties and time restrictions teachers and students faced in completing the course appropriately in the online environment imposed quickly upon schools nationwide. These were a few of the later topics outlined in the Course and Exam Description. For example, on the AB test there were no questions regarding volumes, although this did appear on the BC test.

The quick change to a two question exam gave rise to a scoring system different than the traditional 0 to 9 points for a free response question. For example, the one question (and its variations) that I worked on during most of the reading was worth 16 points. This was a question #1 for AB students. Those students also had a question #2 worth 12 points. The BC test was scored 18 points for one question and 11 points for the second question.

The first question referenced below involved functions, their derivatives and questions about a function maximum and/or minimum, points of inflection, the Mean Value Theorem and/or the Intermediate Value Theorem, the fundamental theorem, and other basic AP Calculus concepts concerning function behaviors, with volume perhaps coming into play on the BC exam. The second question relying on values in a table was given in a context such as motion with time in seconds and distance in meters. This question allowed the context of the problem to enter into student solutions and explanations.

The "mock" questions and scoring guidelines appear at the end of this article. They will be referenced in the discussion of the scoring below. Note that these are worth more points than were on the actual exam. This is in an attempt to address a wide variety of topics, perhaps a bit more than students saw, but of the same type. A variety of questions related to the given information and graphs and tables for both the AB and BC exams show a strong attempt at offering a thorough and comprehensive AP Calculus Exam.

Question #1:
THE AB "MOCK" QUESTION #1:
(a) Critical points occur where the derivative is 0 or undefined. Since $f = g'$ is given as continuous, all that needs to be shown to earn this one point are the values of x where $f = 0$.
(b) To earn these points, a response needs to refer appropriately to the change in signs of g'. If the reference is to the function f then there needs to be the explicit link $f = g'$. If this link was not established, students were still eligible for 2 of the 3 points. Some students referred to signs over intervals. These intervals should be correct. For example, $f < 0$ for $0 < x < 2$ and $f > 0$ for $2 < x < 5$ could be used to justify a relative min at $x = 2$. But stating that $f < 0$ for $x < 2$ and $f > 0$ for $x > 2$ is not correct. This last statement would be interpreted as a global and not local argument for this *relative* minimum.
(c) Both correct intervals need to be stated. The scoring guideline provides a very straightforward way to provide reasoning. Many students connect concavity with the sign of the second derivative. In employing this reasoning, as much information as possible should be given in the student response. For example, a reference to

f' is only acceptable if the link $f = g'$ has been established. Any reference to signs of f' or g'' on intervals would require correct intervals and certainly not references to a sign at a point.
(d) & (e) Both values related to areas below the x-axis. In order to show the computation, an integral has to be shown. Part e is worth two points because the area involved is that of a rectangle less a quarter circle.
(f) To find absolute extrema on a closed interval, examine the values of the function at the endpoints and critical points. Arguments referring to changes in sign are very difficult to use correctly and sufficiently. Note that this question asks for the absolute maximum value of the function. Thus the value of x at that location does not answer the question. Calculate both endpoint values and any critical point value(s) and then state which function value is greatest.
(g) g'' is found from the slopes of f. This is only worth one point. Showing a correct calculation of the slope at the point where $x = 6$ is sufficient in order to earn this point.
(h) Students should be aware that the average rate of change of a function over an interval is part of the conclusion of the Mean Value Theorem. To invoke this theorem (which does not have to be named, but if so the name must be correct) both continuity and differentiability need to be referenced. If intervals are specified, they must be correct in accordance with the hypotheses of the MVT. The verbal description in the scoring guideline below is certainly correct. Also correct would be something like "such a d exists because $g'(d) = \frac{g(2) - g(0)}{2 - 0}$ where $0 < d < 2$."

(i) Since the numerator and denominator in the expression $\frac{3x + g(x)}{\sin(x)}$ are both continuous, it is a simple matter to state that the limits of both are 0. Recognizing the "form" $\frac{0}{0}$ earns the first point. What can lose this point is explicitly using an equal sign as in $\lim_{x \to 0} \frac{3x + g(x)}{\sin(x)} = \frac{0}{0}$. Calculating derivatives of both numerator and denominator earns the second point. Evaluating the limit of the resulting expression earns the answer point.
(j) Computation of h' requires use of both a product rule and the chain rule. Each is worth 1 point in scoring the computation. For example, "$h'(x) = 1 \bullet g\left(x^2\right) + g'\left(x^2\right) \bullet 2x$ would earn a point for the chain rule, but no point for a correct product rule. Evaluating this expression correctly at $x = \sqrt{2}$ earns the third point for this part.

Question #2:

THE AB "MOCK" QUESTION #2:
(a) A local linear approximation requires use of a tangent line near $x = 11.8$. This would be at $x = 12$ where the slope of the line is given by $y'(12) = -5$. Using an equation of this line earns the first point. Using $x = 11.8$ to correctly calculate the approximation earns the second point.
(b) Correctly evaluating the expression $\frac{y'(6) - y'(2)}{6 - 2}$ earns this point.
(c) $y''(4)$ requires an explanation in the context of meters for distance and seconds for time *at* the time $t = 4$. This does not relate to the calculation in part b, and does not occur over an interval, but at a specific time. Since this is the second derivative of the position function, it is acceleration. Units must state meters per second per second to describe acceleration in this context.

(d) The average value of acceleration over [0, 12] is given by $\frac{1}{12-0}\int_0^{12} a(t)dt = \frac{1}{12-0}\int_0^{12} y''(t)dt$. Showing this earns the first point. The antiderivative earns the second point. The answer point is earned by a correct evaluation.

(e) Calculating this Riemann sum requires use of three midpoint values of y', multiplying those by the width of the appropriate intervals, and adding them. This involves the sum of three products. For the last several years on the AP Calculus Exam, the point for this is awarded if all or all but one of the interval and function values are correct. The second point is only awarded for the correct answer, simplified or not.

(f) Since y' is velocity and this definite integral does not involve absolute value, the numerical result is the displacement of the particle over the given time interval. Units in this question are awarded a second point.

(g) The search is for three times where $y' = 0$. This is an intermediate value between times given in the table where the sign of y' changes. Conditions for applying the IVT exist because of the differentiability of y' which implies its continuity. This earns the first point. Further specification of times (in the scoring guideline referred to as *a*, *b* and *c*) and their locations earns the second point.

(h) Note that $y'' = (y')'$ so that the MVT can be applied to y'. Using the three zeros of y' established in part *g*, the MVT can be applied twice to guarantee that y'' is 0 between (taken 2 at a time) the zeros found in part *g* for y'. An alternate solution can be used because of the conditions already mentioned and that y' is 0 at the endpoints of two intervals. Thus Rolle's Theorem can be applied to guarantee that the derivative of y' is 0 somewhere on each of the two intervals.

More detailed interpretations of these scoring guidelines as well as short content presentations associated with each problem part are given in the TI in Focus videos mentioned above, which can be found at education.ti.com.