CSI: Calculus/Statistics Insider

Official Newsletter of the Georgia Association of Advanced Placement Math Teachers

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This publication includes:

Note from the President

First, as a Georgia educator, I would like to say that I am proud of you. We have been faced with the extremely difficult task of redefining how we teach our students as school systems began to close for students in response to COVID-19. I'm sure that you were reassuring to your students as they, and you, realized that the AP exams were coming up. You hadn't covered all of the materials that would be on the test, you weren't exactly sure how you would prepare your students virtually, and you weren't even sure how the AP exams would carry on if schools remained closed past the original few weeks and into May. We are, as has been stated many times, in uncharted territory. It quickly became evident that you were up to the task. Within 24 to 48 hours, you had begun to record short videos, explored the use of video conferencing applications, found ways for students to submit work and have discussions, and kept in touch with your students' progress. Teachers already wear so many hats, as it is, that you adapted quickly and appropriately to the challenge. To their credit, our students have shown that they are up to the challenge, as well.

Perhaps one day in the future, this experience will provide a rich backdrop for discussion in our mathematics classrooms. I can envision a teacher saying, "Hey, you remember when school was closed to help families self-isolate to slow the spread of COVID-19? Well today we are going to discuss exponential growth." Perhaps the teacher will have a simulation where one student is secretly "infected" with the virus. The "infected" student will interact with a number of other students who will then roll a die, or use a random number generator, to see if they were infected. The teacher may also randomize what impact the virus has on the newly infected. The mild cases will continue to interact with other students and the process can continue for many iterations. Maybe they use actual data from the spread of the virus to discuss regression models. We can discuss extrapolation, clinical experimental drug trials, and what exactly "flattening the curve" means. Maybe one day, this experience will benefit future discussions with future students. At the moment, though, school buildings remain closed, most of us are staying home with our families to slow the spread of the virus, and we are doing our best to support our students.

Regardless of what comes, what you are doing now matters. What you are doing for your students matters. Hopefully we will all be able to meet together at Rock Eagle in the fall and share our COVID-19 stories. For now, you are in my thoughts and prayers. Stay safe!

Sincerely,
Billy W. Esra II

A Note from Chuck Garner Concerning the Upcoming Calculus Competition

Out of concern for the spread of the novel coronavirus COVID-19, we are forced to cancel the calculus competition which was scheduled for April 18.

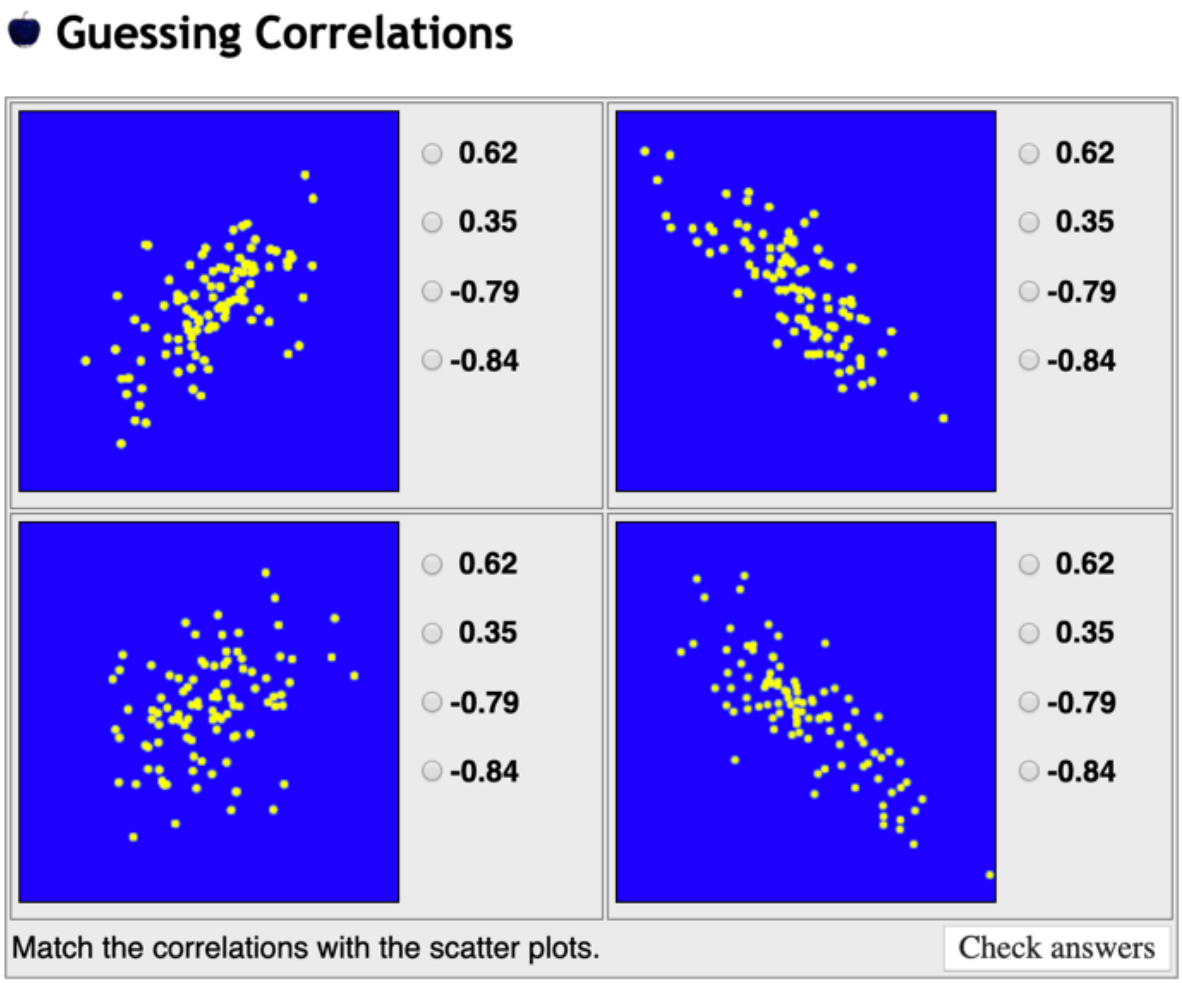
However, mark your calendars for next year: April 17, 2021!

My Favorite Applets!

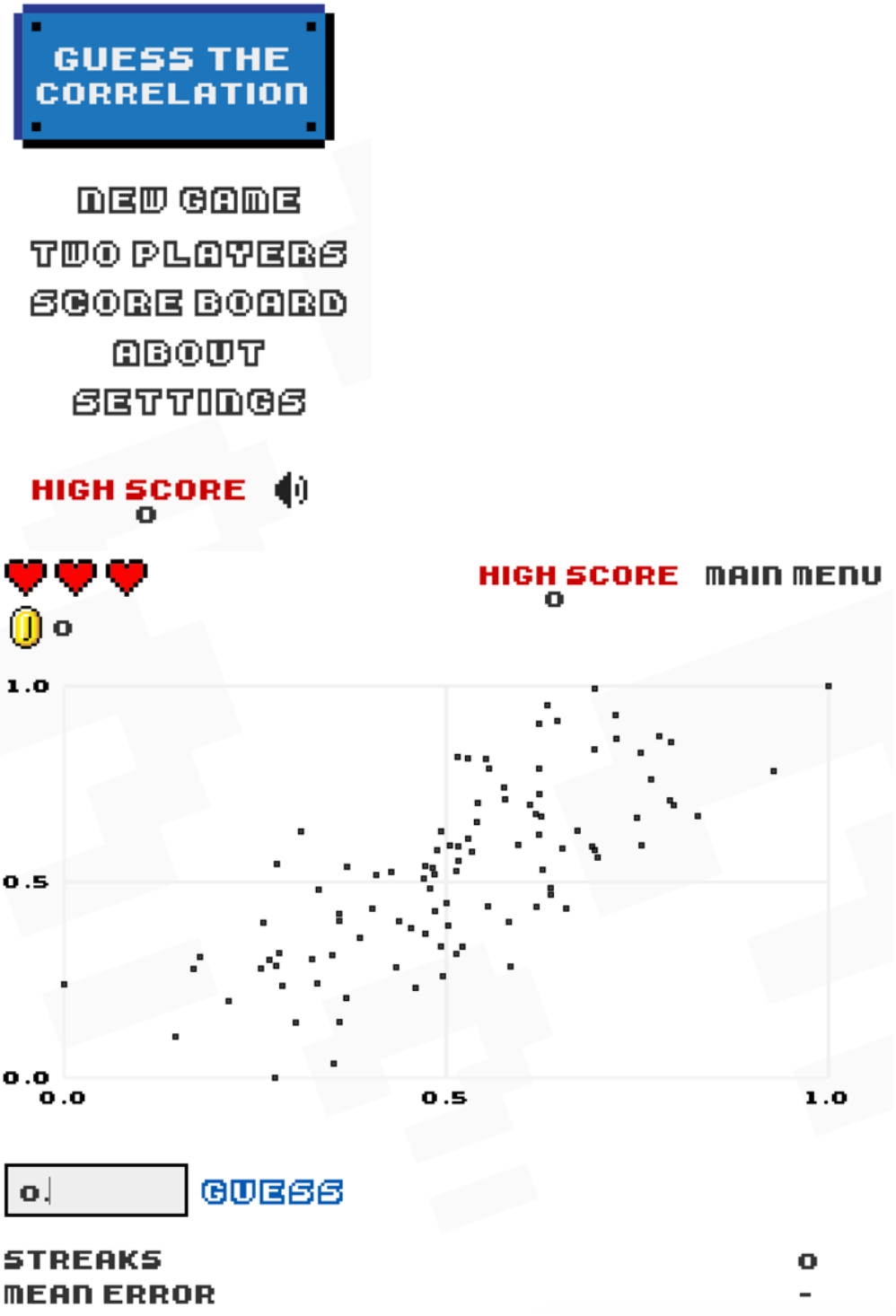
By Vicki Greenberg

I love using applets to illustrate statistical concepts to students. Sometimes I use the applet in the front of the class and other times I have each student on their computers and have them compete in a competition or do a gallery walk to see all their peer's computer screens.

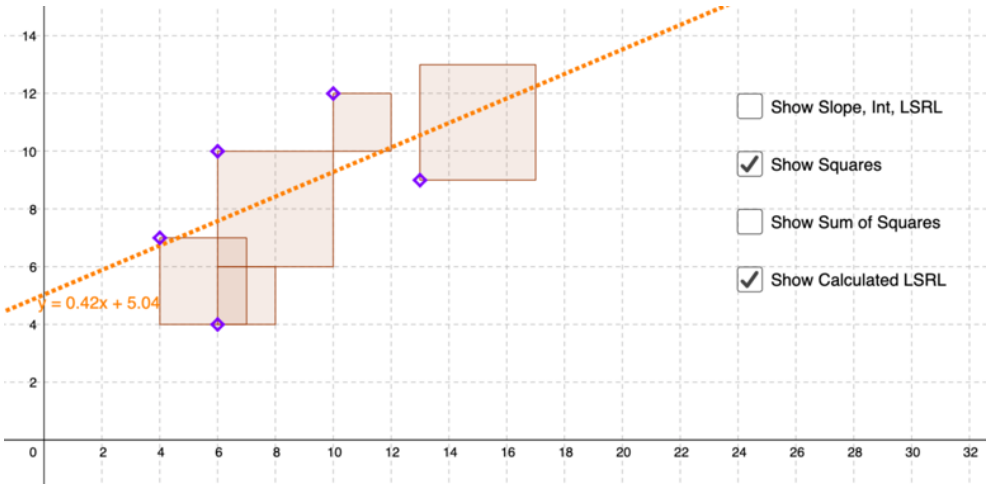
- Guessing Correlations
  http://istics.net/Correlations/
  This applet gives you four graphs and four correlation values and students must match the correct graph to the correct correlation value. You can have competing groups or you can compete for longest streak correct (it also keeps a log of results).



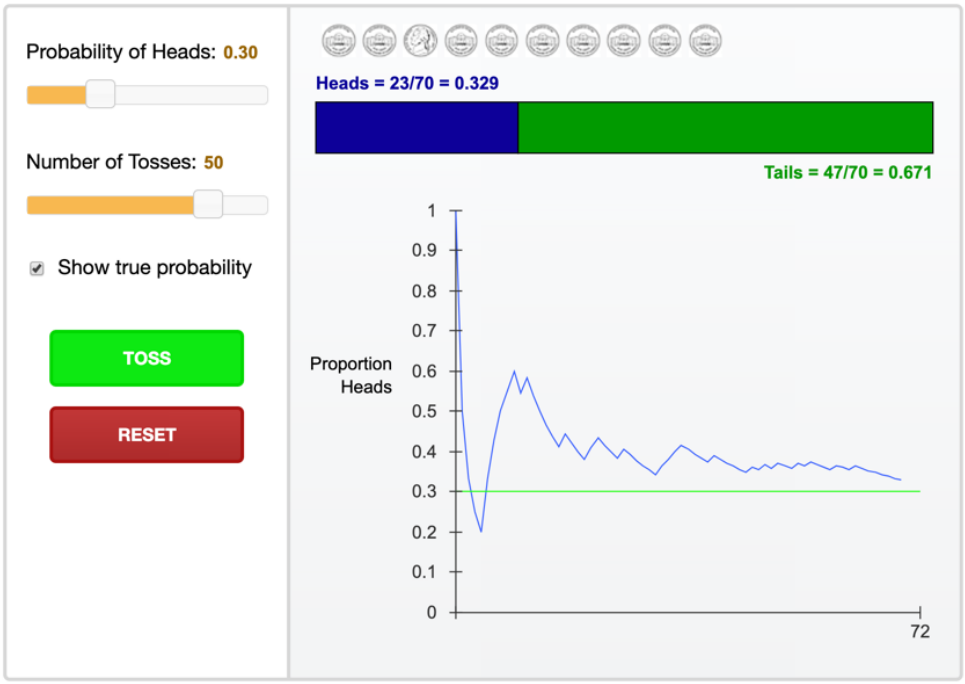
- Guess the Correlation
  http://guessthecorrelation.com/
  This guess the correlation is slightly different in that students actually guess the numerical value of the correlation. Close guesses win “a life” and wrong guesses lose “a life”. Students get better and better with their guesses.



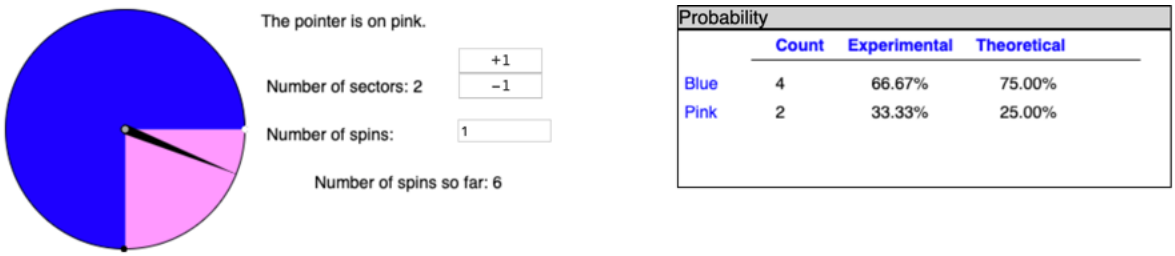
- Least Squares Regression Line
  https://www.geogebra.org/m/XUkhCJRj
  This great GeoGebra graph shows a scatterplot where you can show or hide the your best fit line, squares, sum of squares, and the calculated LSRL. You can move the points around to show the effect that one point can have on the LSRL. I think this applet really helps students understand the difference between outliers and influential points.



- Law of Large Numbers
  http://digitalfirst.bfwpub.com/stats_applet/stats_applet_10_prob.html
  This applet beautifully shows the law of large numbers. One can change the probability of success and number of trials for students to see the erratic behavior at first and then the leveling off around the probability of success.

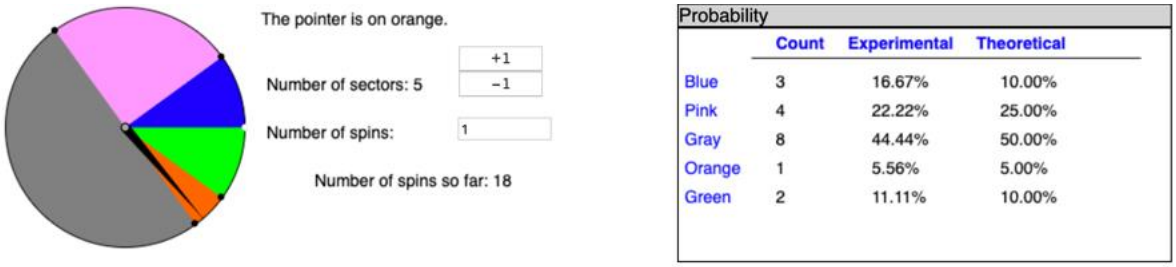


- Adjustable Spinner
  http://www.shodor.org/interactivate/activities/AdjustableSpinner/
  This applet is just a simple spinner. I use this applet when I am introducing hypothesis tests so that the students can see what happens by random chance when the null is true. We then formalize the process. I also use this to introduce confidence intervals (I teach CI's after HT's) to create a list of plausible population values based on a single sample statistic.



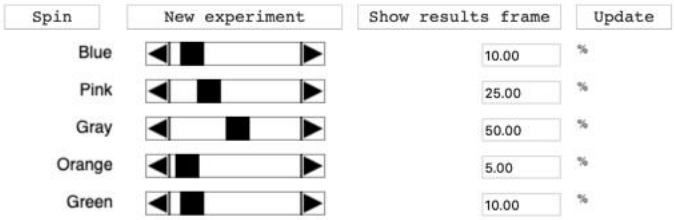
Probability

	Count	Experimental	Theoretical
Blue	4	66.67%	75.00%
Pink	2	33.33%	25.00%

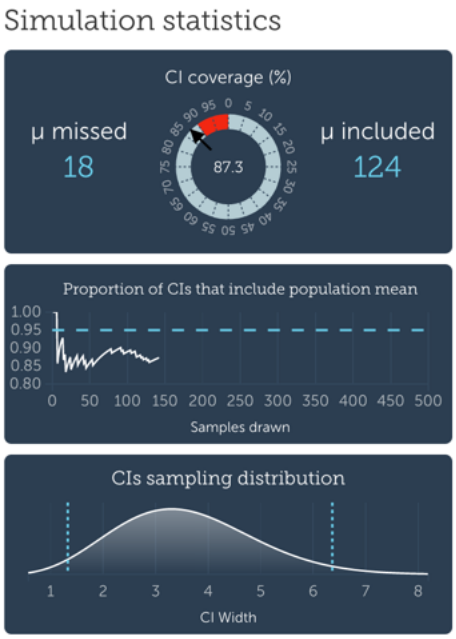


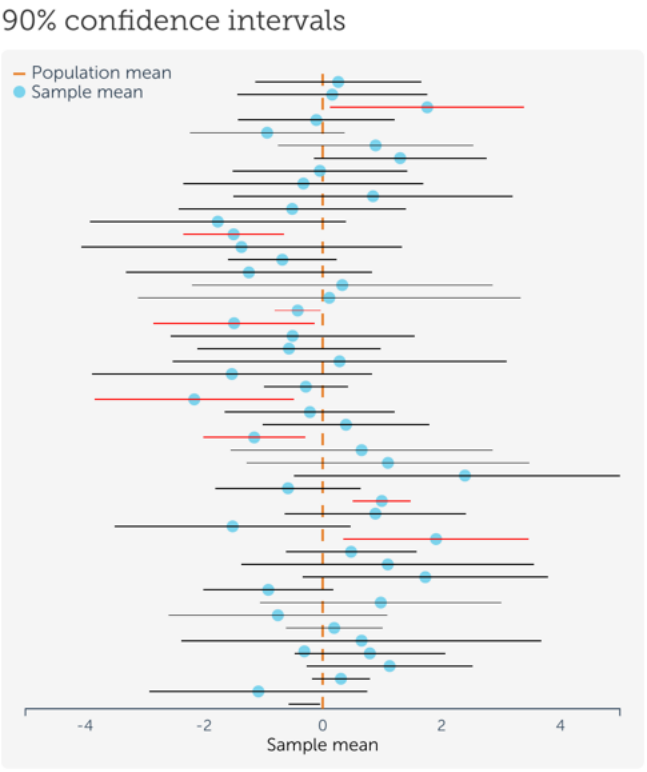
Probability

	Count	Experimental	Theoretical
Blue	3	16.67%	10.00%
Pink	4	22.22%	25.00%
Gray	8	44.44%	50.00%
Orange	1	5.56%	5.00%
Green	2	11.11%	10.00%

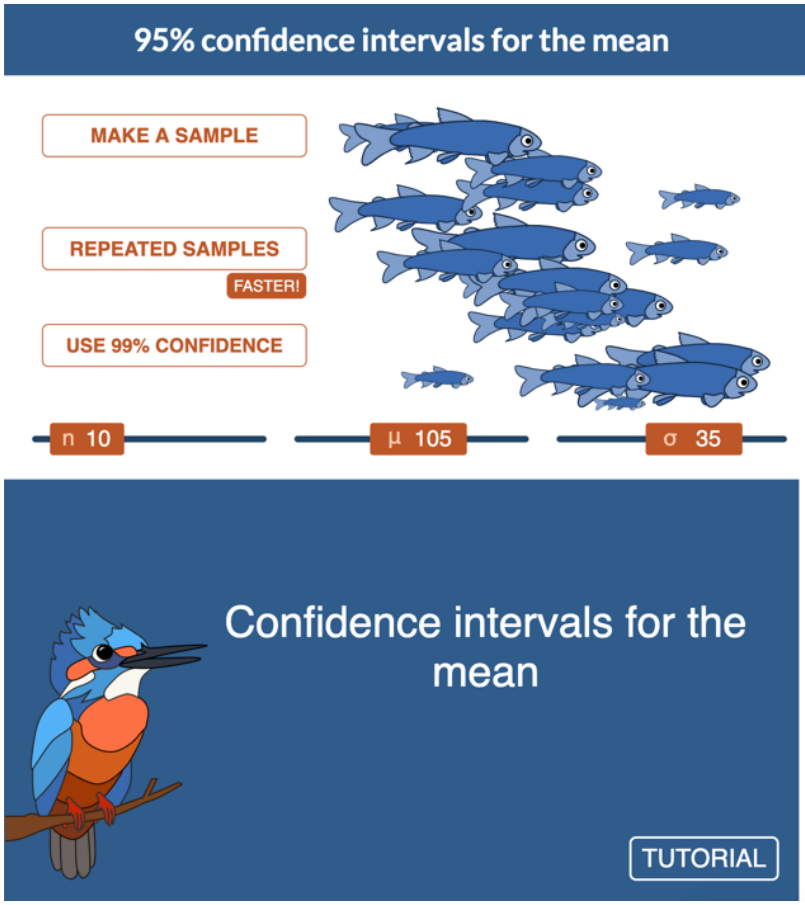


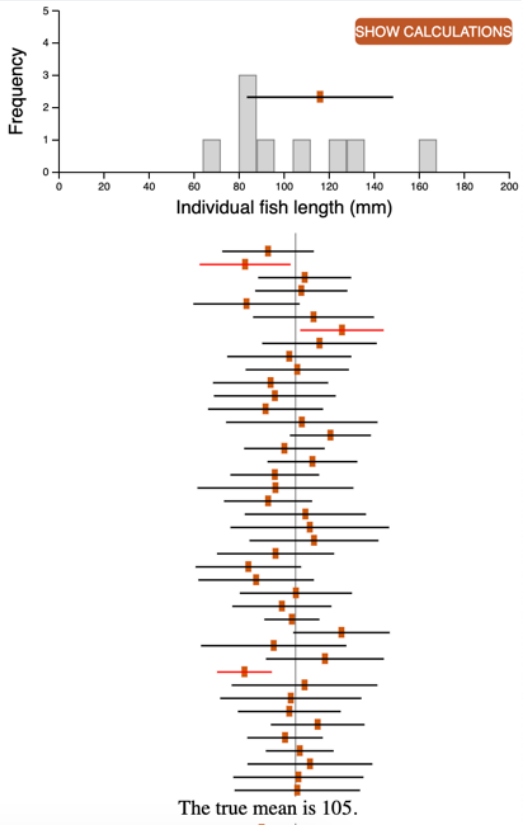
- Dance of the Means – Confidence Intervals for a population mean
  Download an excel file: https://thenewstatistics.com/itns/esci/dance-of-the-means/
  Fully online version: https://rpsychologist.com/d3/CI/
  This is probably my favorite applet! I love the visuals and the ease at which students can see what is happening. I start it running on the board while we are doing something else and let it run for a while. Students can see the intervals that capture the true population parameter and those that do not. The site also shows the capture rate, the number of intervals created, the number of intervals that did not capture the true population parameter. My favorite part about this applet is the second graph it shows – the capture rate as the number of samples increase. This should look very familiar to students – it's the law of large numbers! This helps students make the connection that the confidence level is a probability. We also discuss why the intervals are not all the same width. Change the confidence level slider at the top to see all the intervals change widths. You can also change the sample size to see the confidence intervals change width. You can also control the interval simulation speed.



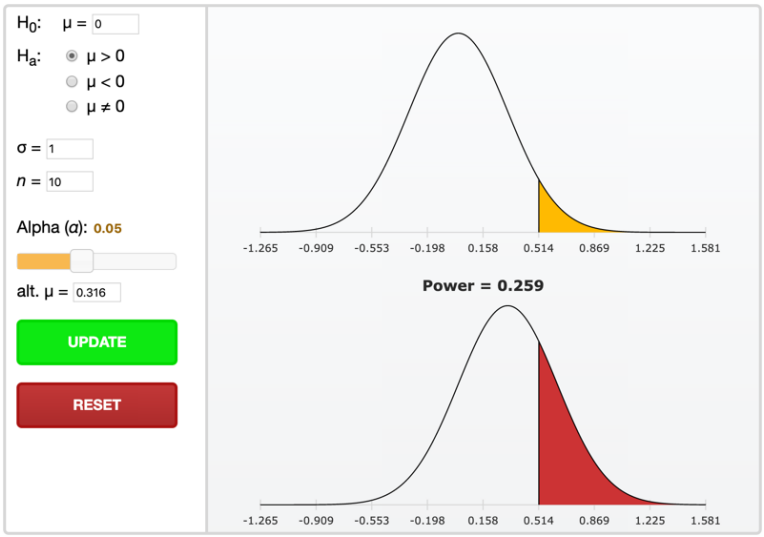


- 95% confidence intervals for the mean
  https://www.zoology.ubc.ca/~whitlock/Kingfisher/CIMean.htm
  I recently learned of this app and it is awesome! It animates (so cool to watch!) taking a sample, calculating the sample mean, and constructing the resulting interval. I love the "show calculations" option – you can click on each symbol and it will show the corresponding value for the most recent sample. The tutorial is also great if you wanted to assign this outside of class.





- Statistical Power
  https://digitalfirst.bfwpub.com/stats_applet/stats_applet_9_power.html
  This applet illustrates power for a population mean. One can change the null hypothesis value, the alternative hypothesis direction, sigma, sample size, alpha, and the alternate hypothesized mean. It is a great illustration of shaded regions for type 1 error and power. I use this applet to show what happens when we change each of these values for the kids to identify the four ways to increase power.



Here is a list of applet collections. These are my go-to places for new and different ways to illustrate statistical concepts.

- Larry Green's Applet Page
  http://www.ltcconline.net/greenl/java/index.html

- STATISTICS THE ART & SCIENCE OF LEARNING FROM DATA
  http://www.artofstat.com/

- Rossman/Chance Applet Collection
  http://www.rossmanchance.com/applets/

- StatKey to accompany Statistics: Unlocking the Power of Data
  http://www.lock5stat.com/StatKey/index.html

- Stapplet
  https://www.stapplet.com/

- Statistical Reasoning in Sports | Student Resources
  https://www.macmillanlearning.com/studentresources/highschool/hsbridgepage/sris.html#

I hope you have found at least one applet to use in your classroom!

Helpful Tips with Simple Anti-derivatives

By Marshall Ransom

Antiderivatives: Provided below are tips for quickly computing antiderivatives that should be easy. That includes *NOT* using a *u*-substitution and associated work if not really needed.

Solutions to the "Try these" exercises are found on the last page.

Helpful tip #1: You can check your work. If I tell you that $\int \sin(x)\,dx = \cos(x) + C$, you can prove me wrong by calculating the derivative of $\cos(x) + C$ which is $-\sin(x)$ NOT $\sin(x)$. The derivative of your antiderivative should be the function you started with, called the integrand. In this case the integrand is sin(*x*).

Example:

Is this statement true? $\int \sqrt{2x-3}\,dx = \frac{1}{3}(2x-3)^{3/2} + C$

Yes, it is because $\frac{d}{dx}\left(\frac{1}{3}(2x-3)^{3/2} + C\right) = \frac{3}{2}\left(\frac{1}{3}\right)(2x-3)^{\frac{1}{3}}(2) + 0 = \sqrt{2x-3}$

Helpful tip #2: Factor out a constant. The anti-derivative depends on the expression in terms of x.

For example, $\int 6\sqrt{x}\,dx = 6\int x^{1/2}\,dx = 6\frac{x^{3/2}}{3/2} = 6\frac{2x^{3/2}}{3} + C = 4x^{3/2} + C$

$\int \sqrt{5x}\,dx = \int \sqrt{5}\sqrt{x}\,dx = \sqrt{5}\int x^{1/2}\,dx = \sqrt{5}\frac{2x^{3/2}}{3} + C$ ⟵ **focus on *x* !!!!**....factor out constants.

$\int \frac{7}{x}\,dx = 7\int \frac{1}{x}\,dx = 7\ln|x| + C$ (We always use absolute value with ln until certain that $x > 0$.)

(Another instance of factoring is shown in an example under **Helpful tip #3**)

Try these:

1. $\int \left(5x^3 - 25x^2\right)dx$
2. $\int 14x^{2/3}\,dx$
3. $\int \sqrt[4]{3x}\,dx$

Helpful tip #3: A reverse chain rule adjustment: divide instead of multiplying IF you see x^1 and no other x's.

For example, we know that $\frac{d}{dx}\sin(5x) = \cos(5x)\cdot 5$, the extra 5 multiplying because of the chain rule. When we calculate *anti*-derivatives, this is a case where all we have to do is divide by the 5 instead of multiply.

We calculate the antiderivative of $5\cos(5x)$ as follows:

$$\int 5\cos(5x)\,dx = 5\int \cos(5x)\,dx = 5\frac{\sin(5x)}{5} + C = \sin(5x) + C$$

Other examples: $\int \sec^2(\pi\theta)\,d\theta = \frac{\tan(\pi\theta)}{\pi} + C$ or $\frac{1}{\pi}\tan(\pi\theta) + C$

$$\int \frac{dx}{3x-5} = \frac{\ln|3x-5|}{3} + C$$

$$\int (9-2x)^5\,dx = \frac{(9-2x)^6}{6(-2)} + C = -\frac{1}{12}(9-2x)^6 + C$$

$$\int (3x-15)^{10}\,dx = \frac{(3x-15)^{11}}{11(3)} + C = \frac{1}{33}(3x-15)^{11} + C$$ OR factor a 3 first, but

be careful because that is 3^{10} ……. $\int (3x-15)^{10}\,dx = 3^{10}\int (x-5)^{10}\,dx = \frac{59049(x-5)^{11}}{11} + C$

Try some more:

4. $\int 3\sqrt[4]{16x^3}\,dx$ 5. $\int \frac{5}{(9x+7)^3}\,dx$ 6. $\int \cos(\pi\theta)\,d\theta$

7. $\int dx$ 8. $\int 8dx$ 9. $\int \frac{dx}{1+5x}$ 10. $\int (2x-7)^{3.9}\,dx$ 11. $\int \left(\frac{1}{2-7x}\right)dx$

12. $\int e^x dx$ $\int e^{2x} dx$ $\int e^{-\frac{x}{4}} dx$ $\int e^{kx} dx$ 13. $\int \frac{dx}{\sqrt{1-8x}}$

Solutions to "Try these" problems: NOTE: "+*C*" is understood, but not written, for each answer.

1. $\int\left(5x^3-25x^2\right)dx=5\int\left(x^3-5x^2\right)dx=5\left(\frac{x^4}{4}-\frac{5x^3}{3}\right)$ 2. $\int 14x^{2/3}\,dx=14\int x^{2/3}\,dx=14x^{5/3}\left(\frac{3}{5}\right)=\frac{42}{5}x^{5/3}$

3. $\int\sqrt[4]{3x}\,dx=\sqrt[4]{3}\int x^{1/4}dx=\sqrt[4]{3}x^{5/4}\left(\frac{4}{5}\right)=\frac{\sqrt[4]{3}x^{5/4}}{5}$ 4. $\int 3\sqrt[4]{16x^3}\,dx=3\sqrt[4]{16}\int x^{3/4}dx=3(2)x^{7/4}\left(\frac{4}{7}\right)=\frac{24x^{7/4}}{7}$

5. $\int\frac{5}{(9x+7)^3}dx=5\int(9x+7)^{-3}dx=5\frac{(9x+7)^{-2}}{-2(9)}=-\frac{5}{18(9x+7)^2}$ 6. $\int\cos(\pi\theta)\,d\theta=\frac{\sin(\pi\theta)}{\pi}$

7. $\int dx=x$ similarly, $\int dt=t$, $\int d\theta=\theta$, $\int d(uv)=uv$, etc. 8. $\int 8dx=8x$

9. $\int\frac{dx}{1+5x}=\frac{\ln|1+5x|}{5}$ 10. $\int(2x-7)^{3.9}dx=\frac{(2x-9)^{4.9}}{4.9(2)}=\frac{(2x-9)^{4.9}}{9.8}$ or $\frac{5}{49}(2x-9)^{4.9}$

11. $\int\left(\frac{1}{2-7x}\right)dx=\frac{\ln|2-7x|}{-7}$

12. $\int e^x dx=e^x$ $\int e^{2x}dx=\frac{e^{2x}}{2}$ $\int e^{-\frac{x}{4}}dx=\frac{e^{-\frac{x}{4}}}{-\frac{1}{4}}=-4e^{-\frac{x}{4}}$ $\int e^{kx}dx=\frac{e^{kx}}{k}$

13. $\int\frac{dx}{\sqrt{1-8x}}=\int(1-8x)^{-1/2}dx=\frac{(1-8x)^{1/2}}{-8}\left(\frac{2}{1}\right)=-\frac{\sqrt{1-8x}}{4}$

AP Updates for Schools Impacted by Coronavirus

Please continue to monitor the College Board changes to AP testing in response to COVID-19. You can find information at https://apcentral.collegeboard.org/about-ap/news-changes/coronavirus-update. Some of the changes to AP Testing this year includes online at-home testing for students, shortened curriculum for this year's exams, increased availability of official study materials online, changes to the format of AP Tests and the length of time for testing, changes to testing dates, etc.

John Neff Award

The John Neff AP Teaching Award is an award that is usually given annually to recognize the contributions of an AP mathematics teacher in Georgia. To qualify, an educator must

- have taught AP Calculus and/or Statistics for at least five years,
- have contributed to the teaching of AP mathematics beyond their classroom, and
- demonstrates excellent teaching within their classroom.

2019 John Neff Award Winner!!!

Jean Linner

Find GAAPMT on social media!

GAAPMT has a Facebook group (https://www.facebook.com/groups/1574708359285045/) and follow us on Twitter (@GAAPMT). Both are great ways to network, to share and get ideas and advice, and to support each other as we negotiate these challenging times. Another great forum for more of the same, but focused on AP Calculus, is the Facebook group AP Calculus Teachers - AB/BC (easily findable by searching Facebook), moderated by experienced AP Readers Virge Cornelius, Mark Kiraly, Erica Arrington and Vic Levine.