CSI: Calculus/Statistics Insider

Official Newsletter of the Georgia Association of Advanced Placement Math Teachers

Issue #289 *Published Semiannually* *Oct 2019*

Find on us the internet:

GAAPMT.org

CONNECT WITH US

Current GA²PMT Officers

- President – Dennis Wilson
- President-elect – Billy Esra
- Secretary/Treasurer – Storie Atkins
- VP of Communications – Rand Wise
- VP of AP Information – Billy Esra
- VP of Technology – Jeff McCammon
- Regional Representatives – David Custer, Lina Ellis, Marshall Ransom, Dean Goldgar, Vicki Greenberg

This publication includes:

SAVE THE DATE!
April 18, 2020
Calculus Tournament

Includes a Written Test and a Team Round
Competition questions will only include topics from the AP Calculus AB exam
Official Announcement will be made at Rock Eagle on October 18.

Note from the President

October is upon us, even if cooler weather is not. I know that all of you face a hectic school year, but I hope these first few months have helped create a strong foundation in each of your classes as you move towards the AP exams in May. College Board has introduced many changes along with many resources for each of us as classroom teachers. It can be difficult to navigate every detail when so much is shared at once with little training. This is one reason for which I am thankful for our organization, the Georgia Association of Advanced Placement Mathematics Teachers. I am thankful for the friends I have made through the years to whom I can reach out for wisdom, ideas, and encouragement. I invite each of you to take the opportunity to build friendships through our annual meeting at the Georgia Mathematics Conference in two weeks. This will be a time to reconnect face to face as well as reap the benefits of the wisdom of others through our conference sessions. I am very excited that Landy Godbold will be our featured statistics speaker for Friday morning. Information about the conference sessions can be found on the following pages and you can register at gctm.org.

At our annual business meeting on Friday, October 18th, we will serve a free lunch to all members, compliments of the publisher Bedford, Freeman, & Worth. We will vote to approve a new slate of officers selected by our board. This slate is listed below. I hope to see each of you at the meeting.

In your service,
Dennis Wilson

New Slate of Officers
President – Billy Esra
President-elect – Storie Atkins
Secretary/Treasurer – Chuck Garner
VP of Communications – Rand Wise
VP of AP Information – Vicki Greenberg
VP of Technology – Jeff McCammon
Regional Representatives – David Custer, Lina Ellis,
Marshall Ransom, Dean Goldgar, Dennis Wilson

GEORGIA MATHEMATICS CONFERENCE OCTOBER 18, 2019

Meet Our Featured Presenter: Landy Godbold

Landy earned BS and MS degrees in Applied Mathematics from Georgia Tech and taught 40 years at The Westminster Schools in Atlanta, GA, including courses in statistics for over 35 of those years (at the AP level since 1996) before retiring in 2015. In 1984 he was one of the first fifty participants in the Woodrow Wilson Summer Institutes in mathematics at Princeton (that year the topic was Statistics) and returned during the summers of 1988-1993 as Computer Director for that program. He served two three-year terms as an NCTM appointee to the ASA-NCTM Joint Committee on Curriculum in Statistics and Probability, the final two years as chair. He began as a reader of AP Statistics exams in 1997, and his 11 years at the Reading includes six years as table leader. He is a co-author of NCTM's *Navigations in Data Analysis* and of activities in the Statistics section of TI's Math Nspired web site. Honors include Westminster's Alex P. Gaines professorship and Alumni Fellow for Distinguished Teaching, the 2009 John Neff Award, and the national Presidential, Radio Shack, and MAA Edyth May Sliffe awards. He remains actively involved with AP Stats on the College Board electronic discussion group as well as through NMSI and TTT. Apart from statistics, Landy enjoys his family (including three grandsons under the age of six), gardening, music, cycling, and table tennis.

	GAS Room 1	GAS Room 2
8:00-9:30	**8:00-9:30** **Answering Your AP Calculus Questions** **Panel**	**8:00-9:30** **A Deep Look at Sampling** **Landy Godbold** *The Westminster School (Retired)*
9:45-11:15	**10:00-11:30** **Scoring Free Response AP Calculus Exam Questions** **Marshall Ransom** *Georgia Southern University* **Chuck Garner** *Rockdale County Magnet*	**10:00-11:30** **A Basket of Simulations** **Landy Godbold** *The Westminster School (Retired)*
11:30-12:15	**11:30-12:15** **GA²PMT Business Meeting & Lunch**	
12:30-2:00	**12:30-2:00** **Report from the AP Calculus Reading** **Provided by Marshall Ransom, Chuck Garner and Dennis Wilson**	**12:30-2:00** **Report from the AP Statistics Reading** **Provided by Billy Esra, Jean Linner**

*Registration for the Georgia Math Conference is required to attend the GA²PMT annual meeting. Visit www.gctm.org to register for the conference.

Lunch will be provided during the AP reading session for current GA²PMT members.

Annual GA²PMT membership is $10, and on-site registration will be available.

All sessions are located in the GAS Building.

2019 Sessions and Descriptions

A Deep Look at Sampling **Landy Godbold**

We will use the (familiar?) River scenario to examine the differences among several different sampling methods, with an eye towards how each is carried out, when each "pays off," what is lost when an inappropriate method is used, and WHY all this is so.

A Basket of Simulations **Landy Godbold**

This session will use a variety of small (and maybe not-so-small) simulations to "see" a number of concepts and rules related to statistical inference.

Answering Your AP Calculus Questions **Panel**

Do you wonder how to approach certain topics in AP Calculus? Do you wonder which topics to stress? Do you wonder how to ensure there is plenty of time to review before the AP exam? This Q&A panel is for those new to teaching AP Calculus and for experienced teachers looking for fresh insights. Bring these questions and any others you have to this diverse panel of AP Calculus teachers.

Scoring Free Response **Marshall Ransom**
AP Calculus Exam Questions **Chuck Garner**

Participants will be trained, as readers are when scoring the AP Calculus Exam, on two problems from recent AP Calculus Exams. Then this training will be put into action by scoring actual student samples of work on these two problems. The presenters are experienced readers who scored these problems, and wrote an article about each one, currently posted at gaapmt.org under "Reader Reports.

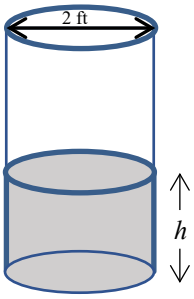
2019 AP Statistics **Billy Esra**
Reading Report **Jean Linner, Matt Tumlin**

This session will be presented by teachers who attended the 2019 AP Statistics reading. Come to learn how the 2019 exam was graded, common student errors and possible ways to improve student learning in AP Statistics based on last year's exam.

2019 AP Calculus **Marshall Ransom**
Reading Report **Chuck Garner, Dennis Wilson**

A review of all nine problems on the operational exam, including discussion of solutions and how the questions were graded. Come to learn how the 2019 exam was graded, common student errors and possible ways to improve student learning in AP Calculus based on last year's exam. The speaker has been a reader and table leader since 1991. Other speakers are also graders for the exam.

Problem Overview:



The cylindrical barrel shown above with diameter 2 feet contains collected rainwater. Water is draining out through a valve (not shown) in the bottom of the barrel. The rate of change of the height h of the water in the barrel with respect to time t is modeled by $\frac{dh}{dt} = -\frac{1}{10}\sqrt{h}$ where h is measured in feet and t is measured in seconds. Students were given that the volume of a cylinder with radius r and height h is $V = \pi r^2 h$.

Part a:

Students were asked to find the rate of change of the volume of water in the barrel with respect to time when the height of the water is 4 feet and indicate units of measure.

Part b:

Students were asked if the rate of change of the height of the water is increasing or decreasing when the height of the water is 3 feet and to explain their reasoning.

Part c:

At time $t = 0$ seconds the height of the water is given as 5 feet. Students were asked to use separation of variables to find an expression for h in terms of t.

Comments on student responses and scoring guidelines:

Part a: worth 2 points

The first point was for calculating $\frac{dV}{dt}$ in terms of *h*. This is perhaps most easily done by substituting 1 for *r* yielding $V = \pi h \rightarrow \frac{dV}{dt} = \pi \frac{dh}{dt}$. This could also be expressed as $\frac{dV}{dt} = \pi\left(-\frac{1}{10}\sqrt{h}\right)$. Students could also report a correct product rule as in $\frac{dV}{dt} = 2\pi rr' + \pi r^2 h'$ and earn this first point. The most common error using the product rule was omitting r'. Another common error was assuming that $r = 2$. Students *showing* $r = 2$ somehow in their work arrived at $V = 4\pi h \rightarrow \frac{dV}{dt} = 4\pi \frac{dh}{dt}$ and could earn the first point but not the second. The second point was for the answer with units and was only awarded for the correct $-\frac{\pi}{5}\,ft^3/s$. Notation could play a role as in $\frac{dV}{dt} = \pi \frac{dh}{dt} \rightarrow \frac{dV}{dt} = \pi - \frac{1}{5}\,ft^3/s$ where it is not clear if the student is subtracting from π or multiplying because of missing parentheses. This last example would not earn the second point.

Part b: worth 3 points

The rate at which the height is changing is the first derivative of *h*. In order to determine whether or not this is increasing or decreasing, $\frac{d^2h}{dt^2}$ has to be calculated. The first step in this calculation is to compute $\frac{d}{dh}\left(-\frac{1}{10}\sqrt{h}\right) = -\frac{1}{20\sqrt{h}}$. This earned the first point. Work showing the chain rule being applied earned the second point. Many students had $-\frac{1}{20\sqrt{h}}$ somewhere in their work, earning the first point, but neglected to complete the calculation by applying the chain rule as in $\frac{d^2h}{dt^2} = \frac{-1}{20\sqrt{h}}\frac{dh}{dt}$ and could not earn the second or third points. The third point was for the answer with reasoning. The mere presentation of a positive second derivative of *h* accompanied by the word "increasing" could earn this point. Unfortunately, many students went further to explain in words beyond the word "increasing" and often did not earn this third point. An arithmetic error in calculating the value of $\frac{d^2h}{dt^2}$ kept students from earning the third point. There is no required arithmetic or simplification here, only the search for and display of a positive value of $\frac{d^2h}{dt^2}$. For example, "$\frac{d^2h}{dt^2} = -\frac{1}{20\sqrt{h}} \cdot \frac{-1\sqrt{h}}{10} \rightarrow$ increasing" earned all three points in part b.

Part c: worth 4 points

Eligibility for points in solving a separable differential equation on the AP Calculus Exam always requires a correct or "almost" correct separation of variables. For example, $\frac{dh}{\sqrt{h}} = -\frac{1}{10}dt$ and $\int \frac{1}{\sqrt{h}} = \int -\frac{1}{10}$ earned the first point for separation. Work showing $h^{-1/2}dh = kdt$ where $k = \pm\frac{1}{10}, \pm 1, \pm 10$ was still eligible for the second and third points even using one of the five constants that are not correct; In fact, $k = -\frac{1}{10}$. Students launching into antiderivatives as in $2\sqrt{h} = -\frac{1}{10}t$ or $2\sqrt{h} = -\frac{1}{10}t + C$ earned both the first point for separation and the second point for antiderivatives. Both antiderivatives needed to be correct in order to earn the second point. If an antiderivative showed h^1 or $\ln(h)$, students could not earn any of the second, third or fourth points. Not showing one of these forms of a bad antiderivative, students having at least one of the antiderivatives correct could earn the third point for having $+C$ in a timely manner *and* trying to use the initial condition correctly. Students showing $\frac{dh}{\sqrt{h}} = -\frac{1}{10} \rightarrow \sqrt{h} = -\frac{1}{10}t + C \rightarrow C = \sqrt{5} \rightarrow h = anything!$ earned the first point, not the second, because of the incorrect antiderivative on the left side, the third point because $C = \sqrt{5}$ comes from correctly using the initial condition, but were not eligible for a correct answer fourth point. Most students more obviously showed the use of $t = 0$ and $h = 5$ if they made it this far in the work, but solving for our "$+C$" was not part of the third point, it being considered part of the fourth point for the expression for *h*. The reason that $C = \sqrt{5}$ is looked at by readers in this last example is because it provides evidence of *using* the initial condition. A trivialization of the problem kept students out of all four points. This could be shown using the separation of $\sqrt{h}dh = -\frac{1}{10}dt \rightarrow anything$ where not having a negative exponent on *h* was considered too much trivialization even if correct work followed.

Observations and recommendations for teachers:

(1) The rate of change of a function is the first derivative of that function with respect to time. In part (a) there were two variables, which would require the calculation to use a product rule had not the given information indicated that *r* is constant. In that case, the value of *r* may be substituted before the derivative calculation. Giving students the value of the *diameter* created difficulties for a number of students in this part of the problem. Many students used the value of the diameter as the value of the radius. Students should read the problem carefully.

(2) The reporting of a numerical answer on the AP Calculus Exam does not require any arithmetic simplification. Students who automatically yield to the urge to simplify often lose a point on the exam and could do so in all three parts of this question. Note that $\frac{d^2h}{dt^2} = -\frac{1}{20\sqrt{h}} \cdot \frac{-1\sqrt{h}}{10}$ indicates that the value of h in $\left.\frac{d^2h}{dt^2}\right|_{h=?}$ is completely irrelevant and that $\left.\frac{d^2h}{dt^2}\right|_{h=?} > 0$ in the part (b) calculation. No simplification is required.

(3) In part (b), students were asked to determine if a rate (a first derivative) was increasing or decreasing. All that needs to be shown to justify the answer is a correct calculation, clearly either positive or negative. The sign of the result of that calculation is assumed to be justification of the choice of either "increasing" or "decreasing." Writing more explanation invites incorrect information such as adding a criterion for justification that is not required. Students writing additional explanatory language after displaying calculations leading to a positive second derivative often lost the third point in part (b).

(4) When solving a separable differential equation and showing work, students should be taught to include the differentials. There must be only one "variable" on each side of the equation resulting from separation. After this separation, this is merely an exercise in antidifferentiation and solving for C using the given initial condition. On the AP Calculus Exam, the appropriate (timely) appearance of C and the substitution of the given initial condition has for several years before 2019 been awarded one point after readers examine the separation and the antiderivatives. Subsequent work in solving for C and finding an expression for the function can often involve several steps, and the result is worth only one point. No algebra or simplification need be done until after C has been determined. It is not a bad test taking strategy to save this last work (for only one point) until the entire exam has been worked because of the time and work it can take.

(5) An all too common error is for students to report that the antiderivative of any "1 over A" function is the natural log of A. This common error should be shown and shunned more than once when teaching after students have knowledge of the fact that $\int \frac{dt}{t} = \ln|t| + C$. In part (c), this error showed its ugly head when students wrote that $\int \frac{dh}{\sqrt{h}} = \ln\left|\sqrt{h}\right| + C$. This error took students out of all points save the possible award of the one point for a correct separation.

(6) The final answer giving an expression for h in part (c) did not require that much attention be paid to the domain of h. Because of the context of the problem, both $h, t \geq 0$ eliminating any concerns that might have arisen because of the presence of $\sqrt{h}$. It is often the case that the domain of the final expression must be considered and has at times appeared as a specific question on the AP Calculus Exam. Functions such as $\ln|x|$, $\sin^{-1}(x)$ or $|\text{expression}|$ require a closer look. Also, a final algebraic step as in solving for y if we have $y^2 = \text{expression}$ can require a look at which branch, $\pm\sqrt{\text{expression}}$, should be chosen. Problems involving these considerations should be used by teachers in the classroom. For some examples from past AP Calculus Exams see the following: 2006AB5 part (b), (b) 2010AB6 part (c), 2011AB/BC5 part (c), 2013AB6 part (c), and 2014AB6 part (c).

AP STAT DEBRIEF – Question 2

Billy Esra
Bishop Hall Charter School

2019 QUESTION 2: Experimental Design

Question 2: INTENT OF THE QUESTION

The primary goals of this question are to assess a student's ability to (1) identify components of an experiment; (2) determine if an experiment has a control group; and (3) describe how experimental units can be randomly assigned to treatments.

SAMPLE SOLUTION and COMMENTS:

Reading (grading) questions like question two can be a challenge for readers. There is a very precise easy to read version of the answers to this question, but students tend to write way more than is required by the rubric. In general on this question, students included extra information (like double blinding the insects and the researchers or calling the 0 ml/L a placebo) that did not negatively impact their score, though they occasionally would add something that changed the intent of their correct answer and result in a lower score. My overall suggestion for teachers is to remind students to: 1. answer the question, 2. be concise, and then 3. stop writing.

"Researchers are investigating the effectiveness of using a fungus to control the spread of an insect that destroys trees. The researchers will create four different concentrations of fungus mixtures: 0 milliliters per liter (ml/L), 1.25 ml/L, 2.5 ml/L, and 3.75 ml/L. An equal number of the insects will be placed into 20 individual containers. The group of insects in each container will be sprayed with one of the four mixtures, and the researchers will record the number of insects that are still alive in each container one week after spraying.

a. Identify the treatments, experimental units, and response variable of the experiment. Treatments:
Experimental units:
Response variable:

b. Does the experiment have a control group? Explain your answer.

c. Describe how the treatments can be randomly assigned to the experimental units so that each treatment has the same number of units."

a. Students had to identify the concentrations of fungus mixtures as the treatments, either explicitly or by listing the four treatments. Students had to identify the 20 containers as the experimental units and they had to identify the number of insects alive in each container after being sprayed by the mixtures as the response variable.
b. The simplest answer to this question was "Yes, 0 ml/L," though students could include more information about why this is the case (and often did).
c. The three components of the answer to part (c) included:
    - 1. an appropriate labeling strategy
    - 2. the description of an appropriate randomization application process
    - 3. that the randomization results in an equal number of containers for each treatment.

NOTES/COMMON MISTAKES

1. Treatments and experimental units needed plural language for a correct response. Some students lost credit for saying things like "concentration of fungus and container" instead of their plural counterparts.
2. Many students incorrectly identified the insects as the experimental units, instead of the containers (or groups of insects). They did not get credit for this component of the rubric, but if they made this mistake they could get credit for saying that the response variable was "whether or not the insect lived or died" and they could get credit in part (c) by correctly randomizing the insects into groups for treatment.
3. Some students did not recognize the containers receiving the 0 ml/L treatment as control group in part (b). These students often felt this way because all containers were sprayed. They mistakenly believed that a control group necessitated that nothing at all was sprayed.
4. Another common mistake in part (b) was to claim that the 0 ml/L concentration formed a control group (which is correct) but also include that these containers did not get a treatment (which is incorrect).
5. Students could get credit in part (a) by listing out the four treatments. Many students that made the mistake in note 4 also failed to list 0 ml/L as one of the treatments in part (a).
6. Defining the response variable without language like "number of …" was a mistake some students made. "Insects alive in each container" is not a variable and was not accepted.
7. In order to get full credit for part (c), students had to create appropriate labels for the units/treatments, describe how to implement the random assignment process, and have a process that results in an equal number of experimental units assigned to each treatment. Students could use many different methods to earn full or partial credit, but, in my opinion, the students who used "the hat method" tended to consistently do well. This method usually went something like, "label all of the containers from one to twenty. Write the label for each container on an index card. Put the index cards in a large hat and shake the hat to randomize the cards. The first 5 cards pulled would represent containers that get the 0 ml/L concentration. The next 5 cards pulled would represent container that got the 1.25 Ml/L concentration, etc." More complicated randomization strategies usually introduced a concept that wasn't quite correct. Random number tables, rolling dice, and randomly assigning insects to treatments (instead of containers) would often result in a lower score.

TEACHER SUGGESTIONS

Experimental design questions on the AP Statistics exam require students to understand the vocabulary of experimental design in context. The language students use should be precise with attention to the details of the question asked. For example, when asked for the response variable, students often responded with something akin to "insects still alive." They understood, to some degree what the question was asking them, but did not earn any credit for this response. With that response, was the student referencing the number of insects alive overall, which is not a variable? Or those alive in each container, which is closer to correct? Even if they included "insects still alive in each container"

they are still not referencing a measurable variable, so still no credit. Students had to refer to “the number of insects still alive in each container” to get credit. I think this practice of answering the question precisely requires constant practice throughout the school year, with real consequences when students aren’t precise. I know that when I am grading student responses, I often unconsciously tend to see what the student intends and not necessarily what they wrote. Students must understand that language and precision to details is important. Sometimes using the wrong tense or wrong plurality can lower a student’s score, even when they know the correct answer.

Also, especially in part c., students seem to think that their answer must be overly complicated to get full credit. Or they are expecting the test to try to throw them a curve ball. My experience is that most questions are straight forward. The test writers are not trying to trick students with overly complicated questions. They ask a straightforward question and are just expecting students to answer with a straightforward answer. If students are writing and writing and writing on a question, there is a good chance that they have gone off the rails of the intent of the question. Caution students to “answer the question and then get out.” They don’t need to throw everything that they know at every question. They simply just need a simple precise answer.