2019 AB/BC-1 Rand Wise, Marist School

Problem Overview:

Students were given a model for the rate of fish entering a lake, $E(t) = 20 + 15\sin\left(\frac{\pi t}{6}\right)$, and a model for the rate at which they leave given by $L(t) = 4 + 2^{0.1t^2}$. Both functions are measured in fish per hour, with t measured in hours since midnight ($t = 0$).

Part a:

Students were asked to find to the nearest whole number how many fish enter the lake between midnight and 5 A.M.

Part b:

Students were asked to find the average number of fish leaving the lake per hour during the same time interval from midnight to 5 A.M.

Part c:

Students were asked to find the time in the interval $0 \leq t \leq 8$ when the number of fish in the lake was greatest, and to justify their answer.

Part d:

Students were asked to determine whether the rate of change in the number of fish in the lake was increasing or decreasing at 5 A.M., with an explanation of their conclusion.

General scoring guidelines for the problems:

Part a: (2 points)

The first point in this part is for the integral $\int_0^5 E(t)dt$. An indefinite integral did not earn the point. Some copy errors were allowed if students used the expression for the function instead of just calling it $E(t)$. Neither point required that units be specified. If students had the degree mode answer in part a, they were inoculated from the same mistake in later parts.

The second point was for the answer. Either 153 or 154 was acceptable. A bald answer earned no points.

Part b: (2 points)

The first point in this part was for the integral $\int_0^5 L(t)dt$ (with or without the multiple $\frac{1}{5-0}$).

The second point in this part was for the answer 6.059 fish per hour. Students had to have their value correct to three decimal places, with a decimal presentation error inoculating them for later parts.

<u>Part c:</u> (3 points)

The first point in this part was for setting $E(t) - L(t) = 0$, or an equivalent clear attempt to determine where $E(t) = L(t)$. Language such as "the functions intersect at…" or "the equations intersect at…" was acceptable for the point but not "the graphs intersect at…" unless the student specified what graphs they were referring to.

The second point in this part was for the answer $t = 6.204$ (or 6.203). This value had to be correct to three decimal places unless the student was inoculated for decimal presentation error earlier in the problem. Simply writing $E(t) = L(t) \rightarrow t = 6.204$ was sufficient to earn the first two points.

The third point was for justifying the maximum value reported. The vast majority of students who earned the third point used one of two approaches: i) a candidates test approach; or ii) a sign test approach. If a student chose the former, values of $A(t)$ had to be correct. If a student chose the latter, readers took care to see whether the student made a local argument (a sign change at $t = 6.204$) or argued globally on the entire interval $0 \leq t \leq 8$. Local arguments did not earn the point.

Many students did not know how to handle the fact that the number of fish in the lake was not given for any *t*. Readers generally ignored claims or assumptions students made to this effect.

<u>Part d:</u> (2 points)

The first point in this part was for considering $E'(5)$ and $L'(5)$. Clear and valiant attempts at analytically differentiating the functions earned the first point if evaluated at $t = 5$. Simply reporting from the calculator the values of $E'(5)$ and $L'(5)$ also was sufficient to earn the point.

The second point was for correctly concluding that the rate of change in the number of fish in the lake was decreasing at $t = 5$ <u>and</u> providing a valid explanation that included a direct comparison of $E'(5)$ and $L'(5)$. Generally, the two acceptable formulations were "because $E'(5) - L'(5) < 0$, the rate of change of the number of fish is decreasing at $t = 5$", or the equivalent "because $E'(5) < L'(5)$ ...". Considering the signs of $E'(5)$ and $L'(5)$ separately did not earn the point.

<u>Observations and recommendations for teachers</u>

Students were guilty of not reading carefully in the first two parts—many read the instructions about rounding in the first part as carrying over to part B without considering that a decimal answer made sense (or remembering that the rule of thumb is always three decimal places).

Teachers should take care to teach students the difference between local and global arguments, and when each is appropriate. Many students lost the point in part C by making a local argument.

Being the first question in the calculator section, students should be aware of calculator-specific issues like decimal presentation expectations, being in radian mode, and reporting bald answers from the calculator. Readers did see many decimal presentation errors and bald answers, but by and large students did well about calculating in radians and not degrees.

In part D, many students had difficulty understanding which level of derivative was relevant to the problem, for example comparing $E(5)$ to $L(5)$ or even $E''(5)$ to $L''(5)$. Even among students who used the correct level, many failed to verbally express their conclusion correctly (e.g., "therefore the number of fish is decreasing" as opposed to "therefore the rate of change in the number of fish is decreasing"). Teachers should give students ample opportunities to practice saying and writing the interpretation of the meaning of various levels of derivatives.