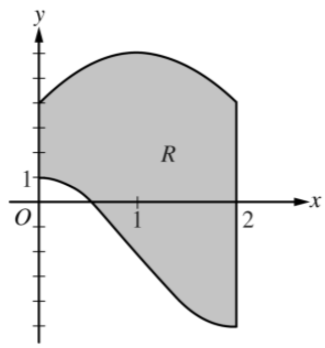
2019 AB-5 “The Lake” Rand Wise, Marist School

Problem Overview:



Students were given the graph of the region *R* enclosed by the graphs of $g(x) = -2 + 3\cos\left(\frac{\pi}{2}x\right)$ and $h(x) = 6 - 2(x-1)^2$, the y-axis, and the vertical line $x = 2$, as shown above.

Part a:

Students were asked to find the area of *R*.

Part b:

Given that region *R* is the base of a solid with cross sections perpendicular to the x-axis with each cross section of area $A(x) = \frac{1}{x+3}$, students were asked to find the volume of the solid.

Part c:

Students were asked to write, but not evaluate, an integral expression that gives the volume of the solid generated when *R* is rotated about the horizontal line $y = 6$.

General scoring guidelines for the problems:

Part a: (4 points)

The first point in this part is for an integrand. At a minimum, the point required the bare integrand $h(x) - g(x)$ (without the integral symbol), plus some subsequent evidence of integration. $\int h(x) - g(x)dx$ was sufficient. Note that students were not told which was greater, $h(x)$ or $g(x)$, so either order of subtraction in the integrand earned the first point and kept them eligible for the second and third points (and even the fourth if they handled the sign of the answer appropriately at the end (with no linkage errors)). There were two special cases where students could lose the first point but remain eligible for the second and third points: integrands of the form $h(x) + g(x)$ did not earn the first point, but allowed readers to consider the second and third points, and students who *initially* presented either $h(x) - g(x)$ or $g(x) - h(x)$ with simplification errors also lost the first point but were eligible for the second and third. Omitting *dx* was not penalized.

The second point was for the antiderivative of $3\cos\left(\frac{\pi}{2}x\right)$. This antiderivative had to be correct, with one exception: students were still allowed to earn this point if they made a copy error, omitting the coefficient 3 in $g(x)$.

Many students attempted u-substitution when integrating $3\cos\left(\frac{\pi}{2}x\right)$ and made errors with the coefficient.

The third point was for the antiderivative of all the remaining terms. Students who were careful with parentheses and signs were highly successful at earning this point.

The fourth point was the answer point. Limits of integration were not considered until this fourth point. Students could earn all four points by breaking the problem into separate integrals as long as they had correct limits of integration (this approach was rare). It was fairly common for students not to get the fourth point even though they had the correct answer because of making errors that cancelled each other out.

Part b: (2 points)

The first point in this part was for the integral, with correct limits of integration. Students were not penalized for omitting *dx*.

Despite being given the expression for the cross-sectional area, many students struggled to formulate the integrand. Students who had a constant factor in the integrand (usually π, but could have been any $k \neq 1$) could earn the first point, but were ineligible for the second.

Some students used u-substitution, but to get both points they had to handle the limits of integration correctly.

The second point in this part was for the answer.

Part c: (3 points)

The first point in this part was for the limits of integration and constant multiple (π), but was only granted in the context of integration (i.e., there had to be an integral symbol and an integrand).

The second point in this part was for the form of the integrand, which had to be a difference of squares $R^2 - r^2$. Students could earn this point with a different axis of revolution as long as it was used consistently with both functions, but this took them out of consideration for the final point (as did having the order of subtraction reversed). Students could also earn the second point with a parentheses error if at least one of the terms was completely correct. If students began with a simplified integrand, it had to be completely correct to earn the second point.

The third point was for a correct integrand. Assuming students did not make errors with the limits, did not forget the constant π, and had the correct axis of revolution, the biggest source of error for students who earned the first two points was incorrect usage of parentheses, usually forgetting to close parentheses. Many students seemed to think that there is a distinction between brackets and parentheses such that closing a bracket could compensate for mismatched parentheses. Readers made no such distinction.

Observations and recommendations for teachers

(1) A major issue in scoring was student attempts at simplification of the integrand prior to finding the antiderivatives, and many of these involved errors in the signs of the terms arising from students failing to use parentheses or using them incorrectly (for example, dropping negative signs or distributing negatives incorrectly). Another common error was also algebra-related: incorrectly expanding $(x-1)^2$. Students also commonly compounded algebra-related sign errors by making another sign error when integrating $3\cos\left(\frac{\pi}{2}x\right)$. The difficulty of negotiating all the signs and parentheses sometimes also led to linkage errors. For students who had the order of subtraction reversed in the integrand, these sign errors were more common, often causing students to lose one or both of the antiderivative points. Teachers should give students frequent practice with expressions involving parentheses and check carefully for accuracy. Sloppy algebra does not go unpunished on the AP exam.

(2) The focus of this problem was setting up and/or evaluating integrals in the applied context of area and volume. This set of skills is tested year in and year out, this year being no exception. This particular problem placed a large burden on students to have algebra details correct, especially involving grouping terms in parentheses in the context of subtraction, and distributing negatives correctly. Students often got themselves into trouble by entering the problem further along than necessary, after already having made some algebra mistake.

(3) Many students missed the second part of this question because they persisted in a formulaic approach to setting up an integrand for cross-sectional area, when it was simply given to them. Teachers would do well to use notation to reinforce the meaning of the integrand in cross-sectional area problems.

(4) Students could have earned 3 to 5 points on this problem without simplifying any algebra expressions or finding any antiderivatives simply by relying on notational fluency. Students too often bypassed setting up integrals in their generic forms with given

function names, and jumped into the algebra expressions and integration with errors from the start.