

2017 AP STATISTICS: Question 2

To access the question, please visit <https://secure-media.collegeboard.org/ap/pdf/ap-statistics-frq-2017.pdf>. Be sure to read the College Board Solutions as well as this commentary.

The intent of this question was to assess:

- 1) Students' ability to name a statistical test, check conditions, construct, and interpret a confidence interval for a proportion.
- 2) Apply their results to a real life scenario to determine the monetary loss effects.

Part a)

Step 1: Students needed to identify the confidence interval by name or by formula and check the appropriate conditions.

Either: One-sample z-interval for the population proportion p ; $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Population is all customers who come into the restaurant and ask for a water cup;
 p = proportion of the population who fill the water cup with a soft drink

Conditions:

1. Random sample
2. Large sample size such that $80 \left(\frac{23}{80}\right) = 23 > 10$ and $80 \left(\frac{80-23}{80}\right) = 57 > 10$

Step 2: Used correct mechanics to get the confidence interval:

$$\hat{p} = \frac{23}{80} = .2875 \quad 95\% \text{ Use } z = 1.96$$
$$0.2875 \pm 1.96 \sqrt{\frac{.2875(1-.2875)}{80}} = 0.2875 \pm 0.0992 = 0.1883 \text{ to } 0.3867$$

Step 3: Interpret

We are 95% confident that the true population proportion of customers who come into the restaurant ask for a water cup and then fill it with a soft drink is located between 0.1883 and 0.3867.

Part b:

If one uses the confidence interval from above, and 3000 customers in June, and the loss is \$.25 per cup, the amount loss would be between $3000 * 0.25 * 0.1883$ and $3000 * 0.25 * 0.3867$ which is between \$141.25 and \$290.00.

SCORING

Part a was scored in 3 parts and part b was scored in 1 part. Each essentially correct got 1 point and each partially correct got $\frac{1}{2}$ point.

Part a:

Section 1:

Essentially correct (E): One-sample z-interval for proportion identified by name/formula and both conditions (random sampling and large sample size) addressed

Partially correct (P): identifies the correct procedure BUT addresses only one of the conditions or Identifies both conditions correctly but does not identify the correct procedure

Incorrect (I): the response does not meet the criteria for E or P.

Answers not considered sufficient:

1. Large sample size
2. Inappropriate conditions like $n \geq 30$, or normal population \rightarrow this earns at most a P
3. Checking sample size relative to population is appropriate but not required.
4. Any statement of hypotheses, description of population, and definition of parameter is extraneous. If it is incorrect, it is considered poor communication in holistic grading.

Section 2:

Essentially correct (E): Student has the correct 95% confidence interval. Supporting work not required but must be correct if it is present

Partially correct (P): Student has the correct 95% confidence interval but has incorrect work (but appropriate) work or has incorrect but reasonable confidence interval with appropriate supporting work shown—for example using another value other than 1.96 for the z^* or a different value for \hat{p} other than $\frac{23}{80}$.

Incorrect (I): the response does not meet the criteria for E or P.

CORRECT SUPPORTING WORK IS OF THE FORM: $\text{proportion} \pm (\text{critical value})(SE/SD \text{ of proportion})$
It is not reasonable for a confidence interval to have values outside of 0 – 1.

Section 3: Interpretation

Essentially correct (E): Interpretation of the interval must contain the three components as follows:

1. Indicates inference about a **population proportion**.
2. Conveys understanding that the parameter is the proportion of the water cup population that filled their cups with soft drink.
3. Mentions 95% confidence and interprets the statement correctly—“we are 95% confident”; or “with 95% confidence”.

Partially correct (P): Provides an appropriate interpretation of the interval that includes the first component and one of the other two components or correctly interprets a confidence level in context without interpreting the confidence interval.

Incorrect (I): the response does not meet the criteria for E or P.

Notes:

1. If a student refers to the 80 students sampled in the interpretation, the section was scored as an I. It no longer infers about the population.
2. Unrealistic values, including blanks lowers the score by one level.
3. When the level and the interval are both given, only the interpretation is scored. If the level is incorrect it is considered poor communication and is used in holistic scoring.
4. Any interpretation that indicates 95% chance of capturing the population proportion is an I.

Section 4: Interpretation of Cost

Essentially correct (E): Response gives a correct interval estimate for the cost to the restaurant and shows enough work to determine how it was calculated.

Partially correct (P): Response gives the correct interval estimate for the number of customers (565, 1160) who will ask for water cups, and includes showing work—but fails to determine cost by multiplying by 0.25

Or Response give a correct interval estimate, including work, (\$0.05, \$0.10) for the expected cost to the restaurant of one individual who asked for a water cup, including showing work, but does not multiply by 3000.

Or Response shows a reasonable attempt to get the correct endpoints of the interval, (3000) (0.1883) (\$.25) and (3000) (0.3867) (\$.25) but they use 80 instead of 3000.

Incorrect (I): the response does not meet the criteria for E or P.

Notes:

1. Units (\$) are not required to get an E
2. An “E” can be obtained if the student used an incorrect interval from part A.

TEACHER NOTES

1. This was one of two inference procedures on the AP STAT EXAM. Students should know that when they are asked to determine if there is convincing evidence and they have 1 – 2 pages to show their work, they are doing a significance test in all likelihood.
2. Some students did not identify the test. For any inference procedure, students will be required to identify the test by name or formula. Strange names were given for the test (and not accepted).
3. Conditions → too many students did not check conditions or they did it wrong. They had a lot of trouble with $n\hat{p} > 10$, $n(1 - \hat{p}) > 10$. They didn't check it and didn't know it was the large sample/normality check condition.
4. Many strange wrong references to a normality condition.
5. Many students conducted a “t” confidence interval with degrees of freedom---don't really know why.
6. Many students correctly used technology and had wrong work. This is not a statement to skip work because more work/justification is being required on the exam. It is a statement to make sure that students check to ensure that what they write down actually gives them the correct answer. If not, they will not get credit for that part.
7. Students have misunderstandings about proportions, categorical and quantitative data. They are not confident about counts and proportions and working with categorical data.
8. Interpretations included too many non-standard phrases. As Corey Anderson stated, “If you say something in a different way, you may be saying a different thing.” Therefore, creativity in the interpretation of a CI is probably not for the best.
9. Too many students still do not know the difference between a confidence interval and a confidence level. This is critical!!!! The exam will ask students to interpret one, and that is what students should interpret. If they interpret the wrong one, they will not get credit. If they correctly interpret the confidence interval and wrongly interpret the confidence level, they will lose points.
10. Some students used a single value for a confidence interval. They need to know this is not accurate.

- 11. Proper notation is critical--> require this of your students and be tough!**
- 12. Have students create their own summary sheet of inference methods with conditions and formulas. Many of us have created them and handed them out, but if a student makes their own, and they are checked in a Think-Pair-Share class, it may help tremendously on the exam.**