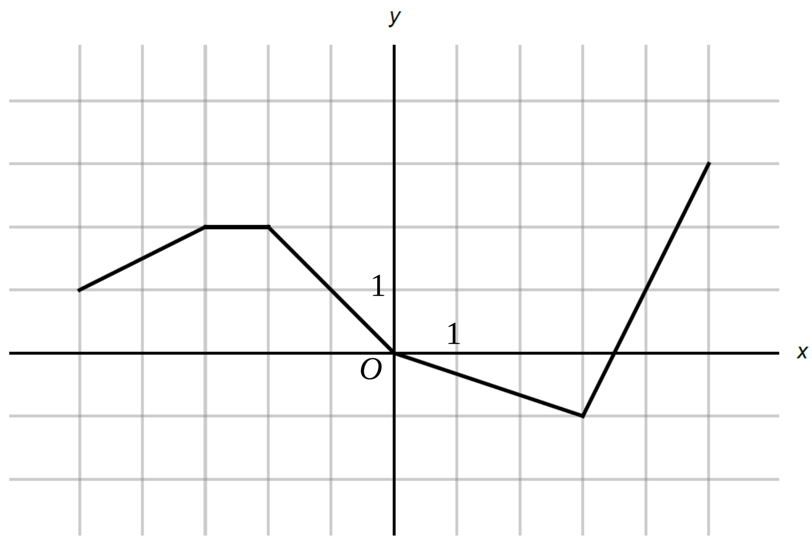
Problem Overview:

The function g is differentiable, and some values of g and g' are shown in the table below. A graph of the function h consists of five line segments, and is also shown below. Another function was defined as follows: $f(x) = \cos(2x) + e^{\sin(x)}$.

x	$g(x)$	$g'(x)$
–5	10	–3
–4	5	–1
–3	2	4
–2	3	1
–1	1	–2
0	0	–3



Graph of h

Part a:

Students were asked to find the slope of the line tangent to the graph of f at $x = \pi$.

Part b:

The function k was defined as $k(x) = h\big(f(x)\big)$. Students were asked to find $k'(\pi)$.

Part c:

Students had to calculate $m'(2)$ if $m(x) = g(-2x) \bullet h(x)$.

Part d:

Students had to determine if there is a number c in the closed interval $[-5, -3]$ such that $g'(c) = -4$ and justify their answers.

Comments on student responses and scoring guidelines:

Part a:

Two points were available in part (a) for showing work with a derivative of f and an evaluation at $x = \pi$. The attempt at a derivative had to show evidence of a chain rule being used in both terms. The result did not have to be evaluated to the final answer of -1. Thus $-2\sin(2\pi) + \cos(\pi)e^{\sin(\pi)}$ was awarded both points. Some students jumped straight to the evaluation, but showed evidence of a correct derivative calculation as in $-2 \bullet 0 + (-1) \bullet e^0$, an answer that also earned both points. While this example shows that readers were to award points based on evidence of the correct derivative calculation, other examples were shown that only earned one or none of these two points. For example, $-2 \bullet 0 + (-1) \bullet 1$ only earned one point, the missing e not showing sufficient evidence of a chain rule in the second term. $0 + (-1) \bullet 1$ earned zero points. Examples of parentheses errors were shown to readers such as the stark answer of $\sin 2\pi - 2 + \cos \pi e^{\sin \pi}$ which earned zero points and $\sin(2\pi - 2 + \cos \pi e^{\sin \pi}$ which earned one of the two points. (See **Observations and recommendations for teachers** #1 below.)

Part b:

Evidence of the chain rule and no chain rule errors were required to earn either of the two points in this part of the problem. Computation errors came off the second point. Since $k'(\pi) = h'\left(f(\pi)\right) f'(\pi)$, students could import an incorrect value for $f'(\pi)$ from part (a) and be eligible for both points unless this resulted in a simplification of work for part (b). Bald answers such as $\left(-\frac{1}{3}\right)(-1) = \frac{1}{3}$ and $k'(\pi) = \frac{1}{3}$ earned zero points. Some students showed errors in calculating $h'\left(f(\pi)\right) = h'(2) = -\frac{1}{3}$ and examples were shown to readers so that in some cases students earned the first but not the second point.

Part c:

There were two points allotted for the derivative of m and one point for the answer. To be eligible for any points, students had to show a product rule structure as in $g'(u)h(x) + h'(x)g(u)$. A chain rule error with the correct product rule structure as in $g'(-2x)h(x) + h'(x)g(-2x)$ was awarded one of the first two points and was eligible for the answer point. The same was true for students presenting a good chain rule and product rule, except showing a difference rather than a sum as in $-2g'(-2x)h(x) - h'(x)g(-2x)$. Eligibility for the answer point required earning one of the first two points and using values for g found on the given table.

Part d:

In the spirit of "MPAC1: Reasoning with definitions and theorems" the justification required verifying the conditions under which the MVT applied in this part of the problem. The first point was for a difference quotient by applying the MVT on the interval $[-5, -3]$. This had to show both a difference and a quotient and reference the table as in $\dfrac{g(-3) - g(-5)}{-3 - (-5)}$ or $\dfrac{2 - 10}{2}$ or some other variation of this. For the justification point, students had to reference *both* continuity and differentiability of g. If a specific interval was referenced, it had to be used correctly. Saying that g is continuous on $(-5, -3)$, would not earn the second point because the function g has to be continuous on the closed interval in order for the MVT to apply. Alone, "differentiability on the closed interval" was not sufficient, as students had to also *specifically* reference continuity. Invoking any theorem other than the MVT did not earn the justification point.

Observations and recommendations for teachers:

(1) The AP Calculus Exam is one that requires students to show work. Students calculating a derivative should not jump to substituting and evaluating at a point in the domain. This makes it very difficult to verify that student work has appropriately calculated the derivative. Showing evidence of an appropriate chain rule calculation, which was required in more than one part of this problem, is essential in communicating good mathematical work. This is best done using function notation and not using values (perhaps "calculated in your head"). When reading student work, readers do not ever know what students were thinking. Readers only know what is presented on the page to be read. If there is any doubt or ambiguity in the student presentation of work, a point will not be awarded.

(2) EVERY derivative calculation requires use of the chain rule. Thus, $\frac{d}{dx}x^2 = 2x \bullet \frac{d}{dx}x = 2x \bullet 1 = 2x$. We as teachers do not emphasize this enough, partially because many applications of the chain rule only involve the number 1. When work has to be shown, the 1 is not necessary to show. But every other instance of the chain rule should be shown, best shown in terms of the variable and not the subsequent evaluation. This has been a hallmark of grading the AP Calculus Exam for many years, and lack of evidence of this work penalizes students on this exam.

(3) When issues arise on the AP Exam regarding improper use/lack of use of parentheses, the term for this at the reading is "presentation error." This may go back to something as simple as whether sin(x) should be presented as sin$x$, in the manner of many textbooks. We represent single variable functions as $f(x)$ or $g(x)$. There is a good argument for using parentheses for the argument of any function. Leaving these parentheses off created presentation problems for students, leading to the loss of points in part (a), for example. It is also impossible to interpret $\sin 2\pi - 2 + \cos \pi e^{\sin \pi}$ as anything other than involving a 2 that is subtracted. The derivative of $\sin(-2x)$, (using parentheses around the argument here) is $-2\cos(-2x)$. Perhaps by some habit of calculation, writing the chain rule result after the cosine function is common, and can result in ambiguity. It is just fine to write this derivative as $\cos(-2x) \bullet (-2)$ or $\cos(-2x)(-2)$ but not as $\cos(-2x)-2$ and certainly not as $\cos -2x-2$. Careful practice using parentheses is important and is probably enhanced by always using parentheses around the argument of a function.

(4) It is clear from the curriculum in effect as of 2017 that students will need to show more information in written work. The hypotheses of theorems must be referenced when invoking a theorem to justify a result. The detail in these hypotheses must be correct. For example, the MVT requires continuity on a closed interval and differentiability on an open interval. Mere mention of differentiability does not guarantee continuity on the closed interval. Specifically pointing out continuity on an open interval is incorrect.