Problem Overview

The student is given information about two particles, P and Q, over the interval $[0, 8]$. Also given is the position of P as $x_P(t) = \ln(t^2 - 2t + 10)$ and the velocity of Q as $v_Q(t) = t^2 - 8t + 15$, along with an initial position for Q: $x_Q(0) = 5$.

Part a

The student is asked to determine when P is moving left on $[0, 8]$.

Part b

The student is asked to determine when P and Q travel in the same direction on $[0, 8]$.

Part c

Once the student finds the value of the acceleration of Q, the student is requested to determine if Q's speed is increasing, decreasing, of neither at $t = 2$.

Part d

The student is asked to find the position of Q when it changes direction the first time.

Comments on Student Responses and Scoring Guidelines

Part a

The student earns one point by finding the derivative of x_P. However, the student who writes

$$\frac{1}{t^2 - 2t + 10}2t - 2$$

did not earn the derivative point due to the ambiguity of the lack of parentheses around $2t - 2$. This was considered a "presentation error"; however, the student remained eligible for the second point. If the student later clarified this presentation error—by writing $(2t - 2)/(t^2 - 2t + 10)$ for example—the student earned the point. To earn the second point, the student must have the correct interval in the presence of the correct derivative. Mistakenly using v_Q did not earn either point. An incorrect part (a) did not make the student ineligible for part (b).

Part b

To earn the first point, the student had to report the correct intervals in the presence of some analysis of the behavior of P and Q. To earn the analysis point, the student has to 1) address P and Q; 2) use $v_Q(t)$ with the roots of 3 and 5; 3) analyze the signs or values of $v_Q(t)$ with some conclusion about the behavior of Q; and 4) use P's interval from part (a). How this analysis was conducted by the student varied, but most used sign charts.

Part c

The student earned the first point only if the student declared $a_Q(2) = -4$. Incorrect values of $a_Q(2)$ did not disqualify the student from the second point as long as the speed answer was consistent with the incorrect value. The student did not earn the second point by saying that velocity and acceleration differ or if the students talked about the directions of the particles. To earn the second point, the student had to declare a negative acceleration and a positive velocity at $t = 2$.

Part d

The student earns the first point with the correct antiderivative. The student earns the second point by writing $5 + \int_0^3 v_Q(t)\,dt$ or by using the $x_Q(0) = 3$ to find the constant of integration. The student earns the third point by evlauating $x_Q(3)$, and can, of course, leave it as an unsimplified expression.

Observations and Recommendations for Teachers

(1) If you were surprised to see sign charts used in part (b) because you thought they were not allowed, you were under a false impression. Sign charts will not be read to justify extrema. In part (b), the student was not asked to justify extrema, but to determine directions. This is perfectly allowed. Most students used sign charts (some explained them in words – good for them), but some did not, and instead used tables of values. This was not acceptable unless their tables included the roots of v_P and v_P and values in each interval determined by the roots and the endpoints. Unlabeled sign charts did not earn the analysis point. Teachers should be careful to read and understand what is and what is not allowed in student work on the AP exam.

(2) Once again, in part (d), we have a problem which is an accumulation with an initial condition. See the comments on AB/BC-1 for more on this.