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Problem Overview:

When a store opens, it has 50 pounds of bananas on display. Bananas are being removed from the display at a rate modeled by $f(t) = 10 + (0.8t)\sin\left(\frac{t^3}{100}\right)$ for $0 < t \leq 12$, where $f(t)$ is measured in pounds per hour and t is the number of hours after the store opened. After the store has been open for three hours, store employees add bananas to the display at a rate modeled by $g(t) = 3 + 2.4\ln\left(t^2 + t\right)$ for $3 < t \leq 12$, where $g(t)$ is measured in pounds per hour and t is the number of hours after the store opened.

Part a:

Students were asked to calculate the number of pounds of bananas removed from the display during the first two hours the store was open.

Part b:

Students were asked to find $f'(7)$, and using correct units, explain the meaning of $f'(7)$ in the context of the problem.

Part c:

Students were asked whether the number of pounds of bananas on display was increasing or decreasing at time $t = 5$ and give a reason for the answer.

Part d:

Students had to determine how many pounds of bananas were on display at time $t = 8$.

Comments on student responses and scoring guidelines:

Part a:

Two points were available in part (a) for showing work with a definite integral of f and an evaluation correct to three digits after the point. An indefinite integral or an integral with incorrect limits could still earn the answer point but not the integral point. This was a calculator active problem and a few students were in degree mode. This resulted in an answer of 20.00089. Thus, readers had to determine if a student showing an answer of 20 had a decimal presentation error (not three correct digits) or a calculator degree mode error. For these errors, students were penalized only one point. Subsequent decimal errors or continued (mathematically correct) use of degree mode were not penalized.

Part b:

The value –8.12 or –8.120 or –8.119 had to be presented for the first point. The second point was for explaining the meaning of that number and had to include three things: (i) correct units (could be used in presenting the value); (ii) appeal to the time $t = 7$ with or without "hours"; and (iii) evidence in the wording of interpreting a rate of a rate in the context of the problem. Units had to be pounds per hour^2 and not bananas per hour^2. "When" or "at" $t = 7$ were acceptable, but "in" 7 hours or "after" 7 hours were not acceptable. The various ways in which students attempted to explain the rate of a rate made this part of the problem time consuming to read. The correct answer had to be explaining that f was decreasing. A student in degree mode found that $f'(7) \approx 0.19128$ and had to be indicating that f was increasing in order to earn the second point. (See **Observations and recommendations for teachers** #4 below.)

Part c:

In order to earn these two points, students had to correctly compare $f(5)$ and $g(5)$ and state what that comparison showed. The first point was for evidence of considering $f(5)$ and $g(5)$. Values could be shown as such evidence, but eligibility for the second point required that the values be correct. The second point was for pointing out the equivalent of $g(5) - f(5) < 0$ and stating that this meant decreasing.

Part d:

The calculation of the total number of pounds of bananas required using two definite integrals. These are $\int_3^8 g(t)\,dt$ and $\int_0^8 f(t)\,dt$. One point was awarded for each integral, and any computation errors came off the third point in this part of the problem. Omitting the dt created problems for some students as readers always assume a dt at the right of the given expression (assumed to be the integrand). The third point required the answer found by correctly using these definite integrals and the initial 50 pounds of bananas.

Observations and recommendations for teachers:

(1) Both parts (a) and (d) required knowledge of the concept that an integral of a rate of change calculates the amount of change over a time interval. Practice is needed so that students comfortably use the correct time interval(s) as limits on the integral. Also, an initial condition had to be used appropriately in part (d). See previous AP Exam questions 2011 AB/BC2 part (d) and 2016 AB/BC1.

(2) An integral of a rate over a time interval gives the amount of change. It is not a bad idea to practice this in both forms: $\int_{t_1}^{t_2} f(t)\,dt = \text{anti}\,f\left(t_2\right) - \text{anti}\,f\left(t_1\right)$ and $\text{anti}\,f\left(t_2\right) = \int_{t_1}^{t_2} f(t)\,dt + \text{anti}\,f\left(t_1\right)$. See previous AP Exam questions 2009 AB/BC2 part(c) and 2012 AB/BC1 part (d).

(3) Once again on the AP Calculus Exam, two functions are explicitly defined and then named without using a prime despite the fact that they are rates of change. This means that they are derivatives of something that is referenced in the problem. Practice with such given information is important. See previous AP Exam questions 2010 AB/BC1, 2015 AB/BC1 and 2016 AB/BC1.

(4) Explaining the meaning of the value of a derivative at a point in time is difficult in the context of a physical situation. Often this difficulty arises due to lack of sufficient information from the student such as the time interval properly referenced or the units properly stated verbally. A third necessity in this problem was correctly referring to f in words because f is what is changing in part (b). An added difficulty was that the derivative of f was the rate of change of a rate of change. Readers were wisely encouraged to look first for words that correctly described the function f.

CORRECT descriptions of f:
(i) The number of pounds of bananas being removed per hour…………(followed by "decreasing")
(ii) The rate of banana removal……….

INCORRECT descriptions of f:
(i) The number of pounds of bananas being removed………..(followed by "decreasing")
(ii) The rate of the rate of bananas being removed ………..(followed by "decreasing")

In writing such a description, the subject of the sentence should be clearly stated, followed by the observation (increasing or decreasing), followed by correct units (and a time interval, if required) as in "The rate at which bananas are being removed (f) is decreasing by 8.12 pounds per hour per hour."
In the units for the above description, the first "per hour" is in the units of f and the second "per hour" is needed because a second derivative with respect to time requires that.