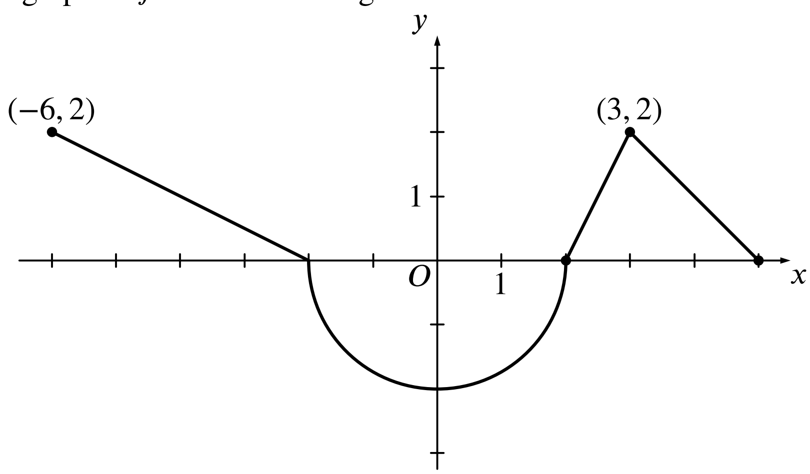
Problem Overview

The student is given the graph of f' on the interval $[-6, 5]$ and told that $f(-2) = 7$. Specifically, the student is told that the graph of f' is three line segments and a semicircle.



Part a

Students were asked to compute $f(-6)$ and $f(5)$.

Part b

Students were asked to determine the intervals where f is increasing, and to justify their answer.

Part c

Students were told to find and justify the absolute minimum of f.

Part d

Students were asked to compute the values of $f''(-5)$ and $f''(3)$, or explain why they don't exist.

Comments on Student Responses and Scoring Guidelines

Part a

Students earned the first point by using $f(-2) = 7$ towards the goal of computing the requested values. The addition of 7 to an attempt at a computation of $f(-6)$ was sufficient to earn the point. The values of $f(-6)$

and $f(5)$ required students to compute areas under the curve, with $f(-6)$ requiring students to consider the area accumulated from $x = -2$ to $x = -6$ as negative. Not considering this area as negative—writing $f(-6) = 7 + 4 = 11$ for example—earned the initial condition point, did not earn the value point, and the student was still eligible for $f(5)$. Writing $7 - 4 = 3$ and $10 - 2\pi$ with no other work earned all three points; however, writing only 3 and $10 - 2\pi$ earned two points (the initial condition point was not earned). Many students found the equations of the segments and semicircle and integrated them. If the student used $f(-2) = 7$ to determine the constant of integration, they earned the first point, and were eligible for the other two points.

Part b

This part required students to make the connection between a positive derivative and an increasing function. Students needed to say that f' is positive on the correct intervals, and so f is increasing on these intervals. Students writing ambiguously did not earn the justification point; i.e., "$[-6, -2)$ and $(2, 5)$ because the slope is positive" or "$[-6, -2)$ and $(2, 5)$ because the derivative is positive" and so on. Both intervals were required for the interval point. Any interval that included a portion of the graph of f' that is negative did not earn either point.

Part c

The consideration point was earned if the student had written somewhere $x = 2$ or $f(2)$ on their paper. The answer point was earned if the student declared $7 - 2\pi$ as the minimum and the student ruled out $f(5)$ as the minimum. If the student used their incorrect values of $f(-6)$ and $f(5)$ from part (a), the student could still earn the answer point as long as these values were greater than $7 - 2\pi$.

Part d

All the student had to write in order to earn the first point was $f''(-5) = -1/2$. The student did not have to write the difference quotient, but an unsimplified difference quotient still earned the first point. To earn the second point, the student had to declare that $f''(3)$ did not exist and provide a reason based on f'. Of course, the fact that the left- and right-hand derivatives of f' at $x = -3$ are not equal is the reason $f''(3)$ does not exist, and this could have been expressed in various equivalent ways to earn the second point.

Observations and Recommendations for Teachers

(1) The accumulation of area from an initial condition is a standard problem on the AP exam. Teachers should reinforce these kinds of problems with students in all manner of forms (algebraic, tabular, graphical). Some students had trouble interpreting the appropriate areas as negative. Some students also had difficulty

computing the areas of triangles and semicircles. This affected both parts (a) and (c). Again, these are standard AP exam problems, and should be practiced.

(2) Many students reported the intervals as "$[-6, -2) \cup (2, 5)$". There was debate over whether to accept this as the answer. The problem requested the intervals where f is increasing, but the union of disjoint intervals is not itself an interval. Technically, this answer is incorrect since it is not an interval. In the end, it was accepted as correct, but teachers should be careful with notation when teaching students how to do problems such as this one.

(3) A surprising number of students who did the work for part (c) correctly failed to declare the absolute minimum value, and did not earn the answer point. Simply listing the values in a table and leaving it at that is not enough. Students must be taught to answer the question being asked. Questions where the student is to find extrema of a function on a closed interval are common on the AP exam and do not require either the first or the second derviative tests. Students who used the first or second derivative tests correctly and also considered the endpoints earned both points (although this was not a common occurrance).

(4) Students attempted to justify why $f''(3)$ did not exist in all manner of ways. Certain descriptions of the nature of the graph of f' at $x = 3$ were accepted as correct (such as "cusp" and "sharp turn"), and others were not ("peak" and "vertex" and "not differentiable"). The take-away from this is that students should be taught to justify that a function is not differentiable at a point in only two ways: the function is not continuous at that point; or the limits of the derivative from the left and the right are not equal. Descriptions of the graph are not appropriate, as there is no way to tell which descriptions will be accepted and which will not year-to-year. A frequent student justification for why $f''(3)$ does not exist was that there is a point of inflection at $x = 3$. This is not correct and it completely befuddles this Reader as to why a student would think this.