PROBLEM OVERVIEW:

A company manufactures model rockets that require igniters to launch. Igniters cannot be reused and sometimes they fail to operate correctly so the rocket does not launch. The company estimates that the failure rate is 15%.

An engineer develops a new super igniter with the intent of lowering the failure rest. The following two step process will be used.

Step 1: One super igniter is selected at random and used in a rocket.

Step 2: If the rocket ignites, another super igniter is chosen at random and used in a rocket.

Step 2 is repeated until a super igniter fails to operate correctly or until 32 super igniters have successfully launched. Assume the super igniters are independent.

Part a:

Students were asked to find the probability that the first 30 super igniters successfully launch rockets given a 15% failure rate.

Part b:

Students were asked to determine the probability that the first failed super igniter occurred on the 31st launch if the failure rate was 15%.

Part c:

Students were asked to determine if it was reasonable that the failure rate was less than 15% given that the first 30 super igniters successfully launched rockets.

Intent of the Questions:

This question determined a student's ability to accurately a) compute a probability with probability rules or using geometric probability, b) recognize that for independent events, a probability calculation does not depend on the previous outcomes and c) use the results of previous calculations to make a reasonable determination about a probability claim.

Solution:

Part a

Students were required to give the correct probability with the correct justification.

Example: Given a failure rate of 15%, the probability of fail is 0.15 and does not fail (success) is 0.85. The probability that the first 30 rockets successfully launch would be $(.85)^{30} = 0.0076$ or $(1-.15)^{30} = 0.0076$.

Reader Notes:

1. Students could justify my using the multiplication ruler OR by defining X to be the trial with the first failure, and recognizes X has a geometric distribution and using that statement to say P(X > 30) OR defining X to be the number of failures for n = 30, p = .15 and indicating P(X=0), using probability rules or the binomial distribution with n = 30 and p = .15
2. Final arithmetic need not be carried out. Students could leave their answers as $(.85)^{30}$ or $(1-.15)^{30}$
3. A response that used the normal approximation to the binomial was acceptable if conditions were checked and it was noted that the conditions were not met so proceed with caution. (np > 5 and n(1-p)>5). This numerical answer was approximately 0.0107
4. Students that reversed the probability of success and failure could receive at most a P.
5. A student who calculated a geometric probability $P(X \geq 30)$ or $P(X< 30)$ could receive a P at most.
6. The following received no credit: normcdf(32,0.15,30).
7. Incomplete parameters for the binomial or geometric distribution could not get an E. It is not assumed that if a student writes calculator speak that they know what the parameters are.
8. An "E" was obtained if the student gave the correct probability and the correct justification.

Part b:

Correct answer: .2725

Justification: Since a failure did not happen on the first 30 rockets, and the probability of failure is independent, P(31 fails) = 0.15 P(32 fails) - .85*.15 = 0.1275, so P(failure on 31 or 32) = 0.15+0.1275 = .2725

Reader notes:

1. Students were allowed to define a geometric probability, use P(X=1 or X=2) = 1-P(X=0) to justify their answer. Parameters had to be stated and could not be assumed.
2. Students could also use the binomial probability, provided the parameters were given, and they wrote a probability statement P(X=1 or X=2).

Part c:

Students had to state that it was reasonable that the failure rate is less than 10% and justify their answer.

Using information already calculated in part a, students had to indicate that the probability that 30 super igniters in a row would not fail is very low, 0.0076. Since this amount is lower than any standard α =.05 or α =.01, it is reasonable to presume that the failure rate of the super igniters is less than15%.

Reader notes:

1. Justification based on the probability may be done by stating a significance level and comparing.
2. Justification may be based on the fact that 0.0076 is very small
3. Justification may be based on the expected number to fail(4.5) if p=0.15, n=30, and 0 failures is more than 2 standard deviations from the mean.
4. Responses that based their justification on the expected value without saying that 0 is far lower than what we expected and gives reason to doubt can only be a P. (SD not discussed).
5. If the probability from a is compared to 0.15, the part is lowered E→P or P→ I.

6. Context could be given by using the words igniters, launchers or rockets.
7. Making a wrong decision lost all credit

Observations:

1. There were lots of students performing binomial or geometric distributions with their calculator and all they wrote down was "calculator speak". This was not accepted, and graders made no assumptions if parameters were not defined. **This was a very common mistake!**
2. Students who were able to use basic probability did better on this question (students who knew that $P(X=30)= 0.85^{30}$.
3. Though some reasonable attempts were accepted for part b, many students had no idea how to deal with only the 31st and 32nd case.
4. Bald answers received a P at most.
5. Students who did two calculations for part b, but did not add them were knocked down one level.
6. Part b cannot be done as a binomial because the binomial accounts for a failure anywhere.
7. Context was a problem for students. A student could have the decision correct and tell why, but with no context, they could not receive an E.
8. Students who used the word "proves" for part c had their scores lowered by 1. (This proves the failure rate is less than 15%).
9. Some student list conditions that are wrong and there scores were lowered by 1.

Recommendations for Teachers:

1. Spend time working problems into the course that require students to think about probability rather than go to a formula.
2. When teaching binomial and geometric probability, ensure that students always write the parameters, as well as a probability statement and their answers.
3. Work on context. Have student's grade AP papers that have context so they can see the rubric and how an analysis section requires context as well as what is acceptable.
4. Spiral a question into tests after the sections have been studied to help students be more comfortable with probability.