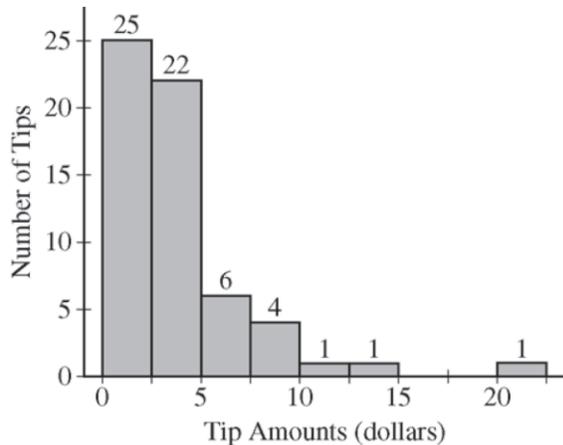


PROBLEM OVERVIEW:

An employee at a restaurant earns tips from customers. This employee recorded the 60 tips received on one particular day and the histogram reflects those tips.



Source: collegeboard.com

Part a:

Students were asked to write a few sentences to describe the distribution

Part b:

Students were asked to determine what effect a change of one tip from \$8 to \$18 would have on the mean and the median and told to justify their answers.

Intent of the Questions:

This question determined a student's ability to accurately a) describe a distribution of a quantitative variable using a histogram and b) determine the effect on the mean and median that would occur if one data value changes.

Solution:**Part a**

Students were required to comment on 5 components to get an "E". These were:

1. Shape of the histogram is skewed right. (or positively skewed, or skewed to large values)
2. Identify a possible outlier OR specifically discuss the gap (one amount higher than \$20, and the next highest is at most \$15). (They may say a probable outlier exists without stating a number).
3. Approximate the center with the median (between \$2.50 and \$5.00) or gives an approximate mean (between \$2.62 and \$5.13). Students had to tell what measurement of center they used.
4. Discuss variability in the tip amounts. They were allowed to state the minimum and maximum to satisfy this component or to say they vary from \$0 to about \$20 or \$22.50. They could also tell

where the majority of tips occurred. If a student said the majority of tips were between \$0 and \$5, that was sufficient for credit. They could also be more detailed in the majority explanation. They could also provide the range with numerical values. The following were accepted: Range [17.5,22.5]; IQR (0,5) (0 could not be included as a value). SD [2.5,5]. The standard deviation values should be an interval of about 2 – 3.

- Context --- students had to discuss the “tips” and not be generic in explanations. Clarity was necessary here. Specifically “amount of tips” could refer to the height of the histogram bars, so reference to the tip amounts had to be clear.

Reader Notes:

- Students who made two comments that contradict each other→ such as a reasonable comment followed by an unreasonable comment for any of the 5 sections lost credit for that section.
- If one or more major incorrect statistical statements were made that was not one of the 5 components, they lost credit for one component.
- All 5 parts correct earned an E.
- 3 or 4 parts correct earned a P.
- 1 or 2 parts correct earned an I.

Part b:

Students had to indicate that the median would not change because the median was between \$2.50 and \$5.00 and both \$8 and \$18 are greater than that. The mean goes up by $\$10/60$ or about 16 or 17 cents.

There were 4 components to this answer.

- Indicating the mean would increase.
- Using correct justification for the increase in mean.
 - States the mean would increase by $\$10/60$ or 16-17 cents
 - Indicates the balance point would increase/move to the right
 - Indicating that the mean is not resistant to extreme values or skewness and adding a value far away from the previous tip amount would add a number in the right tail, increase skewness and pull the mean in the direction of the skewness.
- Median would stay the same.
- Justification for the median required the student to indicate that \$8 is larger than the current median, so replacing it with \$18, another value greater than the median will not change the location of the median.

Reader notes:

- If students “added a new data value--\$18” instead of replacing a value, the following changes in grading occurred.
 - Correct justification for the mean increase would change to $\$10/61$. The other possible justifications for the change in mean are the same.
 - Median might change or stay the same.
 - Justification for the median must indicate with 61 values, the 31st value would be the median. It might be the same number in an ordered list of the data values, or it might be higher.
 - All four parts correct earned an E.
 - 3 or 4 parts correct earned a P.

Observations:

1. Students comment on how the mean and median would change but they do not justify their answers.
2. Variability was difficult for many students to explain.
3. Many students did not mention the gap or the outlier.
4. Context was often not complete. Work on clarity. If it is possible that the way an answer is written can be misconstrued, then it needs to be written a different way. In this question tip amounts was important.
5. A common problem was students not completely describing variability.
6. Students used statistical vocabulary incorrectly.
7. Students contradicted themselves.
8. Did not explicitly state that increasing one value in the tip amounts would increase the sum of the tips when discussing the change in the mean.

Recommendations for Teachers:

1. Use spiral teaching and revisit topics with emphasis on connection, comparison and contrast.
2. Help students clearly distinguish the range between two numbers and the estimation of IQR.
3. Give students ample practice making statistical arguments and giving critical feedback.
4. Give students more exposure to analyzing data with a data value missing or changed.
5. Discuss context---and what that means with respect to variables and a question.
6. Distinguish the ideas of changing 1 value, adding or removing a value and transforming all values.