2016 AB-2 Dennis Wilson, Landmark Christian School

**Problem Overview:**

The problem involved the motion of a particle along the $x$-axis. The students were given the velocity of the particle as $v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$. The students were also given the position of the particle at $t = 4$ as $x = 2$.

**Part a:**

Students were asked to determine if the particle is speeding up or slowing down at $t = 4$.

**Part b:**

Students were asked to find all the times on the interval $0 < t < 3$ when the particle changes direction and to justify their answer.

**Part c:**

Students were asked to find the position of the particle at time $t = 0$.

**Part d:**

Students were asked to calculate the total distance traveled by the particle from time $t = 0$ to time $t = 3$.

**General scoring guidelines for the problem:**

**Part a: (2 points)**

In order to receive any points, the student must have used both the velocity, $v(t)$, and the acceleration, $a(t) = v'(t)$. Students were not required to give numerical values for these quantities but if they did, the quantities should have been correct to at least one decimal place. Integer values for the quantities were not accepted. If the student did not give numerical values, then the student needed to state what the sign of each value is. Students had to include the correct conclusion in order to earn both points. Incorrect signs/values but a consistent conclusion could earn one point.

Common Errors – Students made reference to the position instead of the acceleration. Students did not reference any value other than velocity. Students differentiated the velocity by hand incorrectly (no chain rule).

**Part b: (2 point)**

The first point was earned for the value of $t$ correct to 3 decimal places. While no setup or mathematical work was required for this point, it was helpful to the reader to see $v(t) = 0$. The point for justification was given to any student with a correct value of $t$ and who states in some way that the velocity changes sign at that value. This could be as simple as stating that "the sign changes" or that "the graph crosses the $t$-axis". "$x$-axis" was read as "$t$-axis", assuming that students were referencing the $x$-axis on their calculator. A student with a decimal presentation error could still earn the point for the justification.

Common errors – Student found when $v'(t) = 0$. Students stated that the direction of the velocity changed instead of the sign. Students gave value of $t$ outside the given interval. If students gave the value inside the interval along with a second outside the interval, they were only awarded one point. Students who only gave values outside the interval earned no points.

**Part c: (3 points)**

The first point in this problem was awarded for the initial condition. While students earned this point in various ways, they usually earned the point with the Net Change Theorem or an expression of the Fundamental Theorem of Calculus.

$$2 \pm \int_4^0 v(t)\,dt \qquad\qquad \int_0^4 v(t)\,dt = x(4) - x(0)$$

The second point for this part was earned for a definite integral. The student must have the bounds of 0 and 4 in either order for this point. Errors in the order of the bounds were deducted in the answer point. If students left off the differential, then this point would still be earned in the presence of the correct answer.

The third and final point was awarded for the answer $-3.815$. There was no way for this point to be earned without this answer.

Common errors – Students reversed the order of the bounds of integration. Students mistakenly used $x$ as the independent variable. Students attempted to find an antiderivative for the velocity. The addition of the initial value led some students to link unequal quantities.

**Part d: (2 points)**

The first point for the integral could come one of two ways.
$\int_0^3 |v(t)|\,dt$ or $\int_0^A v(t)\,dt - \int_A^3 v(t)\,dt$ where $0 < A < 3$. The value of $A$ did not need to be the value where velocity changed sign in order to earn the point for the integral.

The second point was awarded for the answer. If the student imported an incorrect value for $A$ from part (b), then the student would still earn this point with a non-negative answer consistent with their incorrect value of $A$.

A student who performed the calculations in degree mode could still earn eight of the nine points if answer consistent with degree mode were given throughout the question.

Common errors – Students incorrectly placed the entire integral in the absolute value. Students added the initial value position to the correct value.

**Observations and recommendations for teachers:**

(1) Splitting the integral for distance on a calculator active question is a poor use of student time. The student should be encouraged to express the integral with the absolute value and then to calculate it directly using their calculator.

(2) The Net Change theorem should be taught in some form. This is helpful when no closed-form expression of the position can be found using an antiderivative. It is also helpful in preventing simple arithmetic errors arising from carelessness.

$$f(b) = f(a) + \int_a^b f'(x)\, dx$$

(3) A calculator active rectilinear motion question represents a chance for some easier points for the struggling student. The various formats of questions on this topic are narrow and extra time reviewing this topic could help borderline students improve their score.