Normal Curve Activity

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This is an activity that I adapted from a data set from a Paul Foerster problem about the lengths of babies born in a hospital. This activity is great for AP Calculus AB or BC students, AP Statistics students who have taken calculus, or Multivariable Calculus students who are studying double integrals. The babies' lengths are approximately normally distributed. Students are asked to find the mean and standard deviation of the data and then write the function for the normal curve that models this data.

The Normal ("Bell") Curve
adapted from a handout by Paul Foerster

The table below shows hypothetical data for the lengths of babies born at Scorpion Gulch Hospital. The lengths are grouped into classes (for example, babies born between 17 and 18 inches long are recorded as 17.5 inches).

Length	Number of Babies
13.5	1
14.5	1
15.5	3
16.5	9
17.5	11
18.5	22
19.5	17
20.5	20
21.5	8
22.5	5
23.5	2
24.5	1

1. Let x = length and y = number of babies. Enter the data into your calculator.

2. Plot a histogram on the graphing calculator. Identify your domain and range:

Domain:
Range:

3. Use the 1-Var Stat option in the Stat menu to find the total number of babies (n), the mean length ($\bar{x}$), and the standard deviation (σ):

$$n =$$
$$\bar{x} =$$
$$\sigma =$$

4. Use the normal curve to model the data, substituting for n, $\bar{x}$, and σ :

$$f(x) = \frac{n}{\sigma\sqrt{2pi}} e^{-(x-\bar{x})^2/(2\sigma^2)}$$

f(x) =

5. Using the first derivative test, find the value of x that gives the maximum value for the normal curve you found in #4. Verify on the calculator. What is the significance of this value?

6. Using the second derivative test, find the value(s) of x where the inflection point(s) occur. What is the significance of these values?

7. Using the general equation for the normal curve given in #4, use the first and second derivative tests to verify your answers to #5 and #6.

8. Find the area under the normal curve. (You may use a calculator).

9. Find the area under the normal curve one standard deviation from the mean. (You may use a calculator.)

Multivariable Calculus Extension:
How do you find the area under the curve algebraically?

The Normal ("Bell") Curve
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KEY

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Hospital. The lengths are grouped into classes (for example, babies born between 17 and
18 inches long are recorded as 17.5 inches).

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18.5	22
19.5	17
20.5	20
21.5	8
22.5	5
23.5	2
24.5	1

3. Let x = length and y = number of babies. Enter the data into your calculator.

4. Plot a histogram on the graphing calculator. Identify your domain and range:

Domain: (13, 25)
Range: (0, 23)

3. Use the 1-Var Stat option in the Stat menu to find the total number of babies (n), the
mean length ($\bar{x}$), and the standard deviation (σ):

n = 100
$\bar{x}$ =   19.2
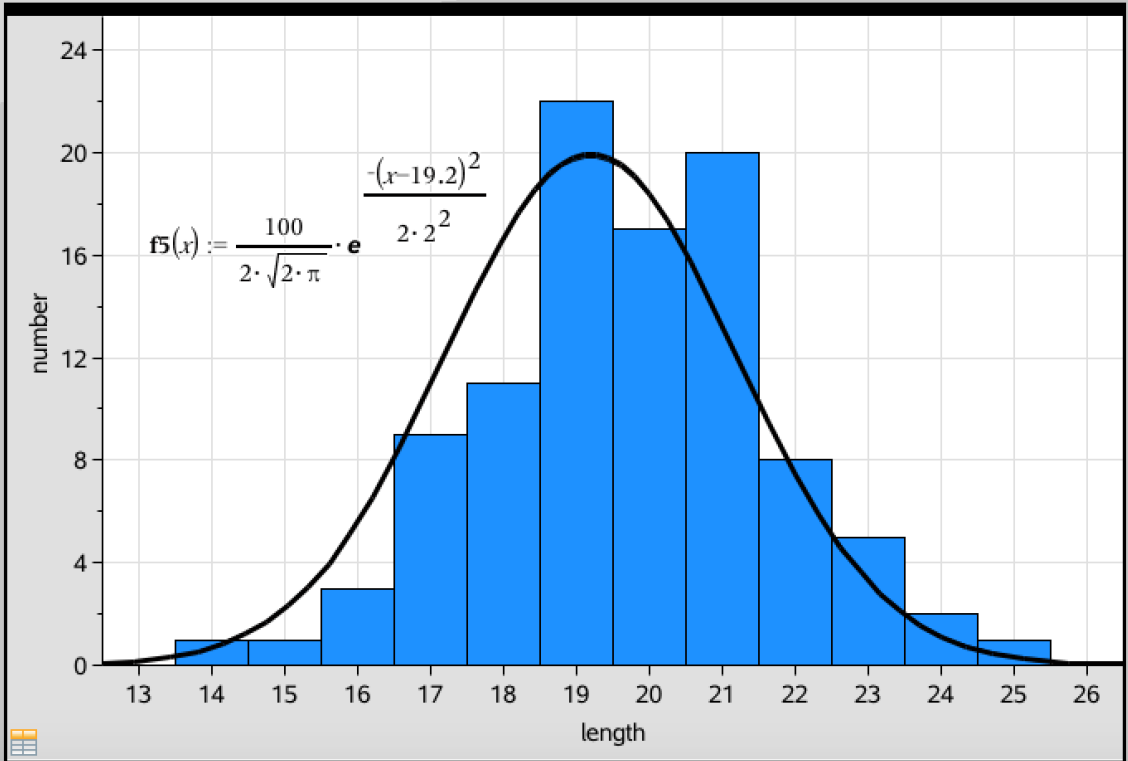σ =   2.01259

4. Use the normal curve to model the data, substituting for n, $\bar{x}$, and σ :

$$f(x) = \frac{n}{\sigma\sqrt{2pi}} e^{-(x-\bar{x})^2/(2\sigma^2)}$$

f(x) = see graph on next page

	A length	B number	C	D	E	F	G
=						=OneVar(	
1	13.5	1			Title	One−Va...	
2	14.5	1			$\bar{x}$	19.2	
3	15.5	3			Σx	1920.	
4	16.5	9			Σx^2	37265.	
5	17.5	11			sx := s$_{n-...}$	2.01259	
6	18.5	22			σx := σ$_{n...}$	2.0025	
7	19.5	17			n	100.	
8	20.5	20			MinX	13.5	
9	21.5	8			Q$_1$X	18.	
10	22.5	5			MedianX...	19.5	

$F1$ ="One−Variable Statistics"

$$f5(x) := \frac{100}{2 \cdot \sqrt{2 \cdot \pi}} \cdot e^{\frac{-(x-19.2)^2}{2 \cdot 2^2}}$$

10. Using the first derivative test, find the value of x that gives the maximum value for the normal curve you found in #4. Verify on the calculator. What is the significance of this value?

x = 19.2

The maximum value occurs at the mean.

11. Using the second derivative test, find the value(s) of x where the inflection point(s) occur. What is the significance of these values?

x = 17.2 and 21.2

The inflection points occur 1 standard deviation on either side of the mean.

12. Using the general equation for the normal curve given in #4, use the first and second derivative tests to verify your answers to #5 and #6.

The maximum value on the normal curve will occur when x is equal to the mean.
The inflection points on the bell curve will occur one standard deviation away from the mean.

13. Find the area under the normal curve.   (You may use a calculator).

100. 100% of the babies lengths will fall under the bell curve.

14. Find the area under the normal curve one standard deviation from the mean. (You may use a calculator.) What percent of the babies' lengths lie within 1 standard deviation of the mean?

Approximately 68% of the babies lengths should fall within one standard deviation on either side of the mean. Approx. 95% of lengths will fall within 2 standard deviations on either side of the mean. Aprrox. 99.7% of the lengths will fall within 3 standard deviations of the mean. This is the 68-95-99.7 Rule and it is true for all normally distributed data.

Multivariable Calculus Extension:
How do you find the area under the curve algebraically?

$$\left(\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-x^2/2}\,dx\right)\left(\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-y^2/2}\,dy\right)=$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(e^{-x^2/2})(e^{-y^2/2})\,dx\,dy=$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(e^{-(x^2+y^2)/2})\,dx\,dy=$$

$$\frac{1}{2\pi}\int_{0}^{\infty}\int_{0}^{2\pi}(e^{-r^2/2})r\,d\theta\,dr=$$

$$\frac{1}{2\pi}\int_{0}^{\infty}2\pi e^{-r^2/2}r\,dr=$$

$$1$$

$$Since\left(\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-x^2/2}\,dx\right)\left(\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-y^2/2}\,dy\right)=1,$$

$$\left(\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-x^2/2}\,dx\right)^2=1$$

$$\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-x^2/2}\,dx=1$$