Georgia Association of Advanced Placement Math Teachers

CSI: Calculus/Statistics Insider

Official Newsletter of the Georgia Association of Advanced Placement Math Teachers

Issue #24 *Published Semiannually* *February 2016*

Slate of GA²PMT Officers for 2015-2017

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This publication includes:

- ✓ Slate of **GA²PMT** Officers
- ✓ Letter from the President
- ✓ Commentary on AP Calculus AB/BC 2015 exam
- ✓ Commentary on AP Statistics 2015 Exam
- ✓ Key Statistical Phrases
- ✓ 2016 Summer Institute dates and locations
- ✓ Link to 2016 curriculum updates and exam format for AP Calculus AB/BC
- ✓ Link to request **GA²PMT** newsletter
- ✓ Mail-in Newsletter Membership Request

GA^{2}PMT
Georgia Association of Advanced Placement Math Teachers
Letter from the President

Welcome to the new and improved GAAPMT! I'm Debbie Kohler and I am the President of GAAPMT. Since this organization was created, we have sought to provide AP Mathematics teachers with the most up to date information about the AP exam, how grading occurs and ways to improve your practice.

The GAAPMT board has made some lofty goals for 2016 including reaching out to every county in the state to get information to AP mathematics teachers. The board is composed of AP Statistics and AP Calculus teachers and CollegeBoard exam readers. We will be producing two newsletters/year and adding helpful information to our website monthly. Our website is located at www.gaapmt.org. Past reading information, newsletters and plans for our annual meeting are located there.

If you would like to receive our newsletter, please complete the google form at http://goo.gl/forms/MbQUWbvQ1L . If you know someone in your county who would benefit from this information, please forward this email to them. Information will not be shared with other organizations or other parties and it will not be sold. The only purpose is to inform you of events related to AP mathematics. We plan to send no more than 1 update per month.

Although it is early in the year, I encourage you to plan to attend the Georgia Mathematics Conference at Rock Eagle October 19 – 21, 2016. Our GAAPMT mini-conference will be on Friday, October 21. We are currently working on guest speakers for AP Calculus and AP Statistics, but it will be worth your time to attend and be enriched!

If you have any questions, please feel free to email me @ dkohler1@kennesaw.edu. We look forward to hearing from you!

Dr. Debbie M. Kohler

AP Calculus

2015 QUESTION AB/BC1

Please view the questions here:

http://apcentral.collegeboard.com/apc/members/exam/exam_information/8031.html

The following overview is written by Marshall Ransom, Georgia Southern University.

> *Marshall Ransom has previously taught AP Calculus at the high school level. He has been teaching in the Department of Mathematical Sciences at Georgia Southern University since 2003 and is currently a Senior Lecturer in the department. Mr. Ransom has graded AP Calculus Exams since 1991 and has been a Table Leader at the AP Calculus reading since 1998.*

Problem Overview:

Rainwater flows into a drainpipe at a rate given by $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t measured in hours, and $0 \le t \le 8$. Water drains out of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8$. At time $t = 0$ there are 30 cubic feet of water in the pipe.

Part a:

Students were asked to calculate the number of cubic feet of rainwater that flow into the pipe for $0 \le t \le 8$.

Part b:

Students were asked to determine whether the amount of water in the drainpipe is increasing or decreasing at time $t = 3$ and give a reason for the answer.

Part c:

Students were asked to determine the time t for $0 \le t \le 8$ when the amount of water in the drainpipe is at a minimum and give a reason for the answer.

Part d:

Water continues to flow into and out of the pipe at the given rates for $t > 8$. The pipe can only hold 50 cubic feet of water before overflowing. Students were asked to write, but not solve, an equation involving one or more integrals to find the time w when the pipe begins to overflow.

Part a:

This is an accumulation of water that can be found using an integral of the rate of change in the water flowing into the pipe. A definite integral $\int_0^8 R(t)\,dt$ is sufficient because the rate given is positive on the given interval and describes water going into the pipe. An indefinite integral was sufficient to earn the first point. The limits on the integral were part of the second point, awarded in the presence of that definite integral and the correct answer.

Part b:

Students needed to compare $R(3)$ to $D(3)$ to answer this question. The first of the two points awarded in this part of the question was for convincing evidence that both of these values were being considered, with the $R(3)$ and $D(3)$ as such, or with numerical values appropriate for these functions at $t = 3$. The

answer for the second point had to be "decreasing" and include some reference to the relationship between $R(3)$ and $D(3)$ such as $R(3) < D(3)$. However, an answer with support such as $R < D$ and no other numerical values (for example, not even the 3 was shown) could only get the second point, not the first. Completely leaving out numerical values and the $t = 3$ was scored as though not enough work was shown by the student and earned at most one of the two points.

Part c:

Since this problem specifies a closed interval, students should have been looking for an absolute minimum on the interval $0 \le t \le 8$. Considering, and even finding, the value of t for which $R(t) - D(t) = 0$ was worth one point. "Consideration" of this could be shown by a sketch of the graphs of R and D intersecting or a graph of $R - D$ intersecting the x-axis or using equations such as $R = D$.

The value of $t = 3.271$ or 3.272 earned the second point. The absolute minimum justification should have shown values of the amount of water in the pipe at the endpoints, $t = 0$ and $t = 8$. Students talking about a change in sign from negative to positive for $R - D$, and not the endpoints, did not often earn the third point. This "relative minimum" argument had to correctly refer to signs over intervals that included values of $t = 0$ and $t = 8$ in their explanations. A few students continued to "talk" in their justifications, making it unclear that 3.271 or 3.272 was the correct (only) value of t and thus lost the last two points.

Part d:

The first of two points awarded in this part of the problem was for a definite integral. This had to be either in an equation or an expression with BOTH either 0 or 8 and a variable as limits AND $R - D$ or $D - R$ as an integrand. Despite a missing differential in an expression, this point was awarded unless of the ambiguous form $\int_0^x (R - D) + 30$. An integral such as $\int_0^x (R - D)$ earned the first point despite not using w. The second point was earned for an equation that would lead to a solution for w. This included the use of values related to the amount of water such as $\int_8^x (R - D)dt + 48.544 > 50$, an unusual way to use the amount of water at $t = 8$ in an equation. Students could earn this point provided they had earned the first point or had only a minor sign error in the setup of the integral.

Observations and recommendations for teachers:

(1) In order to show work in part (a) students needed to start with a definite integral of the rate of change of the water going into the pipe. The setup of a good integrand is worth a point, and that is not unusual on the AP Calculus exam. This can be more involved than in this problem, for example, in cases where the rate of change changes sign. Working with the difference between net change and total change (such as displacement $\int_{t_1}^{t_2} v(t)dt$ vs. total distance $\int_{t_1}^{t_2} |v(t)|dt$) should be practiced in a variety of contexts. Past AP Exam questions such as 2011 AB1 or 2012 AB6 provide examples.

(2) Part (b) is an example of a problem where considering, in writing, an appropriate relationship involving the necessary derivatives earns a point. However, some values need to be shown in order to

get full credit for a justification of the correct answer, either increasing or decreasing. See 2013 AB1 part (c) for an example.

(3) Part (d) is an example of not just locating an extreme value, but finding an absolute extreme value because we are looking at a closed interval. See 2013 AB1 part (d) for another example. If the endpoint and critical point(s) values are shown, the justification is complete without the difficulty of referring to a change in signs over appropriate intervals.

(4) Students should be required to provide the differential in setting up an integral to avoid any ambiguity. A problem such as part (d) is common on the AP Calculus Exam. See 2013 AB2 part (c) for another example.

Please visit the GAAPMT website, www.gaapmt.org for analyses of all six 2015 AP Calculus AB/BC free response questions.

AP Statistics

2015 QUESTION 1: Descriptive Analyses and Graphs
Please view the questions here:
http://media.collegeboard.com/digitalServices/pdf/ap/ap15_frq_statistics.pdf

The following overview is written by Debbie Kohler, Kennesaw State University.

> *Debbie Kohler taught AP Statistics for ten at the high school level in the Cherokee County School System and is currently teaching Elementary Statistics at Kennesaw State University. Dr. Kohler has graded AP Statistics exams since 2008 and is our current President of GA²PMT.*

Question 1: INTENT OF THE QUESTION:

This is a question whose main intent was to assess a student's ability to compare the features of two distributions of data from boxplots and identify statistical measures that are important in data and decision making.

Basic Idea of the Question: The starting salary for an entry-level accountant at two corporations was given, as well as a box and whisker plot of the salaries five years later and sample sizes.

Question 1 Answer:

a. Write a few sentences comparing the distributions of the yearly salaries at each of the companies.

i. **SOLUTION REQUIREMENTS:** Here students must be careful to actually compare the two distributions and not simply state information about each of the two companies. Students were expected to comment of four specific characteristics:

1. Center
2. Spread
3. Outliers
4. Context

ii. **GRADING:** Commenting on all 4 aspects correctly earned an "E" Commenting on 3 of the 4 aspects earned a "P" and anything less received an "I". If a student commented on shape, it was ignored because a box and whisker is not adequate to truly determine shape.

iii. **COMMON MISTAKES/NOTES:**

- Failure to actually compare the two distributions ---only stating information about each.
- Failure to describe center
- Failure to use context
- Working on the question, but failing to identify all the basics---SOCS—shape, outliers, center spread. Although shape was not required or graded, this acronym is very useful in assuring that the students get all the information that is needed (except context---which should be preached!)

b. The second part of the question focused on the salary being offered to someone as a starting salary. Suppose that both corporations offered you a job for $36,000 as an entry level position.

- **SOLUTION REQUIREMENTS**
  1. In part (b-1) a relevant statistical measure is identified or describe or a relevant statistical comparison is provided that supports the choice of Company A.
  2. In part (b-1) an explanation is given for why the measure or comparison is relevant.
  3. In part (b-2) a relevant statistical measure is identified or describe or a relevant statistical comparison is provided that supports the choice of Company B.
  4. In part (b-2) an explanation is given for why the measure or comparison is relevant.

- **GRADING:**
  E – all 4 components addressed
  P – 2 or 3 of the components addressed
  I - 1 or 0 components addressed
- **COMMON MISTAKES/NOTES**
  - Failing to provide a relevant statistical measure and not re-couping this in the explanation. If a student did not give a statistical measure, but the explanation for the comparison could logically lead one to a statistical measure, components the student could still get credit for #2 and #4 above. Example: "At Corporation A, I have the potential to earn a higher salary".
  - Failure to give relevance to the choice. It was not enough to simply give a statistical measure as to why one would choose each company. There must be a connection and relevance of the decision.

SCORING FOR QUESTION 1:

4: EE --- both parts essentially correct
3: EP or PE - one part essentially correct and one part partially correct
2: EI, IE, PP – one part essentially correct and the other incomplete or 2 partially corrects
1: PI or IP – one partially correct and on incorrect

Please visit the GAAPMT website, www.gaapmt.org for analyses of all six 2015 AP Statistics free response questions.

KEY STATISTICAL WORDS AND PHRASES

This is just an attempt to provide some basic statistical vocabulary review. An Advanced Placement Statistics test taker should be familiar with the words and phrases listed below. A number following the word or phrase refers to a page in the Yates, Moore, McCabe text where the word or phrase is referenced.

categorical variable (7)
quantitative variable (7)
bar chart ... as contrasted to **histogram** (7)
outlier (12)
symmetric, **skewed right**, **skewed left** (15)
stemplot (18), **back-to-back stemplot** (20)
mean, **median**, **mode**, **quartiles, five-number summary** (Section 1.2)
standard deviation (43)...remember we have both σ and s. (Page 43 shows s).
variance (43)
density curve (68)
68-95-99.7 rule (75)
percentile (75)
standardized value, or **z-value** (83)
response variable, **explanatory variable** (109)
scatterplot, **positive association**, **negative association** (Section 3.1)
association does not imply causation.
correlation, **coefficient of correlation r** (129)
least-squares regression line (139... understand figures on pages 139, 140)
relation of slope of LSRL and r (144)
r^2 interpretation (149)
residual (151)
influential observation vs. **outlier** (157)
logarithmic transformations (Section 4.1)
extrapolation (207)

lurking variable (208)
causation (211)
common response (211)
confounding (211)... also (247)
Simpson's paradox (224)
types of sampling, bias in sampling (Chapter 5)

- convenience sample
- simple random sample (SRS)
- stratified random sample
- multistage sample design
- voluntary response sample

problems in sampling (257)

- undercoverage
- nonresponse
- response bias
- wording of questions

observation vs. **experiment** (268)
principles of experimental design (276)

- control
- randomization
- replication

statistically significant (276)
double-blind experiment (278)
blocking, **block design** (281)

- blocking is done to reduce variation
- stratification is done to achieve representation

independence, **independent trials** (315, 332, 399)
disjoint events (342)
random variable (368)
discrete random variable, **continuous** random variable (376)
law of large numbers (390)
binomial setting, **binomial distribution** (416)
geometric setting, **geometric distribution** (435)
parameter, **statistic** (456)
unbiased statistic (464)
normal for binomial OK if np and n(1-p) are at least 10. (475)
Central Limit Theorem (488)
confidence interval, **margin of error** (511)
null hypothesis, **alternate hypothesis** (532)
P-value (534)
one-tail vs. **two tail** (538)

Type I Error (569)
Reject Ho when it is true. (A Type I error can only occur if the null hypothesis is true.)
Type II Error (569)
Accept Ho when it is false (A Type II error can only occur when null hypothesis is false.)
Power of a test (574)
1 - probability (Type II error). It is the probability that a false null is correctly rejected.
standard error (587)
t-distribution (588) .... approaches normal as sample size increases
Used for relatively small samples when population standard deviation not known.
degrees of freedom (588)
t used in matched pair design (599)
comparing two sample means (t-distribution) (619)
check distribution of samples to see if use of t OK.
inference for proportions (section 12.1)
normal approximation OK if certain conditions are satisfied. (662)
inference for difference of proportions (679)
normal approximation OK if certain conditions are satisfied. (681)
Chi-Square... goodness of fit (Section 13.1)
inference for two-way tables (717)
conditions for use of Chi-Square (734)
confidence interval for regression slope (762)

2016 CollegeBoard Summer Institutes in Georgia

DATE	AP CALCULUS AB	AP CALCULUS BC	AP STATISTICS
June 6	**Woodward Academy** College Park	**Woodward Academy** College Park	**Woodward Academy** College Park
June 13	**Marist School** Atlanta		
June 19	**University of GA** Athens	**University of GA** Athens	
June 20	**Walton High School** Marietta	**Marist School** Atlanta **Walton High School** Marietta	**Walton High School** Marietta
June 27	**Marist School** Atlanta		
July 5			**Woodward Academy** College Park
July10	**University of GA** Athens		**University of GA** Athens
July 11	**Kennesaw State University** Kennesaw		
July 18	**Woodward Academy** College Park		**Kennesaw State University** Kennesaw **Woodward Academy** College Park

Link to more information about summer institutes:
http://apcentral.collegeboard.com/InstitutesAndWorkshops

Link to information about AP Calculus AB/BC 2016 curriculum changes:
https://advancesinap.collegeboard.org/stem/calculus

Link to receive newsletter: **http://goo.gl/forms/MbQUWbvQ1L**

GA²PMT Membership Request

The benefits of belonging to this organization can make a difference in your students' scores on the AP Exams.

Becoming a member of a professional organization is one of the indicators listed in the Georgia Teacher Keys Effectiveness System (TKES).

Performance Standard #9- Professionalism and Communication: The teacher exhibits a commitment to professional ethics and the school's mission, participates in professional growth opportunities to support student learning, and contributes to the profession.

To join GA²PMT, you may enter your information online using the following web address: **http://goo.gl/forms/MbQUWbvQ1L** or simply scan the QR code.
You may also complete the information below and mail it in with your membership dues, which are $10 per year.

GA²PMT Membership Form
(Oct. 1, 2016 – Sept. 30, 2017)

Name:_____ **School:**_____ **Address:**_____ **City:**_____**State**_____**Zip**_____ **Email:**_____ **Member Status: New**_____ **Renewal**_____	**AP Certification:** ☐ **AP Calculus AB** ☐ **AP Calculus BC** ☐ **AP Statistics**

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