AP Statistics Exam 2015
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Problem 3

The goal of this question was to assess the student's ability to (1) perform a probability calculation from a discrete random variable; (2) calculate the expected value of a discrete random variable; (3) perform a conditional probability calculation from a discrete random variable; and (4) use probabilistic thinking to make a prediction about how an expected value will change given partial information about the random variable.

Probability (a) and Expected Value (b) Calculations

In order to earn full credit, students had to show compute the probability in part (a) and the expected value in part (b) correctly.

Students were generally successful on both parts of this question.

The most common errors were related to notation ($\bar{x}$ vs E(X) or μ_X). Notation errors resulted dropped rubric score from E to P or P to I in each part.

Conditional Probability Calculation (c)

In order to earn full credit, students were required to correctly compute the conditional probability with work shown. Student work had to include correct numerical values for both the numerator and the denominator. If ONE of the numerator or denominator included an incorrect value, the rubric score dropped from E to P or P to I.

Common errors were related to notation and incorrect calculation of the numerator for the conditional probability formula. Students needed to understand that P(X = 3 and X $\geq$ 1) is the same as P(X = 3).

Probabilistic Thinking (d)

In order to earn full credit, students must have correctly answered "greater than" with appropriate justification. Justification could have included any of the following;

- Correct response with work shown: This was rare, as conditional expected value is not a course topic.
- Reasonable explanation based on the fact that with X = 0 eliminated, the probabilities for X = 1, X = 2, and X = 3 will ALL increase. Students had to be very explicit with this explanation.
- Reasonable explanation based on the fact that with X = 0 eliminated, the balance point of the distribution increases.

Students were generally ***unsuccessful*** in answering this question correctly. Even if they responded with the correct answer ("greater than"), many students were unable to successfully defend their choices.

Pedagogical Suggestions

1. Notation, Notation, Notation! Stress the difference between $\bar{x}$ and μ_X (or E(X)).
2. Conditional probability: Work with students to help them conceptually understand the idea conditional probability.
    a. "Given" provides the contextual clue to the condition and the "out of" group for the probability calculation to be placed in the denominator.
    b. The numerator is an "AND" statement. Help students with the idea of intersection of sets. Number sets work well (odd/even, prime/composite) as do die roll outcomes.
3. Probabilistic thinking: Challenge students by offering critical thinking questions. "What happens if…?" Keep in mind that P(S) = 1, where S represents the sample space. If one outcome is no longer possible, the sum of the probabilities of all other outcomes must equal one.