2015 AB/BC3 Marshall Ransom, Georgia Southern University

Problem Overview:
Johanna is jogging along a straight path for $0 \le t \le 40$ where t is in minutes. Johanna's velocity is given by a differentiable function $v(t)$, measured in meters per minute, and some values of v are given in the table below.

t minutes	0	12	20	24	40
$v(t)$ meters per minute	0	200	240	–220	150

Part a:

Students are asked to estimate $v'(16)$ using data in the table.

Part b:

Students are asked to explain the meaning of the definite integral $\int_0^{40} |v(t)|\,dt$ in the context of the problem.

Then students had to estimate the value of $\int_0^{40} |v(t)|\,dt$ using a right Riemann sum with the four subintervals indicated in the table.

Part c:

We get to meet Bob, who is riding his bicycle along the same path as Johanna. For $0 \le t \le 10$, Bob's velocity is given by $B(t) = t^3 - 6t^2 + 300$ where t is in minutes and the velocity is in meters per minute. Students are asked to find Bob's acceleration at time $t = 5$.

Part d:

Based on the model B from part (c), students are asked to find Bob's average velocity during the interval $0 \le t \le 10$.

Part a:

For one point, students had to use the interval between $t = 12$ and $t = 20$. Also, a difference, a quotient, and evidence of use of values from the table had to be shown. Some answers that were acceptable are $\frac{240-200}{20-12}$, $\frac{40}{20-12}$, and $\frac{v(20)-v(12)}{8} = 5$. An unacceptable answer would be something like 5 with no supporting work or $\frac{v(20)-v(12)}{20-12}$, the latter not showing a final answer 5, thus not showing use of values from the table. Units, not required, had to be correct if included.

Part b:

For one point, an explanation needed to include the concept of total distance, the units (meters or m), and the time interval. Readers needed to be convinced that "total distance" was expressed and not some form of "displacement." The time interval could not be expressed as "the whole time" or "over the interval," but could be expressed as on [0, 40] or "in the 40 minutes" or "while Johanna jogged." Meters or m could appear in the approximation work for the Riemann sum. A point was awarded for setting up the Riemann sum. This could be done using the four terms and adding, showing scratch work calculating those areas and then the final sum, or in a most simplistic manner using 2400 + 1920 + 880 + 2400, which would also earn the approximation (answer) point. As usual, students did not need to add the terms and present a simplified final approximation for the value of the definite integral. Students who set up the sum using 4 interval widths and 4 function values got the Riemann sum point if 7 of these 8 were correct. An example of this would be (12)200 + (8)240 + (4)200 + (6)150 where the student has written an incorrect 6 in the last term instead of a 16.

Part c:

For the first point, there had to be evidence of the use of $B'(5)$ or $B' = 3t^2 - 12t$. For both points a student could write "$a(t) = 3t^2 - 12t$" followed by "$a(5) = 15$." Some answers that earned only the first point are $B'(5) = 75 - 60$ (no supporting work) and $B'(t) = 3t^2 - 6t \rightarrow B'(5) = 3(25) - 6(5)$, (an incorrect derivative, but an attempt at the derivative AND using the 5).

Part d:

The average value of a function over an interval can be calculated using a definite integral over that interval and dividing by the length of that interval. A correct definite integral using limits 0 and 10 earns the first point. An indefinite integral with evidence of a late use of the 10 also earns this first point. Because this is not a calculator active question, the antiderivative work must be shown. If the integrand is a trinomial, and 2 of the 3 terms were correct, an antiderivative point and an answer point consistent with the work could be earned. The 1/10 was considered to be part of the third, answer, point.

Observations and recommendations for teachers:

(1) In order to show work in part (a) and in any question with values from a table, students need to show that values from the table are used. Some students started with a correct setup of $\frac{v(20)-v(12)}{20-12}$ followed by an incorrect answer or incorrect values or copy errors from the table, thus making it clear as readers graded exams that a correct setup alone is not sufficient. The setup alone is only correct if already showing correct table values. The AP test grading has been consistent over the years, only allowing use of the smallest interval containing the required value of t when calculating this type of estimation.

(2) In part (b) an explanation requires the following: WHAT is being measured, the units, and the time interval. It is difficult to predict what on the exam constitutes a satisfactory explanation, but this should be practiced in the classroom to include important aspects of the context of a problem. A general comment for teachers: it is no longer simple enough to calculate and say that, for example, "the slope is 2." Such calculations should be practiced as often as possible in the context of a real world situation, and the meaning of the answer should be required and discussed.

(3) Student work in part (c) made it clear that many students know that acceleration is the second derivative of a position function. However, in this part, students were given a velocity without a prime. The search for two primes took some students out of points in this part. Use of a derivative and the value of t earned students a point. Merely declaring "$a(t)$" in the work was not sufficient to indicate work with the acceleration in the context of this problem. There had to be much correct work or a solid connection to B'.

(4) Part (d) should remind us that students usually must do analytic work calculating antiderivatives or derivatives in a non-calculator problem. The setup is an important skill being tested, but so is the symbolic calculation. On the AB exam, this doesn't often involve much more than a power rule, use of an exponential rule with base e, a simple form of an inverse trig function, or at most a very straight-forward u-substitution. It should go without saying (although there are those old-fashioned folks out there who think that calculator use precludes this) that students do indeed need to know basic rules for both derivative and antiderivative calculations.