**Problem Overview:**

Grass clippings are decomposing in a bin over a 30 day period. The amount of grass clippings remaining in the bin, in pounds, is given by $A(t) = 6.687(0.931)^t$, $0 \le t \le 30$ and $t$ is in days.

In part (a), students were asked to find the average rate of change of $A(t)$ over the interval $0 \le t \le 30$ and indicate the units of measure. In part (b) students were asked for the value of $A'(15)$ and, using correct units, an interpretation of this value in the context of the problem. Part (c) asked for the time when the actual amount of grass clippings in the bin was the same as the average amount over the interval $0 \le t \le 30$. It was stated in part (d) that for $t > 30$, $L(t)$, the linear approximation to $A$ at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Students were asked to find the time at which there will be exactly 0.5 pounds of grass clippings remaining.

**Part a:**

The solution involves the slope of a secant line between the two points $\left(0,\ A(0)\right)$ and $\left(30,\ A(30)\right)$. This slope $\frac{A(30)-A(0)}{30-0}$ required students to use their calculator in this calculator active question. The value $-0.197$ (or $-0.196$) could be presented without much supporting work. But to earn the one point awarded for this part, the units had to be lbs/day or an acceptable, sometimes strangely, abbreviated equivalent.

**Part b:**

For the first point the value $A'(15) = -0.164$ (or $-0.163$) had to be presented. The second point in part (b) was for the interpretation. The interpretation required words describing three things related to the grass clippings: (i) "decreasing" or "shrinking" at a rate…. (ii) the units lbs/day…. and (iii) "at time 15 days." Such words as "over the 15$^{\text{th}}$ day" or "during the 15$^{\text{th}}$ day" were not acceptable. Readers were given a variety of samples of student responses that sounded true, but were not quite completely correct, as well as some that sounded cryptic but did indeed explain completely the meaning of $A'(15)$. An incorrect negative answer was eligible for the second, interpretation, point. Students had to be "interpreting" a negative value.

**Part c:**

In order to answer this question, students had to calculate $\frac{1}{30}\int_0^{30} A(t)\,dt$, the presence of which was awarded the first point. Solving for $t$ in this calculator active problem involved student uses of a calculator that were not shown. "Mathematical musings" on the student papers had to show an equation such as

$$A(t) = \frac{1}{30}\int_0^{30} A(t)\,dt \text{ or } 6.687(0.931)^t = \frac{1}{30}\int_0^{30} A(t)\,dt \text{ or } A(t) = 2.7526 \text{ or } A(t) - 2.753 = 0.$$

In the presence of such an equation, the correct answer was awarded the second point in part (c).

**Part d:**

For one point, readers needed to see in student work the attempt to set $L(t) = 0.5$. For two points, students had to present an expression for $L(t) = A(30) + A'(30)(t-30)$. This was expressed in various forms such as $L(t) = 0.782 + -0.055(x-30)$ or $y = f'(30)(x-30) + f(30)$. Such answers were awarded the two points for expressing $L(t)$. Answers somewhat incomplete that were awarded only one of these two points might have looked like $L(t) = A(30) + m(t-30)$ or $y = A'(30)t$. Readers were supplied with other examples and shown student responses that only earned one of the two points for an expression for $L(t)$. The fourth point in this part (d) was for the answer $t = 35.054$.

**Observations and recommendations for teachers:**

(1) An average rate of change of a function *f* on an interval [a, b] is the slope of the secant line $\frac{f(b)-f(a)}{b-a}$. Slopes are rates of change, especially applicable in a real-world context.

(2) Explaining the meaning of a number "in the context of the problem" involves describing the entity (in this case the grass clippings) and what is happening and when it is happening and including correct units. This needs to be practiced. Practice can start with, for example, explaining the meaning of a slope in a simple linear equation model. Past AP exams contain such problems. For example, see 2010 AB2 part (b) and 2012 AB1 parts (a) and (b). A point can be awarded for this explanation even in the presence of an incorrect number, as long as that number is explained correctly.

(3) In a calculator active question, buttons will be pushed that don't have to be described to readers. On the other hand, it is wise to show a setup such as $\frac{1}{30}\int_0^{30} A(t)\,dt$ in part (c). The setup is often awarded one point.

(4) The average value of a function $\frac{1}{b-a}\int_a^b f(t)\,dt$ is an important use of a definite integral, and students should know this well. Also, the distinction between the average value of a function and the average rate of change should be clear to students.

(5) A linear approximation to a function at a point $x = a$ is a simple $f(a) + f'(a)(x-a)$. This often gives good approximations to $f(a \pm \delta)$ if $\delta$ is small. In a class of students moving on to more calculus or the BC curriculum, it does no harm to mention that this is a first order Taylor polynomial centered at $a$.