2018 BC-1 Dennis Wilson, Landmark Christian School

Problem Overview:
Students were given a function $r(t)$ which models the rate people enter a line for an escalator. Students were also told that people leave the line at a constant rate of 0.7 person per second and that at time $t = 0$ there are 20 people in the line.

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & 0 \le t \le 300 \\ 0 & t > 300 \end{cases}$$

Part a:
Students were asked to determine the number of people who enter the line for the escalator in the first 300 seconds.

Part b:
Students were told that the line for the escalator is never empty during the first 300 seconds. They were asked to find the number of people in line at time $t = 300$.

Part c:
Students were asked to find the first time, $t > 300$, for which there are no people in line for the escalator.

Part d:
Students were asked to find the time in the first 300 seconds at which the number of people in line was a minimum, justifying their answer. Students were to find the number of people in line at this time to the nearest whole number.

General scoring guidelines for the problems:

Part a: (2 points)

The first of two points for this part was for the integral. This integrand could be written as $r(t)$ or as $44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7$ but the integral must have the correct limits to earn the point. The point could be earned with a missing differential but only in certain situations. The reader was to assume that the differential occurred at the end of the expression. Therefore, if anything was added to $r(t)$ in the absence of the differential, this first point was not earned.

The second point was earned for the answer of 270. No bald answers were accepted so the integral must be present to earn the second point. An incorrect answer from part (a) could be imported into later parts, as long as the answer was greater than 190. This lower bound on the import prevents the line from containing a negative number of people.

Part b: **(2 points)**

Students earned the first of two points in this part for considering the rate out. Evidence of this consideration could come in many forms which include some of the following:

$$-0.7t \qquad \int_0^{300} -0.7\,dt$$

$$-.7(300) \qquad -210$$

Overall, the point was earned for the product of rate and time provided the rate was negative. Students could also earn this point by simply writing "The number of people leaving the line is 210."

Omitted differentials were scored with the same philosophy as part (a). If a student wrote an expression such as

$$20 + \int_0^{300} r(t) - 0.7t$$

then the point for the rate was not earned because the reader was to assume the differential went at the end of the expression.

The second point was for the answer of 80. Students were not eligible for this point without considering the rate out. No bald answers were accepted. The minimal answer earning both points for this part was simply

$$290 - 210$$

Students importing an incorrect answer above the lower bound of 190 could earn both points for this part. They simply needed to consider the rate out and arrive at a value which was 190 less than their incorrect import.

Part c: **(1 points)**

The lone point for this part was for the rounded answer 414.286 or the truncated answer 414.285. A bald answer was accepted in this part. A student's answers needed to be rounded or truncated to at least 3 decimal places. A point was deducted if the student did not do this. These students gained immunity for further decimal presentation errors in part (d).

Part d: **(4 points)**

The first point for part (d) was earned for considering when the rate of people entering the line was equal to the rate of people leaving the line. Students usually earned this point with an equation such as $r(t) = 0.7$. Some students defined a function involving an integral for the number of people in line.

$$n(t) = 20 + \int_0^{t} (r(x) - 0.7)\,dx$$

If these students then expressed $n'(t) = 0$, they earned the first point. Inequalities did not earn this first point unless one of the two times when $r(t) = 0.7$ was also given.

The second point came for the identification of the time $t \approx 33.013$. This value was not required to be presented with 3 decimal places to earn the point. A bald 33 written in the section for part (d) would earn this second point.

The third point came for the correct answer for the time presented with three decimal accuracy and the minimum numbers of people rounded to the nearest whole numbers. Either 4 or 3 were accepted as the minimum number of people. If the number of people was not rounded, this point was not earned.

The fourth point was for the justification of the answer and was the most difficult for students to earn. Students could only earn this point with a correct answer. Poorly presented (not rounded appropriately) correct answers could still earn this point.

A candidate table was the most common way for students to earn this point. The table alone was not sufficient to earn the point for justification though. Students also had to show work involving an integral to demonstrate how the number of people at the relative extrema were calculated. The values in the candidate table did not need to be presented with 3 decimal places.

Students were able to use the first derivative test to show why $t \approx 158.070$ was ruled out as an absolute minimum and not included in the candidate table.

Observations and recommendations for teachers:

(1) An incorrect answer of 56,700 appeared for part (a) on many student papers. It was discovered that this value would occur when parentheses were improperly placed in their calculator.

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fnInt(44(X/100)^3(1-(X/300
)^7),X,0,300)
                          56700
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It is unfortunate for students to lose the answer point for something small like this when the integral point had been earned for the correct expression on the paper. It is even more unfortunate because most calculators will present mathematics in some form of pretty print that prevents this mistake. While there are only 4 functions required for the graphing calculator, other good practices should be encouraged when instructing the use of technology.

(2) Missing differentials seemed to be an epidemic at times. While this might seem like a case of poor notational fluency, it is more a case of poor comprehension of the integral. Students should be instructed on the meaning of the differential and understand its necessity in the integral. While the reader may have instructions to assume a missing differential, students should not have that option in class or learn that they have that chance on the exam.

(3) Students should consider if their answers make sense in the context of the problem. 56,700 seems like an incredibly large number of people to be waiting to ride an escalator. While this was a large answer, it was far from being the largest. Also, many students reported that there was a negative number of people in line for part (b) despite being told otherwise by the question.

(4) Students need to be careful to remember the rule for rounding or truncating to three decimal places. It seems that the fact that the answers to part (a) and (b) were whole numbers led students to forget to give their answer for part (c) with three decimal places. They should also trust and follow rounding instructions given in the questions. Students presented the minimum number of people to three decimal places despite the instructions to round to the nearest whole number.

(5) It should be remarked that both parts (b) and (c) could be completed without any knowledge of calculus. Many students overthought these problems. Some students indicated the rate of people leaving the line to be proportional to the number of people in it. Some students did not think enough. In part (c), they found the amount of time after $t = 300$ for the line to empty but failed to add this value to 300. There is a good opportunity to develop vertical instruction with members of your department using the ideas in this problem to teach math in context. For example:

> *At a time* $t = 300$ *seconds, it is known that there are 80 people in line for an escalator. Write a linear function modeling the number of people in line for* $t > 300$ *if people are leaving the line at a rate of 0.7 person per second.*