Problem Overview:

A particle moves on the x-axis with a velocity given by $v(t) = \dfrac{10\sin\left(0.4t^2\right)}{t^2 - t + 3}$ for $0 \le t \le 3.5$. It is also given that the particle is at position $x = -5$ when $t = 0$.

Part a:

Students were asked to find the acceleration of the particle when $t = 3$.

Part b:

Students were asked to find the position of the particle when $t = 3$.

Part c:

Students were asked to evaluate both $\int_0^{3.5} v(t)\,dt$ and $\int_0^{3.5} |v(t)|\,dt$ and interpret the meaning of each integral in the context of the problem.

Part d:

A second particle is moving on the x-axis with position given by $x_2(t) = t^2 - t$ for $0 \le t \le 3.5$. Students were asked to find the time t when the two particles are moving with the same velocity.

Comments on student responses and scoring guidelines:

Part a:

This part was worth one point for the answer $v'(3) = -2.118$. The numerical answer had to be connected to $v'(t)$. Thus an answer such as -2.118 or $a(3) = -2.118$ did not earn this point (see **Observations and recommendations for teachers:** (1) below). A number of students computed v' symbolically despite the fact that this was unnecessary since this is a calculator active question. The symbolic calculation had to be done correctly if linked to the answer using an equal sign. Some students made a good connection between their answers and $v'(t)$ but gave an answer of 0.00769 that results from having a calculator in degree mode. In this case the point was not awarded, but students were not penalized for degree mode answers in subsequent parts of the problem once they had lost one point for this type of error.

Part b:

Three points were awarded in this part of the problem, one each for presenting the definite integral $\int_0^3 v(t)\,dt$, using the initial condition, and the answer. Thus $\int_0^3 v(t)\,dt - 5 = -1.76$ was awarded all three points. An answer of the form $\int_0^3 v(t)\ -5 = -1.76$ was scored 0 – 1 – 1, not getting the integral point because of the missing dt. An indefinite integral form such as $-5 + \int v(t)\,dt = -1.76$ was scored 0 – 0 – 1 because the integral point is not awarded, and it cannot be known if the initial condition is used appropriately unless a lower limit of 0 is seen on the integral. Versions of student work were seen using an upper limit of 3.5. A solution such as $\int_0^{3.5} v(t)\,dt - 5 = -1.76$ or -2.156 was scored 0 – 1 – 1 because the 3.5 was assumed to be a copy error. The first answer is correct, the student using 3 in the calculator, and the second answer is consistent with the copy error being both shown on the paper and used in the calculator.

Part c:

Two correct numbers attached to the integrals earned the first of three points in this part of the problem. The remaining two points were awarded for interpretations of the meanings of these integrals. Interpretations needed to include a reference to both $t = 0$ and $t = 3.5$ (see **Observations and recommendations for teachers:** (3) below). Students were not penalized twice if a correct reference to time was omitted in both interpretations. Students had to refer to $\int_0^{3.5} v(t)\,dt$ as displacement and $\int_0^{3.5} |v(t)|\,dt$ as the total distance traveled (see **Observations and recommendations for teachers:** (4) below). Therefore an interpretation such as "$\int_0^{3.5} v(t)\,dt$ is the displacement of the particle's motion and $\int_0^{3.5} |v(t)|\,dt$ is the total distance traveled by the particle" earned one of these two points, the missing reference to time only penalized once.

Part d:

The first point was awarded for $v(t) = x_2{}'(t)$ or $v(t) = 2t - 1$. Confusion resulting from the subscript 2 lost students this point if they introduced x_1 or v_1 without clearly defining those functions in a manner connecting them appropriately to $v(t)$ (see **Observations and recommendations for teachers:** (5) below). In other words, student work showing $v_1(t) = x_2{}'(t)$ did not earn the first point without information connecting v_1 or $x_2{}'$ to $v(t)$. Student work showing a correct answer and showing $2t - 1$ somewhere in the work could earn the answer point even without earning the first point for a good connection to $v(t)$. Of course, the explicit expression for $v(t)$ could be used in place of $v(t)$.

Observations and recommendations for teachers:

(1) It is not assumed in the presence of $v(t)$ as a velocity that $a(t)$ is the derivative of $v(t)$ and is therefore the acceleration. Students must show work. In this case of a calculator active question, all that needs to be shown is something like $v'(3)$ or $v'(t) = a(t)$ or $\frac{d}{dt}\left(v(t)\right)\Big|_{t=3}$. This communicates the task the calculator is performing, using good mathematical notation. Calculating the acceleration symbolically is absolutely not necessary in a calculator active question. This can easily lead to minor errors that cause students to lose this point.

(2) A missing dt as in $\int_0^3 v(t) - 5 = -1.76$ was scored $0 - 1 - 1$. But a student writing $\int_0^3 v(t) - 5$ was awarded zero points, the missing dt not interpreted by readers as located in the appropriate place without the presence of the correct answer. Students should be taught to include the differential when showing an integral.

(3) Interpretations of integrals in which the limits are values of time should say "on the interval $0 \le t \le 3.5$" thereby naming both endpoints. Students who tried to put this in words often did so improperly. For example, "during the first 3.5 units of time" was accepted but "from start to end" and "over 3.5 units of time" were not. "For time between $t = 0$ and $t = 3.5$" also was acceptable. Message: name both endpoints.

(4) Definite integrals of velocity and the absolute value of velocity are classically referred to as "displacement" and "total distance traveled" respectively. These basic terms should be taught and expected in student work. Attempts to use wordy descriptions of these concepts often fell short. For example, "displacement" refers to the signed distance between starting and ending points of particle motion along a line. Any reference to position does not describe displacement unless fairly involved, clarifying language is also used. Often, students did not describe these two concepts correctly when they launched into more elaborate language. An example of an incorrect statement in the absence of the word "displacement": since $\int_0^{3.5} v(t)\,dt \approx 2.844$, a number of students wrote that this means the particle moved about 2.844 units to the right over the interval $0 \le t \le 3.5$. In fact, the particle moved to the right until $v(t) = 0$ at $t \approx 2.802$ for a total motion to the right of about $\int_0^{2.802} v(t)\,dt \approx 3.29$ units.

(5) The introduction of x_2 into the part d of the problem without any information about either x_1 or v_1 caused difficulties for students who used these latter functions in their work. In using either x_1 or v_1, student work introduced functions with new names not already stated in the problem. This requires from the student a connection of the new functions to something specific. Readers will not assume a correct description of an introduced function; students have to describe any introduced function. Some students wrote $v_1 = v$ and usually did well in part d of the problem if a correct answer was shown. Readers may well have known exactly what students were trying to do, but could not award the first point in the absence of an explicit description of a named function if the name was not included in the original problem.