How would these responses be scored on the AP Calculus Exam?

This is 2017 AB6 part (d), and sample "possible" student responses are analyzed.

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The function g is differentiable. Some values of g and g' are shown in the table below. Is there a number c in the closed interval $[-5, -3]$ such that $g'(c) = -4$? Justify your answer. This question is worth 2 points.

x	$g(x)$	$g'(x)$
–5	10	–3
–4	5	–1
–3	2	4
–2	3	1
–1	1	–2
0	0	–3

Solution:

g is differentiable $\Rightarrow g$ is continuous on $[-5, -3]$.

$$\frac{g(-3)-g(-5)}{-3-(-5)} = \frac{2-10}{2} = -4$$

Therefore by MVT there is a value c, $-5 < c < -3$, such that $g'(c) = -4$.

Scoring:

1 – $\dfrac{g(-3)-g(-5)}{-3-(-5)}$

1 – Justification using MVT

Possible scores:

1 – 1, 0 – 0, 1 – 0, 0 – 1

Possible Responses | **Score??**

1. $\dfrac{g(-3)-g(-5)}{-3-(-5)} = \dfrac{2-10}{2} = -4$

because g is differentiable on $[-5, -3]$,
there is a c with $-5 < c < -3$ by MVT

? – ?

2. Since $-5 < -4 < -3$ and g is continuous because it is differentiable, then by IVT there exists such a c.

? – ?

3. $\dfrac{2-10}{2}$; g differentiable $\Rightarrow g$ continuous

So yes, there is such a value for c

? – ?

4. $\frac{-10}{2} = -4$ and g is differentiable and therefore ? – ?
continuous on $[-5, -3]$, so yes there is such a c by MVT.

5. $\frac{g(-3) - g(-5)}{-3-(-5)} = \frac{2-10}{2} = -4$

Because g is differentiable everywhere we also ? – ?
know that g is continuous on $(-5, -3)$ so there
is such a value c by the Mean Value Theorem.

6. $\frac{g(-3) - g(-5)}{-3-(-5)} = \frac{2-10}{2} = -8$

g is differentiable and therefore continuous on ? – ?
$[-5, -3]$, so yes there is such a c by MVT.

7. $\frac{g(-3) - g(-5)}{-3-(-5)}$

g differentiable $\Rightarrow$ g continuous, YES ? – ?

8. $g(-3) - g(-5) / \left(-3-(-5)\right) = -4$

g differentiable $\Rightarrow$ g continuous, YES ? – ?

Correct scoring of possible student responses:

Response #1: score 1 – 0
The difference quotient is correctly presented for the first point. However, the justification does not include any reference to continuity, one of the hypotheses of the MVT.

Response #2: score 0 – 0
The first point is not awarded because the difference quotient is not presented. No point is awarded for justification with reference to any theorem other than MVT. Note another mathematical error in the justification: this response uses the value –4 and compares it to values of x in the domain of g rather than realizing that $-4 = g'(c)$ and comparing that –4 to g' or the slope of the appropriate secant line.

Response #3: score 1 – 1
For the first point, the student has shown a "minimally" correct difference quotient because it uses correct values from the table and shows *both* a difference and a quotient. The student also earns the second point because both hypotheses of the MVT are stated correctly (the theorem does not have to be named if the hypotheses are invoked correctly).

Response #4: score 0 – 0
The work presented offers a quotient, but not a difference and is unacceptable for the first point. While the justification is stated correctly, in the absence of correct work to justify, this response does not earn the second point. This is referred to as "eligibility" for the second point, and this response is not eligible for that point.

Response #5: score 1 – 0
The correct difference quotient is shown, earning the first point. While the justification refers correctly to differentiability, the reference to continuity is made to an open interval, not to a closed interval as required in the hypothesis of the Mean Value Theorem. The second point is not awarded.

Response #6: score 1 – 0
The difference quotient $\dfrac{g(-3)-g(-5)}{-3-(-5)}$ is correct and earns the first point. The incorrect evaluation of this quotient makes the student ineligible for the second, justification, point (even though the justification is correctly stated!). An incorrect answer/value cannot be justified.

Response #7: score 1 – 1
This is a "minimally" completely correct response. According to the scoring guidelines, the presence of the correct difference quotient, evaluated or not, earns the first point. The justification includes correct reference to both hypotheses of the MVT and the answer to the question. The MVT does not have to be named in the presence of correct reference to all the hypotheses.

Response #8: score 0 – 1
The attempt at the difference quotient is incorrect because of missing parentheses in the numerator. Such an incorrect difference quotient would make the student ineligible for the second point except in the case shown where the correct value –4 is also shown. This is referred to as a "presentation error" that costs the student the first point but allows eligibility for the justification point because the correct –4 indicates the student is working correctly. The justification in this response is correct and does earn the second point.

Note that on the AP Calculus Exam, work must be shown. It is not uncommon for students to show "presentation errors" and subsequently have difficulties earning additional points. Students should practice in such areas as the following:

Example #1: $$\frac{d}{dx}\left(\frac{8x}{x^2-x}\right)=\frac{x^2-x\bullet 8-8x(2x-1)}{\left(x^2-x\right)^2}$$

This example shows a "presentation error" because of missing parentheses around the x^2-x term.

Example #2: $$\frac{d}{dx}\left(\frac{\cos(x)}{e^x+5}\right)=\frac{\left(e^x+5\right)-\sin(x)-e^x\cos(x)}{\left(e^x+5\right)^2}$$

This example shows a "presentation error" because of the ambiguity implied by the "$-\sin(x)$" …..
is "$-\sin(x)$" subtracted from e^x+5 or is this multiplied?

Example #3: $$\int_1^2 5x+420=420+\frac{5}{2}x^2\Big|_1^2 \text{ vs. } \int_1^2 5x+420=\frac{5}{2}x^2+420x\Big|_1^2 \text{ vs. } \int_1^2 \left(5x+420\right)dx=\frac{5}{2}x^2+420x\Big|_1^2$$

This example shows ambiguity because of the missing *dx* which apparently is applied only to the 5*x* term in the first case. If not so expressed, correctly using *dx*, this can lead to ineligibility for the student regarding subsequent work or even for a point for showing this integral. Students should be taught to use the differential *dx* appropriately, and with parentheses around the integrand, as in the third example above.