A Differential Equation

Marshall Ransom, Georgia Southern University mransom@georgiasouthern.edu

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Separate variables: $e^y dy = xdx$ *GOOD!* 1 pt. for separation

Antiderivatives: $e^y = \frac{x^2}{2}$ *GOOD!* 1 or 2 pts. for antiderivatives

Attempt #1: $y = \ln\left(\frac{x^2}{2}\right)$ *INCORRECT!*........ no use of +*C*…….

No more points possible

Attempt #2: $e^y = \frac{x^2}{2} + C \rightarrow e^0 = \frac{4^2}{2} + C$ *GOOD!* 1 pt. for + *C* **and** using (4, 0)

$C = e^0 - \frac{4^2}{2}$ *NO MORE Pts. YET…………*

$e^y = \frac{x^2}{2} + e^0 - \frac{4^2}{2}$ *NO MORE Pts. YET…………*

$y = \ln\left(\frac{x^2}{2} + e^0 - \frac{4^2}{2}\right)$ *PERFECT!..* .1 more point

The domain of the function is all possible values of the independent variable; in this case x is that variable. This often involves *excluding* values not allowed such as those that require division by zero, that require a square root of a negative number, or, in this case, because of ln we *require* that $\frac{x^2}{2} + e^0 - \frac{4^2}{2} > 0$. Solving for x we get $x^2 > 14 \rightarrow x < -\sqrt{14}$ or $x > \sqrt{14}$. However, the solution to a differential equation with initial condition requires that the value of x for that initial condition point be in the domain. We must choose $x > \sqrt{14}$ for the domain since $x = 4$ is in that interval. NOTE that this is a lot of work solving for y for one point and a lot of work and thinking for the domain. I recommend postponing such work until all the easier questions on the exam have been answered.