2017 BC-5 Dennis Wilson, Landmark Christian School

<u>Problem Overview</u>:

The problem involved the rational function $f(x) = \frac{3}{2x^2-7x+5}$.

<u>Part a:</u>

Students were asked to find the slope of the line tangent to the graph of f at $x = 3$.

<u>Part b:</u>

Students were asked to find each critical point of f on the interval $1 < x < 2.5$. They were then asked to classify each location as a relative minimum, relative maximum, or neither with justification for their answer.

<u>Part c:</u>

Students were given the partial fraction decomposition of the function f and asked to use it to evaluate the improper integral $\int_5^\infty f(x)\,dx$.

<u>Part d:</u>

Students were asked to determine whether the series $\sum_{n=5}^\infty f(n)$ converges or diverges. The students were told to state the condition used for determining convergence or divergence.

<u>General scoring guidelines for the problems:</u>

<u>Part a:</u> (2 points)

Students earned 2 points on this part for a correct numerical expression of $f'(3)$. There were several scenarios for which a student would only earn one of these points. If students incorrectly substituted $x = 3$ into their expression, they only earned one point. Some students incorrectly differentiated $f(x)$ for various reasons, usually algebra errors or carelessness. For certain eligible forms of incorrect $f'(x)$ with correctly substituted $x = 3$, students earned one point. Eligible forms of incorrect $f'(x)$ were:

$$f'(x) = \frac{ax+b}{(2x^2-7x+5)^2}, a \neq 0 \text{ or } f'(x) = \frac{-3(4x-7)}{2x^2-7x+5}$$

Some students gave the equation for the line tangent to $f(x)$ at $x = 3$. If students presented this equation with a correct slope calculated outside of the equation, the student could earn both points. If the student placed the equation of the tangent line in a box or circled it though, they only received one point.

Common Mistakes: Students unfortunately lost points for incorrect arithmetic. This is unfortunate as they lost points despite correct calculus and they lost valuable time performing calculations that are not required on the AP exam. Students lost points for linkage errors. This is when $f'(x)$ is set equal to $f'(3)$ during the substitution.

<u>Part b:</u> (2 points)

The first point was earned for correctly identifying $x = \frac{7}{4}$ as the x-coordinate of the critical point. This point could also be earned by students who imported an acceptable incorrect form of $f'(x)$ from part (a).

The second point was earned for classifying the critical point as a relative maximum and properly justifying the answer. To receive the point, students must specifically refer to $f'(x)$ changing sign from positive to negative. Students could not simply state "the derivative", or "the slope", or "it" changes from positive to negative. Students could also justify using the second derivative test if the second derivative was explicitly calculated. Students could get this point with an incorrect but importable form of $f'(x)$ if their critical point was in the given interval and their justification was correct.

Common Mistake: Students referred to $f'(x)$ vaguely instead of explicitly. Some students incorrectly phrased information by confusing $f(x)$ and $f'(x)$.

<u>Part c:</u> (3 points)

The first two points were awarded for the antiderivative and the limit expression respectively. To earn the antiderivative point, both anitderivatives must be correct. To earn the limit expression point, the limit must be attached to a definite integral or an antiderivative. Late or wandering limits did not prevent students from earning this point. Due to the complexity of the calculations involved, linkage errors were not deducted. To earn the third and final point for this part, students must give the correct answer and also must have earned the limit expression point. The only copy error permitted was changing subtraction to addition in the integrand.

Common Mistakes: Students did not deal properly with the antiderivative of $\frac{2}{2x-5}$ because the derivative of the denominator was not equal to one. Students struggled with evaluating the limit because of the indeterminate difference $(\infty - \infty)$ from the natural logarithms, failing to use log properties to achieve an indeterminate ratio (∞/∞).

<u>Part d:</u> (2 points)

Students could determine convergence of the series using either the integral test or a comparison test. In either case, the conditions for the test must be stated to earn the points.

Integral Test: Student must state that the series is positive, continuous and decreasing. The student did not have to prove this and the reader was to ignore any efforts made at such proof. The student should have said that this is true for all $n \geq 5$, but "all n" was read as "all *our* n".

Student could reference their work in part (c), but their conclusion should be consistent with the result they reached in that part. Students could earn one of two points using the integral test by clearly stating the conditions for which the test applies but incorrectly executing the test.

Limit Comparison Test: To earn any points, students must clearly state that the terms of the given series are positive. Once again, the student should have said that this is true for all $n \geq 5$, but "all n" was read as "all *our* n". Students must then properly execute the limit comparison test This included a correct limit set up, a correct limit value, and a correct conclusion. Most students chose $\sum \frac{1}{n^2}$ for their comparison. To earn both points students must state that this series converges. Students who state $\frac{1}{n^2}$ converges, only earned 1 point.

Direct Comparison Test: Students who attempted this test were rare. There were many difficulties faced by students in choosing an appropriate series and communicating the results of the test. Few students tried Direct Comparison and even fewer were able to earn the points.

Common Mistakes: Students failed to state the conditions for the test despite being instructed to do so in the problem. Some students did not state all the conditions for the Integral Test. A fairly large number of students attempted to use ratio test. Poor communication was the key reason students with good calculus work lost points though.

Observations and recommendations for teachers:

1. Students should make sure they only answer the question that is asked. It is unfortunate when students perform extra unnecessary work. It takes up valuable time that could be applied on other questions. It also qualifies the student to lose points for circling an answer that was not requested as in part (a).

2. Students should be ready to justify extrema using words and correct explicit expressions for the derivatives. Students should be strongly discouraged in their use of pronouns or the word derivative without a reference. A good calculus student should be able to write sentences using words and calculus notations.

3. Students should be taught to correctly link expressions with variables with those which contain substituted values. Teachers should encourage their colleagues in previous classes to model the usage of a vertical bar for evaluation.

$$f'(3) = \left.\frac{-3(4x-7)}{(2x^2-7x+5)^2}\right|_{x=3} = \frac{-3(4(3)-7)}{(2(3)^2-7(3)+5)^2}$$

4. While all teachers want their students to have good arithmetic skills, the use of arithmetic for simplification should be deemphasized when reviewing for the exam. Too many good calculus students lose points for arithmetic errors. It is doubtful that these students are good at calculus and poor at arithmetic. On the contrary, good arithmetic students are spending too much time on complex arithmetic, and due to expiring time, are simply making careless

mistakes. The reader can easily recognize the unsimplified answer and award the answer point.

5. Communication was a key point of emphasis for the reading of this exam. Students will use good communication in their solutions only if their teachers model it themselves and require it on assessments. Teachers should dedicate some time on every concept for communication, especially for series.

6. Students should understand the conditions for which each convergence test is used. Understanding these conditions will help student to understand which tests can be applied. Students should also develop a sense for when the ratio test is best applied.