CSI:
Calculus/Statistics Insider

Georgia Association of Advanced Placement Math Teachers

Official Newsletter of the Georgia Association of Advanced Placement Math Teachers

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This publication includes:

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- ✓ John Neff Award
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GA²PMT
Georgia Association of Advanced Placement Math Teachers
Letter from the President

This school year seems to have flown by! It is already March, so most of us AP teachers are looking at our plans and trying to make certain that we accomplish all of our standards and leave some time for review prior to the students' taking the AP Exam. We know you are very, very busy, but we would like to offer you a few resources to aid in your goals of preparing your students for their AP examination.

In this newsletter you will find an analysis of the first question from the AP Calculus AB/BC exam and from the AP Statistics exam. These analyses were written by teachers who attended last year's reading and spend many, many hours grading last year's exam. They have tried to give you some advice on common mistakes that students make nationwide and possible ways to help improve these issues. Only the first question is included in the newsletter. The remaining questions may be found on our website, www.gaapmt.org

This year we will be electing officers for a two year term, and we have several vacancies. If you are interested in working on the newsletter, helping with the John Neff Award and being an officer please email me at dkohler1@kennesaw.edu. We typically meet once per year in person and twice through a conference call. We are an all-volunteer organization, so unfortunately no money is involved!

Our website is constantly changing, but it requires that you go within your field to see the changes. We have been adding articles and best practices for you to use in your classroom. Though you may not have seen this before, you may find an activity to use for review, or one you would like to try out next year!

We are already planning our annual meeting that will be held at Rock Eagle next October. If you have specific requests for what you would like our guest speakers to feature in their presentations, please send me an email at dkohler1@kennesaw.edu. We will work together to plan the best conference for you that we can. Remember, when you come to the annual meeting and join/renew your GAAPMT membership, lunch is on us that Friday!

If you would like to receive our newsletter, and you have not completed the form yet, please visit http://goo.gl/forms/MbQUWbvQ1L . If you know someone in your county or another colleague who would benefit from this information, please forward this email to them. Information will not be shared with other organizations or other parties and it will not be sold. The only purpose is to inform you of events related to AP mathematics. We plan to send no more than 1 update per month.

If you have any questions, please feel free to email me @ dkohler1@kennesaw.edu. We look forward to hearing from you!

Dr. Debbie M. Kohler

AP Calculus
2016 QUESTION AB/BC1
Please view the questions here:
http://apcentral.collegeboard.com/apc/members/exam/exam_information/232050.html

Problem Overview:

Water is pumped into a tank at a rate given by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \le t \le 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour. It is given that R is differentiable and decreasing on $0 \le t \le 8$. Some values of R are shown in the table below. It is also given that at time $t = 0$ there are 50,000 liters of water in the tank.

t (hours)	0	1	3	6	8
$R(t)$ (liters/hour)	1340	1190	950	740	700

Part a:

Students were asked to estimate $R'(2)$, showing the work that leads to the answer and indicating units of measure.

Part b:

Students were asked to estimate the total amount of water removed from the tank during the 8 hours using a left Riemann sum with four subintervals indicated by the table. Students were also asked if this was an over or under estimate of the water removed and to give a reason for their answer.

Part c:

Students were asked to find an estimate of the total amount of the water in the tank at the end of 8 hours, to the nearest liter, using the answer from part (b),

Part d:

It was asked whether there was a time for $0 \le t \le 8$ when the rate at which water is pumped into the tank is the same as the rate at which it is removed, and to explain why or why not.

Comments on student responses and scoring guidelines:

Part a:

In calculating this estimate, the interval $[1, 3]$ had to be used, and both a difference and quotient had to be seen in order to earn the first point. Thus, $\dfrac{R(3) - R(1)}{3 - 1} = -120$ or $\dfrac{950 - 1190}{2}$ earned the first point. The expression $\dfrac{R(3) - R(1)}{3 - 1}$ without the final answer was not acceptable. However, the final answer connected to this expression was considered evidence that values were correctly pulled from the table. The units point was earned only in the presence of an answer and had to be a correct version of L/hr/hr.

Part b:

There are four intervals in the table, and the widths of these intervals had to be shown in demonstrating an attempt at a left Riemann sum. The first point was for this demonstration and had to show a sum of products using 8 numbers, at least 7 of which had to be correct. If only 7 were correct, students did not earn the second point for showing the actual Riemann sum value, but were eligible for the third point in this part of the problem. Thus, $1340 + 2(1190) + 3(950) + 2(740)$ earned the first two points, the students not needing to simplify this answer and provide the number 8050. The expression $1340 + 2380 + 2850 + 1480$ does not show the multiplication by the correct interval widths, not enough work shown, and this earns only the second point, but not the first. Even in the presence of a right Riemann sum of 6710, students were eligible for the third point. The sum is always an overestimate when using a left sum for a decreasing function.

Part c:

The integral giving an estimate of the water removed from the tank was estimated in part (b). To get into this part of the problem for the first point, students had to show evidence of trying to calculate $\int_0^8 W(t)\,dt$.

The integral value is approximately 7836 liters. For the second point, students had to show appropriate use of this value, along with 50,000 and the estimate from part (b). Thus, use of an incorrect value imported from part (b) allowed students to earn the second point. Readers checked the arithmetic for this point.

Part d:

A nice way to deal with this part of the problem is to consider the function $f(t) = W(t) - R(t)$, noting that at the point where $t = 0$ this is greater than 0 and noting also that $W(8) - R(8) < 0$. Combining this information with the fact that $W(t) - R(t)$ is continuous or that IVT applies to this function verifies that at some point t on the interval $(0, 8)$ $W(t) - R(t) = 0$. Not many students used this approach. Students who appeared to understand what was happening here often looked at $W(t)$ and $R(t)$ separately at the endpoints. These students had to identify the facts that $W(0) > R(0)$ and $W(8) < R(8)$ as well as the fact that BOTH these functions were continuous (or IVT applied) in order to earn both points. Consideration of the endpoint values did earn the first point, but most students using this approach to the problem did not earn both points in part (d). Students who appealed to the average value

$\dfrac{1}{8}\displaystyle\int_0^8 W(t)\,dt \approx 979.52$ often noted, correctly, that this value is between $R(0)$ and $R(8)$. But since the

time at which this average value is attained may not be the time at which the functions are equal, this argument earned zero points.

Observations and recommendations for teachers:

(1) This problem is a version of finding information about a total amount, given rates of input and output as well as an initial condition, the 50,000 liters. Examples of this type of problem are common on AP Calculus exams. Refer to 2010 AB/BC1, 2013 AB/BC1 or 2015 AB/BC1.

(2) The estimate of a derivative of a function, given tabular data values for the function, is common on AP Calculus exams. This estimate should be done by calculating the slope of a secant line using the narrowest interval in the table that includes the point in question, in this case in part (a) where $t = 2$. To show work, values from the table must be used in a rational expression for this secant line slope, showing some evidence of subtracting those values as well as the quotient. Although the numerical answer is easy to determine without pencil and paper, work must be shown on the AP Calculus exam.

(3) Knowing how to calculate a left, right, or mid-point Riemann sum is a requirement in the AB/BC curriculum. To show work, students need to show not just arithmetic results, but the use of interval widths as well as summing the products of those widths with appropriate function values. A reminder: Do Not Simplify arithmetic!! Some students had a nice setup, but made arithmetic mistakes in simplifying.

(4) Right and left Riemann sums are predictably either over- or under-estimates depending upon the function being increasing or decreasing. This should be illustrated and practiced in the classroom.

(5) Finding a total amount (as in part (c)) involves the initial amount, the amount removed, and the amount put in. These three quantities are found in different ways. But integrals of rates of change do calculate accumulated amounts. Since this was a calculator active problem, the amount of water that

went into the tank is found using a definite integral functionality of a calculator. The setup, using correct mathematical notation, must be shown so that readers can see where this amount comes from. The point that students are awarded is for seeing that correct notation, rather than seeing the actual correct numerical amount of water. An error there comes off the final answer point.

(6) Asking about an intermediate value, as in part (d), requires students to invoke the IVT or the condition of continuity and to show two function values that as an interval contain the value in question. Part (d) was particularly difficult for students because looking at R and W separately required a more complicated argument. When two quantities are supposed to be equal at some point, it is not a bad idea to check the meaning of $R - W = 0$, as in this problem, and look at the implications of that equation. In particular, that equation gives us a new function to consider, $R - W$, with its own properties. Not many students looked at this function. A message to teachers might be to look at that form of any equation where it is desired to solve for x or t or whatever when $A = B$.

(7) An example of (6) above is provided by considering two functions that have the same derivative as in
$f' = g'$. If we form the function $h = f - g$, we see that $h'(x) = f'(x) - g'(x) = 0$. By the MVT we can easily prove that if $h'(x) = 0$, then $h(x) = C$, a constant. (Note that this must be proven, not assumed merely because $\dfrac{d}{dx}(C) = 0$ because it is the **converse** of that fact). Since
$h(x) = f(x) - g(x) = C$, we establish the fact that two functions with equivalent derivatives differ by a constant.

Please go to GAAPMT.ORG to view all the 2016 AP Calculus AB/BC readers' report and analyses by readers at last year's exam.
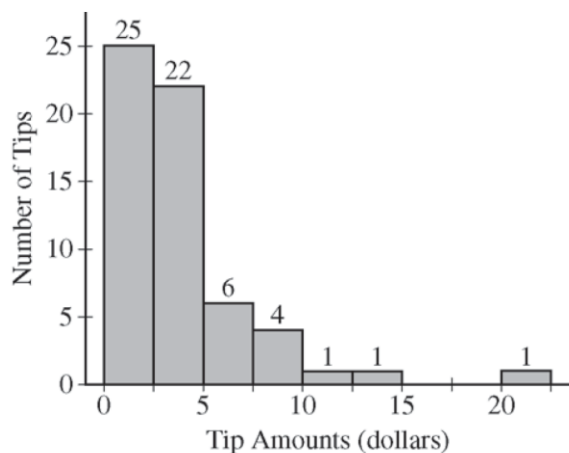

AP Statistics
2016 QUESTION 1
Please view the questions here:
http://apcentral.collegeboard.com/apc/members/exam/exam_information/8357.html#free-response

Problem Overview:

An employee at a restaurant earns tips from customers. This employee recorder the 60 tips received on one particular day and the histogram reflects those tips.

Source: collegeboard.com

Part a:

Students were asked to write a few sentences to describe the distribution.

Part b:

Students were asked to determine what effect a change of one tip from $8 to $18 would have on the mean and the median and told to justify their answers.

Intent of the Questions:

This question determined a student's ability to accurately a) describe a distribution of a quantitative variable using a histogram and b) determine the effect on the mean and median that would occur if one data value changes.

Solution:

Part a

Students were required to comment on 5 components to get an "E". These were:

1. Shape of the histogram is skewed right. (or positively skewed, or skewed to large values)
2. Identify a possible outlier OR specifically discuss the gap (one amount higher than $20, and the next highest is at most $15). (They may say a probable outlier exists without stating a number).
3. Approximate the center with the median (between $2.50 and $5.00) or gives an approximate mean (between $2.62 and $5.13). Students had to tell what measurement of center they used.
4. Discuss variability in the tip amounts. They were allowed to state the minimum and maximum to satisfy this component or to say they vary from $0 to about $20 or $22.50. They could also tell where the majority of tips occurred. If a student said the majority of tips were between $0 and $5, that was sufficient for credit. They could also be more detailed in the majority explanation. They could also provide the range with numerical values. The following were accepted: Range [17.5,22.5]; IQR (0,5] (0 could not be included as a value). SD [2.5,5]. The standard deviation values should be an interval of about 2 – 3.

5. Context --- students had to discuss the "tips" and not be generic in explanations. Clarity was necessary here. Specifically "amount of tips" could refer to the height of the histogram bars, so reference to the tip amounts had to be clear.

Reader Notes:
- Students who made two comments that contradict each other→ such as a reasonable comment followed by an unreasonable comment for any of the 5 sections lost credit for that section.
- If one or more major incorrect statistical statements were made that was not one of the 5 components, they lost credit for one component.
- All 5 parts correct earned an E.
- 3 or 4 parts correct earned a P.
- 1 or 2 parts correct earned an I.

Part b:

Students had to indicate that the median would not change because the median was between $2.50 and $5.00 and both $8 and $18 are greater than that. The mean goes up by $10/60 or about 16 or 17 cents.

There were 4 components to this answer.
a. Indicating the mean would <u>increase.</u>
b. Using correct justification for the increase in mean.
   - States the mean would increase by $10/60 or 16-17 cents
   - Indicates the balance point would increase/move to the right
   - Indicating that the mean is not resistant to extreme values or skewness and adding a value far away from the previous tip amount would add a number in the right tail, increase skewness and pull the mean in the direction of the skewness.
c. Median would stay the same.
d. Justification for the median required the student to indicate that $8 is larger than the current median, so replacing it with $18, another value greater than the median will not change the location of the median.

Reader notes:
1. If students "added a new data value--$18" instead of replacing a value, the following changes in grading occurred.
   - Correct justification for the mean increase would change to $10/61. The other possible justifications for the change in mean are the same.
   - Median might change or stay the same.
   - Justification for the median must indicate with 61 values, the 31st value would be the median. It might be the same number in an ordered list of the data values, or it might be higher.
   - All four parts correct earned an E.
   - 3 or 4 parts correct earned a P.

Observations:

1. Students comment on how the mean and median would change but they do not justify their answers.
2. Variability was difficult for many students to explain.
3. Many students did not mention the gap or the outlier.
4. Context was often not complete. Work on clarity. If it is possible that the way an answer is written can be misconstrued, then it needs to be written a different way. In this question tip amounts was important.
5. A common problem was students not completely describing variability.
6. Students used statistical vocabulary incorrectly.
7. Students contradicted themselves.
8. Did not explicitly state that increasing one value in the tip amounts would increase the sum of the tips when discussing the change in the mean.

Recommendations for Teachers:

1. Use spiral teaching and revisit topics with emphasis on connection, comparison and contrast.
2. Help students clearly distinguish the range between two numbers and the estimation of IQR.
3. Give students ample practice making statistical arguments and giving critical feedback.
4. Give students more exposure to analyzing data with a data value missing or changed.
5. Discuss context---and what that means with respect to variables and a question.
6. Distinguish the ideas of changing 1 value, adding or removing a value and transforming all values.

Please go to GAAPMT.ORG to view all the 2016 AP Statistics questions and for the readers' report and analyses by readers at last year's exam.

The John Neff Award

It's never too early to begin thinking about the John Neff Award. Dr. John Neff was a faculty member at the Georgia Institute of Technology and a former President of the Georgia Council for Teachers of Mathematics. He was a mentor who truly inspired teachers and was a delight to talk to. The Georgia Association of Advanced Placement Teachers created an award in his memory due to his legacy in mathematics, advanced placement mathematics and education. We encourage you to nominate teachers who have taught AP Calculus and/or AP Statistics for at least five years, have demonstrated excellent teaching in their classrooms, and demonstrate a willingness to mentor others. This person also contributes to the teaching of mathematics beyond their classroom. Nominations will be taken in the spring and early fall for this honor. You need to submit a brief email that states who you would like to nominate, their qualifications and why you believe they deserve this award. If you would like to send a nomination now, please send it to dkohler1@kennesaw.edu,

AP Reader – Comments from an Acorn (1st Year Reader)

Dennis Wilson

"Not in a million years!"

"The last place I will ever spend my summer."

"It would be torture."

"Never!"

These are ways I have responded in the past when someone suggested that I apply to become a Reader for the AP Calculus exam. Scoring student papers has never been task that I call enjoyable. Grading responses from my own students was taxing enough, I could not imagine grading responses from others.

For years, my resolve against becoming an AP Reader only seemed to grow until I met two people. The first was Marshall Ransom, an AP Table Leader who always gives the reader report our annual GAAPMT conference. The second was Sergio Stadler, the retired AP Calculus teacher from Marist who was my instructor at an AP summer institute. I highly respect the wisdom of both of these gentlemen. It was on their recommendation that I changed my mind and applied to become an AP Reader.

This past June, I served as an Acorn, the nickname given to first year Readers. I would like to share some of my experience from this past summer for both those considering becoming a Reader and for those who are adamantly against the thought.

In January of 2016, after two years of waiting, I received an invitation from ETS. After accepting the invitation, I soon booked my flight to Kansas City on the Reader website. I even chose a friend to be my roommate. Preparing for an eight-day trip caused some logistical struggles, but once I was in Kansas City, the accommodations were great. The AP readers stay in one of two four-star hotels which are across the street from each other. The actual reading takes place at the Kansas City convention center which is also where meals are served. The meals are well-prepared and come with a variety of options for each day.

In addition to sleeping and eating, the reading provides other opportunities. There is a dine out night, and readers eat at a restaurant and ETS reimburses the expense up to \$25. There is also a professional night at which a topic of mathematical interest is presented. On another evening, The College Board answered questions about the exam and a reception followed. There was even a morning run called the Goose Residue Classic. I participated in all these activities and more. While the after-hours activities are optional, these opportunities to unwind and connect with people were much welcomed after a day of scoring exams.

The scoring is every bit as grueling as I thought it would be. Except for the briefings from the Question Leaders, all eight hours of the work day are filled by sitting at a table scoring questions. During the week, each Reader will usually score three different problems. Before each problem, the Readers attend a briefing given by the Question Leader. The briefing moves quickly, without questions, and a great amount of information is given. The goal is that each exam question is consistently scored, regardless of the Reader or the day on which it was scored. Even after teaching calculus for 15 years, I still learned things in the briefing that improved both my understanding and my teaching of the subject.

After being briefed on a question, you sit at a table with a folder of 25 tests. When you score the test, the only thing you mark is the score on a separate bubble sheet. If you are puzzled by any responses, you have a table partner with whom you can discuss the scoring. If you cannot reach clarity with your table partner, you also have two Table Leaders who can make the call. The Table Leaders

also "back read" your folders in help ensure consistency in scoring. After you finish your folder of 25, you exchange it for yet another. The monotony of scoring is broken up by two breaks, morning and afternoon, as well as by lunch. Some days seem especially longer than others, particularly those without a briefing. The days gradually wore me down as the week went on and I thought that the reading had gotten the best of me. At my weakest, my table partner and table leaders were highly encouraging which kept me energized.

The things I liked most about the reading were the people. At each meal, I tried to eat with someone new. I encountered friends that I knew from conferences and became friends with their friends. Since no outside conversations occur at the scoring table, the breaks offered great opportunities to build friendships with those at your table. As a Calculus teacher, I can always have enriching math conversations with my fellow department members at school. It is rare, though, for these discussions to include Calculus. The AP reading afforded me many opportunities to have such discussions.

When I left Kansas City after 7 days of scoring, I was undecided if I would return the next year. While the friendships and the non-scoring activities were fulfilling, the actual scoring had drained me. Given just a few days of rest afterward though, I knew I would return. The $1600 stipend deposited in my bank account at the end of the month certainly helped as well.

While I was challenged during my week in Kansas City, it was certainly worth my time, as I was enriched by experiences that are helping me grow as a calculus teacher. I strongly encourage you to give serious consideration to become a Reader yourself.

2017 CollegeBoard Summer Institutes in Georgia
*A= all teachers **N = New teachers

DATE	AP CALCULUS AB	AP CALCULUS BC	AP STATISTICS
June 5	**Woodward Academy** College Park (A)	**Woodward Academy** College Park (A)	**Woodward Academy** College Park
June 12	**Marist School (N)** Atlanta		
June 19	**University of GA** Athens (A & N)	**University of GA** Athens (A & N)	
June 20	**Walton High School** Marietta (A)	**Walton High School** Marietta (A)	**Walton High School** Marietta
June 26	**University of TN** Chattanooga (A & N)		
July 10	**University of GA** Athens (A)		**University of GA** Athens (A)
July 10	**Kennesaw State University** Kennesaw (A)	**Kennesaw State University** Kennesaw (A)	**Kennesaw State University** Kennesaw (A)
July 17	**Woodward Academy** College Park (N)	**Woodward Academy** College Park (A)	**Kennesaw State University** Kennesaw **Woodward Academy** N,E

Link to more information about summer institutes:
http://apcentral.collegeboard.com/InstitutesAndWorkshops

Link to information about AP Calculus AB/BC 2016 curriculum changes:
https://advancesinap.collegeboard.org/stem/calculus

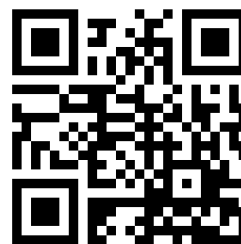**Link to receive newsletter:** **http://goo.gl/forms/MbQUWbvQ1L**

GA^{2}PMT Membership Request

The benefits of belonging to this organization can make a difference in your students' scores on the AP Exams.

Becoming a member of a professional organization is one of the indicators listed in the Georgia Teacher Keys Effectiveness System (TKES).

Performance Standard #9- Professionalism and Communication: The teacher exhibits a commitment to professional ethics and the school's mission, participates in professional growth opportunities to support student learning, and contributes to the profession.

To join GA²PMT, you may enter your information online using the following web address: **http://goo.gl/forms/MbQUWbvQ1L** or simply scan the QR code.
You may also complete the information below and mail it in with your membership dues, which are $10 per year.

GA²PMT Membership Form – next page

GA²PMT Membership Form
(Oct. 1, 2017 – Sept. 30, 2018)

	AP Certification:
Name:_____	
	☐ **AP Calculus AB**
School:_____	
Address:_____	☐ **AP Calculus BC**
City:_____**State**_____**Zip**_____	
	☐ **AP Statistics**
Email:_____	
Member Status: New_____ Renewal_____	

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