2016 AB6 Marshall Ransom, Georgia Southern University

Problem Overview:

The functions f and g have continuous second derivatives. Some values of these functions and their derivatives are given in the table below.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	–6	3	2	8
2	2	–2	–3	0
3	8	7	6	2
6	4	5	3	–1

Part a:

Students were asked to write an equation of the line tangent to the graph of k at $x = 3$ if $k(x) = f\left(g(x)\right)$.

Part b:

Students were asked to find $h'(1)$ if $h(x) = \dfrac{g(x)}{f(x)}$.

Part c:

Students were asked to evaluate $\int_1^3 f''(2x)\,dx$.

Comments on student responses and scoring guidelines:

Part a:

The first two points in this part of the question were for finding the slope, the third point for the equation. Evidence of using the chain rule in calculating the slope had to be shown. Any work without the chain rule or an unsupported $k'(3) = 10$ earned neither of the first two points. Something like $f'\left(g(3)\right)g'(3) = 10$ earned both of these points. Various ways to earn both or only one of these two points were shown to readers. For example, $f'\left(g(x)\right)g'(x) = 10$ earned only one of the two points since there was no evidence of pulling values from the table. The point for the tangent line required use of the point (3, 4) as well as a connection between the derivative of k and the slope used in the equation of this line. A somewhat minimal connection could be established by stating that $k'(3) = f'\left(g(3)\right)g'(3)$ and $y - 4 = 10(x - 3)$ which earned all three points. However, $k'(x) = f'\left(g(x)\right)g'(x)$ and $y - 4 = 10(x - 3)$ earned only one of the first two points, as well as the third point. Equations of lines not including the point (3, 4) did not earn the third point.

Part b:

Students were expected to show the equivalent of $\dfrac{f(1)g'(1)-g(1)f'(1)}{\left(f(1)\right)^2}=-\dfrac{3}{2}$ which would earn all three points in this part of the problem. The expression $\dfrac{-6(8)-3(2)}{(-6)^2}$ was also acceptable since the values pulled from the table were correct for the quotient rule and gave the (unsimplified) correct answer. In the presence of -6^2 without parentheses, student work was penalized one of the first two points. A correct quotient rule $\dfrac{f(x)g'(x)-g(x)f'(x)}{\left(f(x)\right)^2}$ earned only one of the first two points, values from the table or use of $x=1$ not being shown. A numerator sum, a reversal in the numerator, or a correct quotient rule for $\dfrac{f(x)}{g(x)}$ could earn at most one of the three points in part (b). No square in the denominator earned zero of these points.

Part c:

A correct anti-derivative earned the first two of three points in part (c). This could be shown as $\frac{1}{2}f'(2x)$ or as some students jumped right into evaluation, this could be shown as $\frac{1}{2}\left(f'(6)-f'(2)\right)$. Mishandling the $\frac{1}{2}$ as in $f'(2x)$ or $2f'(2x)$ allowed the student one of the first two points, but not the third point for the answer.

Observations and recommendations for teachers:

(1) Use of tabular data about functions and their derivatives is a staple of recent AP Calculus exams. Past tests abound with such examples. Some more practice with problems involving tabular data is available at tinyurl.com/mransom/APAB/APCalculusReview.html

(2) Calculus is used in generating an equation of a tangent line, usually because the slope is a derivative of something. In part (a) students had to calculate the derivative of a composition of two functions. There is an "inside" function and an "outside" function in part (a) which makes the derivative calculation a simple two step process. This type of derivative should be practiced both using specific functions as in $\sin\left(2x^3-5x\right)$ as well as symbolically as in $\dfrac{d}{dt}g\left(h(t)\right)$. (I do not recall seeing on an AP Calculus exam a more extended chain rule such as $\dfrac{d}{dt}\sin\left(e^{\cos\left(\tan(3t)\right)}\right)$, but of course these should be practiced in class as well).

(3) Work must be shown on the AP exam. Students who know well the derivative of a composition of functions must still show evidence of calculating the derivative AND using values in the table.

(4) As in part (b), a quotient rule demonstration leading to a numerical answer must show the correct quotient rule AND correct values leading to a numerical answer. Errors on the AP exam have been graded as mentioned above for a number of years. Reversals and a "+" sign can possibly get one point, but leaving off the square in the denominator gets zero points. Clearly, students should memorize this rule correctly.

(5) The chain rule used in calculating the derivative looks at the derivative of the "inside" function, and then multiplies. An anti-derivative "chain rule" does only one thing differently: divide instead of multiply, provided that the "inside" function is linear. For simple functions such as $\sin(5x)$, $\sec^2(\pi x)$ or $f'\left(\frac{x}{3}\right)$, it is not a bad idea to practice (after formally doing all the work as in "let $u = 5x$, etc.") in the following manner:

$$\int \text{trig}(kx)\,dx = \frac{\text{anti-trig}(kx)}{k} + C \quad \text{or} \quad \int f'(kx)\,dx = \frac{f(kx)}{k} + C$$

The quick division by k saves work that does not need to be seen when these problems as in part (c) are graded. In the classroom also, this work does not need to be seen when this simple division is all that is necessary, provided that students are given sufficient practice with u-substitution to realize this.

For additional AP examples, see 2014 AB5, 2011 AB/BC2 and 2009 AB/BC5.