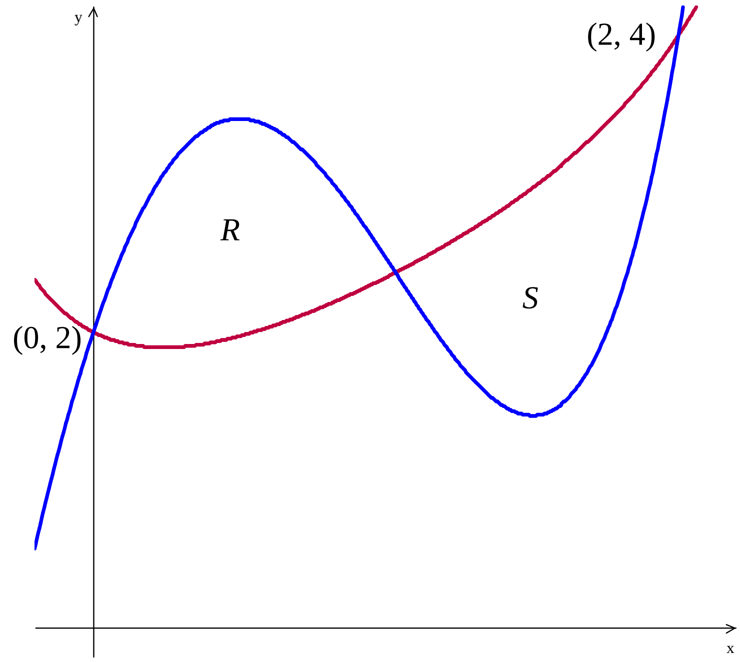
Problem Overview:



Students were given the graph at right of $g(x) = x^4 - 6.5x^2 + 6x + 2$ and $f(x) = 1 + x + e^{x^2 - 2x}$.

In part (a) students were asked to calculate the sum of the areas of regions R and S.

Part (b) announced S as the base of a solid with cross sections being both perpendicular to the x-axis and squares. Students were asked to find the volume of this solid.

Part (c) introduced h as the vertical distance between the graphs of f and g in region S. Students were asked to find the rate at which h changes with respect to x when $x = 1.8$.

Part a:

Students had to determine which of the functions graphed was f and which was g as well as find the point of intersection to completely solve this part of the problem…… if they used the point of intersection in their work. The intersection value of $x = 1.032$ or 1.033 could also be expressed as 1.03 in order to earn the limits point. But the lower limit had to be 0 and the upper limit had to be 2. A few students avoided the need for the intersection point and earned the limits point by using a correct absolute value expression for $|f - g|$ or $|g - f|$ as long as they used a lower limit of 0 and an upper limit of 2.

Part b:

Students using $(f - g)^2$ or $(g - f)^2$ earned both integrand points even if they were also calculating in region R. Students who left off the square or added the functions, but had our answer, lost both integrand points but did get the answer point. Students who had used expressions, rather than f and g, but had an incorrect expression, were eligible for points provided that the incorrect expression had been imported from part (a) and a point had already been deducted for that error. The use of a constant such as π was ignored in awarding the integrand points. Our answer with a lower limit (an intersection point) of 1.032, 1.033 or 1.03 was awarded the answer point. Students with an incorrect lower limit, but a correct 2 as upper limit, were awarded an answer point for an answer consistent with their work, provided that the error had been imported from part (a).

Part c:

The first point is a “considers” point which means that the student is beginning to correctly set up h and at least trying to calculate a derivative. Any form of $h = f - g$ or $h = g - f$ and any attempt at a derivative earned the first point. An answer such as $h(1.8) = -3.812$ in the absence of any supporting work did earn the answer point. An answer with no supporting work, no $h(1.8)$, or work including an integral earned neither of the two points. The answer point could be earned by showing intermediate values such as $g'(1.8) = 5.9200$ and $f'(1.8) = 2.11628$ along with a final answer such as 2.11628 – 5.9200.

Observations and recommendations for teachers:

(1) For all parts in this problem, students could have used f and g rather than writing out the entire expressions, which some students did to their detriment by making copy errors or doing incorrect simplifications. If a function is named in the stem of the question, that name may be used rather than the entire expression. Although students could check the graphs on their calculators, it would be nice if students were very familiar with key differences between the graphs of polynomials and exponentials. In other words, know the (possible) properties of those functions and how they affect a graph.

(2) In the past several years, a volume created by cross sections has appeared on the AP test. This sometimes involves cross section areas of familiar figures such as semi-circles and types of triangles. The cross section area constitutes the integrand. Limits clearly in the region need to be a part of this work for a limits point; or in some cases that involve volume, a “limits and constant” point, where the constant is either of π or 2π.

(3) Please do NOT have students do analytic work calculating antiderivatives or derivatives in a calculator problem. The setup is the important skill being tested. In the case of finding a portion of an area (or perhaps a volume), a presentation of a numerical value must be justified somewhere in work on the exam.