Problem Overview:

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	–120	–150

Train A is moving back and forth on an East/West section of track. Its velocity is given by a differentiable function $v_A(t)$ measured in meters per minute. Some values for $v_A(t)$ are given in the table above.

In part (a), students were asked to find the average acceleration of train A over the interval $2 \le t \le 8$.
Part (b) asked if the data in the table supported the conclusion that train A's velocity is –100 at some time t such that $5 \le t \le 8$ and for students to provide a reason for the answer given.
In part (c) students were given the information that at $t = 2$ train A is 300 meters east of Origin Station, and the train is moving to the east. Students were asked to write an expression involving an integral that would give the train's position in meters from Origin Station at time $t = 12$. To approximate this position, students were asked to use a trapezoidal sum, involving three subintervals indicated by the table.
Part (c) made students aware of a second train, train B, traveling north from Origin Station. The velocity of train B at time t is given by $v_B(t) = -5t^2 + 60t + 25$. At time $t = 2$, train B is 400 meters north of the station. Students were asked to calculate the rate at which the distance between the two trains was changing at time $t = 2$.

Part a:

The solution involves the slope of a secant line between the two points $(2, v_A(2))$ and $(8, v_A(8))$. This slope $\dfrac{v_A(8) - v_A(2)}{8-2}$ required students to use values found in the table. The value $\dfrac{-120-100}{8-2}$ would earn this point, presenting evidence of a difference quotient and the correct value all in one expression……. remember: Do Not Bother to Simplify !! A few students correctly set up the average acceleration using $\dfrac{1}{8-2}\int_2^8 v_A{}'(t)\,dt$.

Part b:

For the first point the values $v_A(8) = -120$ and $v_A(5) = 40$ had to be presented or at least considered, possibly as follows: $v_A(8) < -100 < v_A(5)$ or $40 > -100 > -120$. The second point in part (b) was for the reason, which needed to cite the IVT or the fact that the function is continuous. It was unacceptable to cite the MVT or EVT or that v is differentiable or decreasing. A student could have a copy error or mistaken value from the table and still earn this second point. A student who said "No" would receive neither of these two points.

Part c:

Students needed to present, somewhere in their work, an expression equivalent to $300+\int_2^{12} v_A(t)\,dt$. This earned the first point. The second point was for showing a trapezoidal sum involving 3 heights and 3 partial sums. If 5 out of 6 of these were correct, students received this point but were not eligible for the answer point. An acceptable sum could be $\frac{3}{2}(v_A(2)+v_A(5))+\frac{3}{2}(v_A(5)+v_A(8))+\frac{4}{2}(v_A(8)+v_A(12))$. A misplaced parenthesis would make the student ineligible for the sum point, but eligible for the answer point, if our correct answer. A sum such as $3\frac{100+40}{2}+3\frac{40-120}{2}+4\frac{-120-150}{2}$ earned both the sum and the answer points…… remember: Do Not Bother to Simplify !! Readers were presented a variety of student solutions that may have left out the 300 or used the integral $\int_0^{12} v_A(t)\,dt$, and were given specific scoring guidelines.

Part d:

The implicit differentiation earned two points. To get one of these points, readers needed to see that at least two of three variables were squared such as $\frac{dH}{dt}=2A\frac{dA}{dt}+2B\frac{dB}{dt}$. However, this was not eligible for the answer point. Some students entered the problem with an implicit differentiation rather than explicitly writing $z^2=x^2+y^2$, and this was read for the possibility of both points and the answer point.

Observations and recommendations for teachers:

(1) An average rate of change of a function *f* on an interval [a, b] is the slope of the secant line $\frac{f(b)-f(a)}{b-a}$. Slopes are rates of change, especially applicable in a real-world context. It is helpful to know that a rate of change in velocity is acceleration = $\frac{v_A(8)-v_A(2)}{8-2}=velocity\ \div\ time$.

(2) The IVT guarantees that a function continuous on an interval assumes all values between those at the endpoints of the interval. The MVT makes a statement about a derivative of a function, not applicable in the case of this part (b). Students need to be clear on this distinction.

(3) A trapezoidal sum involves adding the area of trapezoids, which may be found using various triangles and rectangles. A formula, the trapezoidal rule, does not help much if the intervals are of unequal width.

(4) A related rates problem is not always on the written response section of the AP exam. But this is a classic, using the Pythagorean Theorem. Calculate derivatives implicitly with respect to time and then substitute known values. What has worked for me is to have students practice a number of these simple setups, and save the more involved problems for group work or take home work. In more involved problems, it is usually the setup of the equation that is the difficulty rather than the calculus.