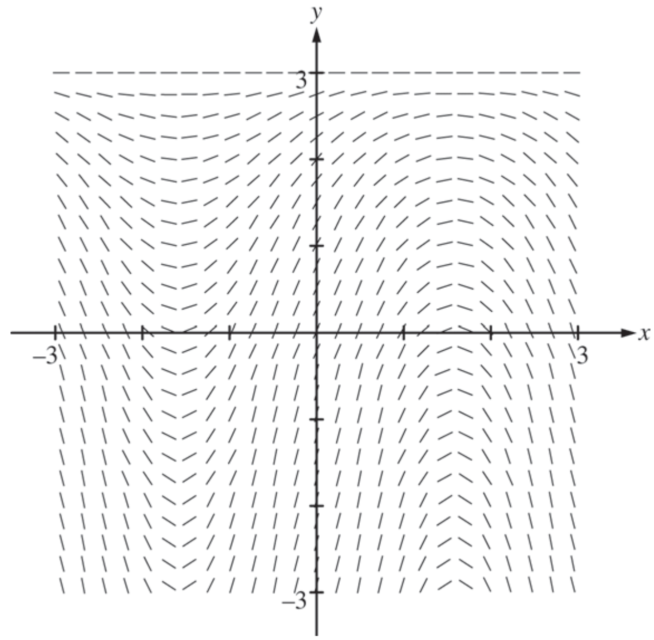

Problem Overview:

The differential equation $\frac{dy}{dx}=(3-y)\cos(x)$ is given. $y=f(x)$ is the particular solution with the initial condition $f(0)=1$. The function f is defined for all real numbers.

In part (a) students are given a portion of the slope field as shown at right and asked to sketch the solution curve through the point (0, 1).

In part (b) students are asked to write an equation of the line tangent to the solution curve in part (a) at point (0, 1).

Part (c) asked students to find $y=f(x)$, the particular solution to the differential equation with the initial condition $f(0)=1$.

Part a:

A reasonable, continuous, curve that passed through (0, 1) was acceptable. "Reasonable" should mean the general shape of the curve, and this did not have to extend completely to the outer limits of the slope field. Presentation of $f(0.2)\approx 1.4$ or $f(0.2)=1.4$ earned the second point.

Part b:

To get the first point, an equation of a tangent line had to be presented, along with an algebraic connection (show work) to our differential equation in order to earn the 1st point. This connection shows how the slope of 2 is calculated. This equation had to be present (or obviously used) in order to earn the 2nd point.

Part c:

For years on the AP exam, the separable differential equation solution work has been graded on a 5 or 6 point scale, depending upon whether there are 1 or 2 points for the antiderivatives. In this problem, there were 6 points. The first is for separating the variables. If students failed to do this, they received 0 of the 6 points. An example of such failure is $\int \frac{1}{3-y}dy=\int 1dx$. Because of the initial condition, absolute value was not necessary in the antiderivative $-\ln(3-y)$, but missing parentheses would not get this point. To be eligible for the fourth, "+C," point, students had to have earned at least one of the antiderivative points. Using the initial condition in the presence of a correct "+C" earns the fifth point, but actually solving for C is part of the answer point. Students could not earn the answer point unless they had earned all of the first 5 points or only missed the one because of a missing parenthesis **and** had our correct answer.

Observations and recommendations for teachers:

(1) Sketching a slope field with several mini-segments is something that should be practiced as well as sketching an entire curve containing an initial condition. Such curve sketching shows the changing position of the curve as the initial condition changes.

(2) Showing work (this is a calculus test, so in a tangent line question, that usually involves showing how the slope is obtained) is important on the AP exam. The mere presentation of the correct tangent line will not earn a point. The tangent line equation can be used, as in problem #1 part (d) on this test, to approximate values of a function near the point of tangency. This involves a simple substitution, into the equation of the tangent line, of the value of x near the point of tangency or the substitution of the directed distance between the value of x at the point of tangency and the value of x near that point.

(3) Since the scoring process hasn't changed in quite some time, I advise that students should KNOW how the solving of a separable differentiable equation is scored. It should be obvious that the final solving for the explicit function $y = f(x)$ is almost as much work as preparing for this solution. In the interests of time, I would advise students taking the test to postpone this work for the answer point until they have tried all other problem parts on the test…… then come back to this.