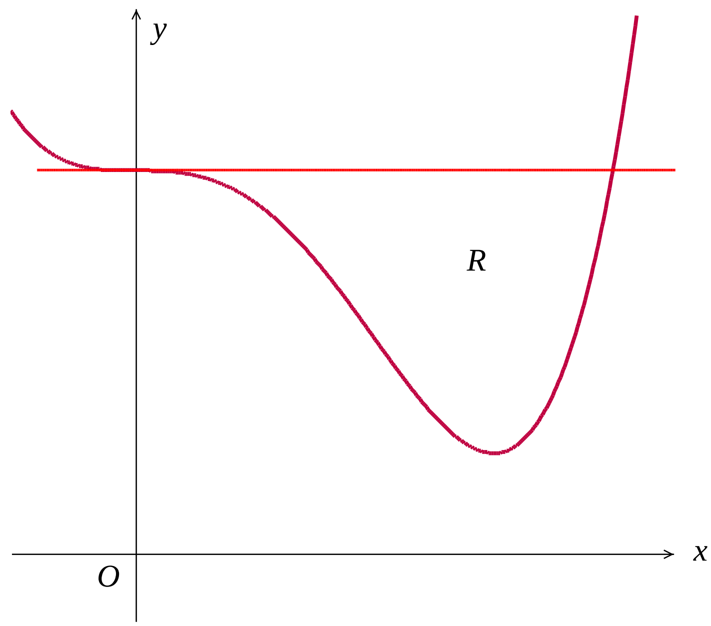
Problem Overview:



Students were given the graph at right of $y=4$ and $f(x)=x^4-2.3x^3+4$.

In part (a) students were asked to calculate the volume of the solid generated when R is rotated about the line $y=-2$.

Part (b) announced R as the base of a solid with cross sections being both perpendicular to the x-axis and isosceles right triangles with one leg in R. Find the volume of this solid.

Part (c) introduced a vertical line $x = k$ into the discussion. Students were requested to write, but not solve, an equation involving integrals whose solution gives the value of k if the line $x = k$ divides R into two regions with equal areas.

Part a:

Finding this volume involved using washers, and the classic "π times (Big R squared – little r squared)." This definite integral required limits, which could be found without a calculator as $x = 0$ and $x = 2.3$. The final answer did not actually require the use of a calculator, but could be found that way much more easily as either 98.868 or 98.867 or 31.471π or 31.470π. The work-by-hand, analytic, method resulted in $\pi\left(-\frac{1}{9}(2.3)^9+\frac{4.6}{8}(2.3)^8-\frac{5.29}{7}(2.3)^7-\frac{12}{5}(2.3)^5+\frac{27.6}{4}(2.3)^4\right)$ which is, of course, correct without simplification. Various methods by students involving incorrect uses of parentheses were shown in detail to readers so that they would know whether to award 1, 2 or zero points for the integrand setup. For example, a "presentation" error might involve a missing parenthesis: $(-2-\left(x^4-2.3x^3+4\right)^2$ rather than $\left(f(x)-(-2)\right)^2$ or $\left(x^4-2.3x^3+6\right)^2$. This error was eligible for a score of $1-1-1$, losing only one of the integrand points. **<u>If</u>** the limits and the answer were correct, this student received all three of those points for a score of 3 out of 4 in part (a). Students leaving off the π were not penalized for any integrand points. This merely rendered the answer incorrect. An indefinite integral, with a correct integrand, earned the first two points but no other points.

Part b:

The length of a leg of this triangle had to appear in an integral in some form such as $4-f(x)$ in the presence of multiplying with another non-constant factor in order to earn one of the two integrand points. For the second point, everything else including the ½ had to be present in the students' answers. Consideration for the answer point required that the lower limit be 0 and that the upper limit be a positive number in [0, 4]. If incorrect limits were imported from part (a), readers looked for the answer calculation for one more point. A "presentation error" such as the missing parenthesis discussed in part (a) above, could earn a student one of the integrand points and the answer point.

Part c:

In order to be eligible for points, students had to use k in either the lower or upper limit of at least one integral. Any work students did to try to actually solve for k was ignored. To earn the first point, at least one correct expression $\int_0^k (4-f(x))\,dx$ or $\int_k^{2.3} (4-f(x))\,dx$ had to be presented in student work. Some students used the area values $A=3.21817$ or $\frac{A}{2}=1.60909$, which values could have been imported from earlier work in parts (a) or (b) as in $\int_0^k 4-f(x)\,dx=1.60909$. But students did need to show where that numerical value came from if not imported from part (a) or (b).

Observations and recommendations for teachers:

(1) For part (a), students need to know the basic forms for calculating volumes of revolution. This should also be practiced when the line about which the revolution is done is in another quadrant. Practice and emphasis on use of all required parentheses is important in communicating correct mathematics.

(2) In the past several years, a volume created by cross sections has appeared on the AP test. This sometimes involves cross section areas of familiar figures such as semi-circles and types of triangles. The cross section area constitutes the integrand. Limits clearly in the region need to be a part of this work for typically a limits point or in some cases involving volume, a "limits and constant" point where the constant is either of π or 2π.

(3) Please do NOT have students do analytic work calculating antiderivatives in a calculator problem. The setup is the important skill being tested. In the case of finding a portion of an area (or perhaps a volume), a presentation of a numerical value must be justified somewhere in work on the exam.